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Experimental Application of Transmissibility Operators to Fault Detection

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Abstract— **This paper considers a technique for fault detection called sensor-to-sensor identification. Sensor-to-sensor identification takes advantage of freely available and unknown external (ambient) excitation to identify a sensor-to-sensor model (i.e., a transmissibility operator), which is independent of the excitation signal and the initial conditions of the underlying system. In the presence of unknown external excitation, the identified transmissibility operator is used to compute the sensor-to-sensor residual, which is the discrepancy between the predicted sensor output (based on the transmissibility operator) and the actual measurements. The sensor-to-sensor residuals are used to detect and diagnose faults in sensors and system dynamics. We consider an experimental setup consisting of a drum with two speakers and four microphones. Each speaker is an actuator, and each microphone is a sensor that measures the acoustic response at its location. Measurements from the four microphones are used to construct transmissibility operators, which in turn are used to detect changes in the dynamics of the drum by computing the resulting one-step residual.**

I. INTRODUCTION

The study of structural vibration focuses on the displacement, velocity, and acceleration response of a structure to force and torque inputs. This response can be studied in terms of either transfer functions or differential equation models. In the former case, the frequency response of the structure can be estimated by computing the ratio of the transforms of the forcing and response signals. These procedures are based on the assumption that the input and output signals are stationary, and thus initial conditions and transient effects are either assumed to be absent or are ignored. In the case of differential equation models, time-domain methods can be used to estimate the model parameters assuming that the input is sufficiently persistent. In this case, initial conditions and transient effects can enhance rather than degrade the accuracy of the estimated model.

An alternative approach to force-to-motion models is the class of models known as *transmissibilities*. A transmissibility is a relationship between identical variables, such as force-to-force, position-to-position, velocity-to-velocity, and acceleration-to-acceleration [1, 2]. The "input" and "output" of the transmissibility, referred to as the *pseudo input* and *pseudo output*, respectively, are thus outputs of the underlying system. Although force-to-motion transfer functions can be identified using either time-domain or frequency-domain methods, transmissibility estimates are traditionally obtained using only frequency-domain methods, and this remains an active area of research [3–8].

Interestingly, time-domain methods have not been traditionally used to identify transmissibilities. The reason for this is partly due to the fact that the meaning of a transmissibility in the time domain is suspect. In particular, the input (forcing) to the underlying system plays no visible role in the transmissibility, and the states of the transmissibility have no physical interpretation. Nevertheless, for applications in which response data have a significant transient, nonstationary component, it is of interest to use time-domain methods to estimate transmissibility functions. A first step in this direction was taken in [9–11], where time-domain transmissibility models were obtained. These models, called *pseudo transfer functions* (PTF), are formulated in terms of the differential operator $\mathbf{p} = \mathrm{d}/\mathrm{d}t$ rather than the Laplace "s". This framework, which requires some care due to the fact that $\mathbf{p}$ is not a complex number, accounts for the free response in the time domain, but requires special attention due to issues of causality, stability, and order. In particular, a time-domain transmissibility model may be noncausal (in either continuous time or discrete time), where its "numerator" and "denominator" dynamics are the zero dynamics of the underlying transfer functions from the forcing to the respective motion sensors.

Various fault-detection techniques have been introduced in the literature [12–15]. In some cases, health monitoring can be performed by exciting the system in a controlled manner, using a plant model and observer to predict the response, and comparing the measured response to the prediction [16–18]. This approach, known as active fault detection, is based on residual generation. In contrast, passive fault detection detects faults by analyzing the sensor signals alone and searching for anomalies [19–21].

In the present paper we focus on a technique for fault detection called *sensor-to-sensor identification* (S2SID). S2SID is neither active nor passive as defined above. Instead, S2SID takes advantage of freely available and unknown external (ambient) excitation to identify a sensor-to-sensor model (i.e., a transmissibility operator), which is independent of the excitation signal. In the presence of subsequent unknown external excitation, the identified PTF is used to compute sensor-to-sensor residuals, which are used to detect and diagnose faults in sensors and system dynamics. The sensor-to-sensor residual is the discrepancy between the predicted sensor output (based on the PTF) and the actual measurements.

The ability to take advantage of unknown external excitation along with the fact that the PTF is independent of the excitation gives the method flexibility in practice by alleviating the need for a known or controlled excitation. This feature is the key benefit of the proposed approach relative to fault-detection methods that require known external excitation.

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A useful feature of this approach is the ability to exploit external excitation in order to identify a PTF between sensor signals. In particular, the external excitation, whether it is provided by the environment or by actuators, need not be measured or precisely controlled. Consequently, freely available ambient noise (such as flow around an aircraft wing) can play a useful role in PTF identification. Most importantly, the identified PTF is independent of the excitation; this means that the PTF identified using one data set can be used for fault detection with a different data set; for both data sets, the external excitation can be completely unknown.

Since transmissibility operators may be noncausal, unstable, and of unknown order, we consider a class of models that can approximate transmissibility operators with these properties. This class of models consists of noncausal finite impulse response (FIR) models based on a truncated Laurent expansion [22]. These models are shown to approximate the Laurent expansion inside the annulus between the asymptotically stable pole of largest modulus and the unstable pole of smallest modulus. By delaying the measured pseudo output relative to the measured pseudo input, the identified finite impulse response model is a noncausal approximation of the transmissibility operator. The causal (backward-shift) part of the Laurent expansion is asymptotically stable since all of its poles are zero, while the noncausal (forward-shift) part of the Laurent expansion captures the unstable and noncausal components of the transmissibility operator. Subspace identification is also used with the measured pseudo input and the delayed measured pseudo output to identify a noncausal state space model.

Since the transmissibility operator has the form of an input-output time-series model, methods developed for ARMAX models are candidates for this application. Note that some caution is needed since the pseudo input and pseudo output of the pseudo transfer function are both noisy, leading to an errors-in-variables problem with noise on both the input and output signals [23, 24]. In addition, neither the pseudo input nor the pseudo output signals (irrespective of the noise) can be expected to be white, thus complicating the consistency analysis. Due to the difficulty of the EIV problem, we compare the one-step prediction error for the estimates of the transmissibility operator obtained using least squares (LS), prediction error methods (PEM), instrumental variables (IV), and subspace identification, and we compare the estimates based on the one-step prediction error.

Transmissibility operators were used in [25] for rate-gyro health monitoring in aircraft. In the present paper, we consider an experimental setup consisting of a drum with two speakers and four microphones. Each speaker is an actuator, and each microphone is a sensor that measures the acoustic response at its location. Measurements from the four microphones are used to construct transmissibility operators, which in turn are used to detect changes in the dynamics of the drum by computing the resulting one-step residual.

The contents of the paper are as follows. In Section II, we derive MIMO transmissibility operators. In Section III, we show that transmissibility operators can be noncausal, unstable, and of unknown order and thus noncausal FIR models can be used to approximate transmissibility operators with these properties. In Section IV, we use noncausal FIR models with least squares, prediction error methods, and instrumental variables to identify transmissibility operators. We also use subspace identification to estimate noncausal state space models of transmissibility operators. In Section V, we use the identified transmissibility operators for fault detection of an acoustic system. Finally, we give conclusions in Section VI.

II. MIMO Transmissibility Operators

We consider the linear system

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

$$x(0) = x_0, \tag{2}$$

$$y_{\rm i}(t) = C_{\rm i}x(t) + D_{\rm i}u(t) \in \mathbb{R}^m, \tag{3}$$

$$y_{\rm o}(t) = C_{\rm o}x(t) + D_{\rm o}u(t) \in \mathbb{R}^{p-m}, \tag{4}$$

where $u \in \mathbb{R}^m$, p is the number of measurements, m is the number of pseudo inputs, and $p-m$ is the number of pseudo outputs. The coefficient matrices have dimensions $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C_{\rm i} \in \mathbb{R}^{m\times n}, C_{\rm o} \in \mathbb{R}^{(p-m)\times n}, D_{\rm i} \in \mathbb{R}^{m\times m}$, and $D_{\rm o} \in \mathbb{R}^{(p-m)\times m}$.

The goal is to obtain a relation between $y_{\rm i}$ and $y_{\rm o}$ that is independent of both the initial condition x_0 and the input u. Suppose that $m=1$ and $p=2$. Transforming (3) and (4) to the Laplace domain yields

$$\hat{y}_{\rm i}(s) = C_{\rm i}(sI-A)^{-1}x_0 + [C_{\rm i}(sI-A)^{-1}B + D_{\rm i}]\hat{u}(s), \tag{5}$$

$$\hat{y}_{\rm o}(s) = C_{\rm o}(sI-A)^{-1}x_0 + [C_{\rm o}(sI-A)^{-1}B + D_{\rm o}]\hat{u}(s), \tag{6}$$

respectively, and thus

$$\frac{\hat{y}_{\rm o}(s)}{\hat{y}_{\rm i}(s)} = \frac{C_{\rm o}(sI-A)^{-1}x_0 + [C_{\rm o}(sI-A)^{-1}B + D_{\rm o}]\hat{u}(s)}{C_{\rm i}(sI-A)^{-1}x_0 + [C_{\rm i}(sI-A)^{-1}B + D_{\rm i}]\hat{u}(s)}. \tag{7}$$

Note that, if x_0 is zero, then $\hat{u}(s)$ can be cancelled in (7), and $\hat{y}_{\rm o}(s)$ and $\hat{y}_{\rm i}(s)$ are related by a transmissibility that is independent of the input. However, if x_0 is not zero, then $\hat{u}(s)$ cannot be canceled in (7).

Alternatively, we consider a time-domain approach using the differentiation operator $\mathbf{p}$. Define

$$\Gamma_{\rm i}(\mathbf{p}) \stackrel{\triangle}{=} C_{\rm i}\mathrm{adj}(\mathbf{p}I-A)B + D_{\rm i}\delta(\mathbf{p}) \in \mathbb{R}^{m\times m}[\mathbf{p}], \tag{8}$$

$$\Gamma_{\rm o}(\mathbf{p}) \stackrel{\triangle}{=} C_{\rm o}\mathrm{adj}(\mathbf{p}I-A)B + D_{\rm o}\delta(\mathbf{p}) \in \mathbb{R}^{(p-m)\times m}[\mathbf{p}], \tag{9}$$

$$\delta(\mathbf{p}) \stackrel{\triangle}{=} \det(\mathbf{p}I-A) \in \mathbb{R}[\mathbf{p}]. \tag{10}$$

Then, the *transmissibility operator* from $y_{\rm i}$ to $y_{\rm o}$ is the operator [26]

$$\mathcal{T}(\mathbf{p}) = \Gamma_{\rm o}(\mathbf{p})\Gamma_{\rm i}{}^{-1}(\mathbf{p}). \tag{11}$$

Note that (11) is independent of both the initial condition x_0 and the input u. As in [26],

$$y_{\rm o}(t) = \mathcal{T}(\mathbf{p})y_{\rm i}(t) \tag{12}$$

represents the differential equation

$$\det \Gamma_{\rm i}(\mathbf{p}) y_{\rm o}(t) = \Gamma_{\rm o}(\mathbf{p}) \left[\operatorname{adj} \Gamma_{\rm i}(\mathbf{p})\right] y_{\rm i}(t). \tag{13}$$

The transmissibility operator (11) is in continuous time. Henceforth, we assume that measurements of the output signals are obtained in discrete time, and we consider discrete-time transmissibility operators in the forward-shift operator $\mathbf{q}$ [27].

III. Noncausal FIR Approximation of Transmissibility Operators

Expression (11) show that $\mathcal{T}(\mathbf{p})$ contains information about the zeros of the system and not the poles. Therefore, a nonminimum-phase zero in the input channel of a transmissibility operator results in an unstable transmissibility operator. Moreover, if the output channel of a transmissibility operator has more zeros than the input channel, then the transmissibility operator is improper, that is, noncausal. Since this information may not be known in advance, we consider a class of models that can be used to approximate transmissibility operators that may be both unstable and improper.

FIR models can be used to describe linear asymptotically stable systems [28]. In addition, noncausal FIR models can be used to describe systems both noncausal and unstable. A noncausal FIR model, which approximates the Laurent series of an unstable system, involves both positive and negative powers of the Z-transform variable z. The negative powers approximate the stable part of the plant outside of a disk (that is, inside a punctured plane), whereas the positive powers approximate the unstable part of the plant inside a disk. Inside the common region, which is an annulus, the Laurent series represents a noncausal model, as evidenced by the positive powers of z [22].

Consider the transmissibility operator whose input is $y_{\rm i}$ and whose output is $y_{\rm o}$. We can write

$$y_{\rm o}(k) = \sum_{i=-\infty}^{\infty} H_i y_{\rm i}(k-i), \tag{14}$$

where $H_i \in \mathbb{R}^{(p-m)\times m}$ for all $i \in (-\infty, \infty)$ are the coefficients of the Laurent series of $\mathcal{T}(z)$ in the annulus of analyticity that contains the unit circle. Let r, d be positive integers and define

$$\theta_{r,d} \stackrel{\triangle}{=} \begin{bmatrix} H_{-d} & \ldots & H_r \end{bmatrix} \in \mathbb{R}^{(p-m)\times \mu m}, \tag{15}$$

where $\mu \stackrel{\triangle}{=} r+d+1$. Let $\mathbf{q}^{-1}$ be the backward shift operator and define the noncausal FIR model for the transmissibility (11) as [22]

$$\mathcal{T}(\mathbf{q}^{-1}, \theta_{r,d}) \stackrel{\triangle}{=} \sum_{i=-d}^{r} H_i \mathbf{q}^{-i}. \tag{16}$$

Then, the noncausal FIR model output can be defined as [22]

$$y_{\rm o}(k|\theta_{r,d}) \stackrel{\triangle}{=} \mathcal{T}(\mathbf{q}^{-1}, \theta_{r,d}) y_{\rm i}(k). \tag{17}$$

IV. Identification of Transmissibility Operators Using Noncausal FIR and State Space Models

To identify transmissibility operators that are possibly improper, unstable, and of unknown order, we first use noncausal FIR models with least square (LS), prediction error methods (PEM) [29], and instrumental variables (IV) [30].

For each choice of transmissibility coefficients

$$\bar{\theta}_{r,d} \stackrel{\triangle}{=} \begin{bmatrix} \bar{H}_{-d} & \cdots & \bar{H}_r \end{bmatrix} \in \mathbb{R}^{(p-m)\times (r+d+1)m}, \tag{18}$$

it follows that

$$\mathcal{T}(\mathbf{q}^{-1}, \bar{\theta}_{r,d}) = \sum_{i=-d}^{r} \bar{H}_i \mathbf{q}^{-i}. \tag{19}$$

The residual of the transmissibility $\mathcal{T}(\mathbf{q}^{-1}, \bar{\theta}_{r,d})$ at time k is defined to be the one-step prediction error

$$\begin{aligned} e(k|\bar{\theta}_{r,d}) &\stackrel{\triangle}{=} y_{\rm o}(k) - y_{\rm o}(k|\bar{\theta}_{r,d}) \\ &= y_{\rm o}(k) - \mathcal{T}(\mathbf{q}^{-1}, \bar{\theta}_{r,d}) y_{\rm i}(k) \\ &= y_{\rm o}(k) - \sum_{i=-d}^{r} \bar{H}_i y_{\rm i}(k-i). \end{aligned} \tag{20}$$

The accuracy of $\bar{\theta}_{r,d}$ is measured by the performance metric

$$V(\bar{\theta}_{r,d}, \ell) \stackrel{\triangle}{=} \frac{1}{\ell - d - r + 1} \sum_{k=r}^{\ell-d} \|e(k|\bar{\theta}_{r,d})\|_2, \tag{21}$$

where$\|\cdot\|_2$ is the Euclidean norm and $\ell+1$ is the number of data samples. Then, the least squares estimate $\hat{\theta}_{r,d,\ell}$ of $\theta_{r,d}$ is given by

$$\hat{\theta}_{r,d,\ell} \stackrel{\triangle}{=} \underset{\bar{\theta}_{r,d}}{\arg\min}\, V(\bar{\theta}_{r,d}, \ell), \tag{22}$$

where

$$\hat{\theta}_{r,d,\ell} \stackrel{\triangle}{=} \begin{bmatrix} \hat{H}_{-d,\ell} & \cdots & \hat{H}_{r,\ell} \end{bmatrix} \in \mathbb{R}^{(p-m)\times (r+d+1)m}. \tag{23}$$

We use the Matlab functions `pem` and `iv4` to obtain PEM and IV estimates $\hat{\theta}_{r,d,\ell}$ of $\theta_{r,d}$, respectively.

It follows from (20) that the residual of the identified transmissibility $\mathcal{T}(\mathbf{q}^{-1}, \hat{\theta}_{r,d,\ell})$ at time k is given by

$$\begin{aligned} e(k|\hat{\theta}_{r,d,\ell}) &= y_{\rm o}(k) - y_{\rm o}(k|\hat{\theta}_{r,d,\ell}) \\ &= y_{\rm o}(k) - \mathcal{T}(\mathbf{q}^{-1}, \hat{\theta}_{r,d,\ell}) y_{\rm i}(k) \\ &= y_{\rm o}(k) - \sum_{i=-d}^{r} \hat{H}_{i,\ell} y_{\rm i}(k-i), \end{aligned} \tag{24}$$

and thus,

$$V(\hat{\theta}_{r,d,\ell}, \ell) = \frac{1}{\ell - d - r + 1} \sum_{k=r}^{\ell-d} \|e(k|\hat{\theta}_{r,d,\ell}, \ell)\|_2. \tag{25}$$

For all $r \leq k \leq \ell - w - d$, define

$$E(k|\hat{\theta}_{r,d,\ell}, w) \stackrel{\triangle}{=} \sqrt{\sum_{i=k}^{w+k} \|e(i|\hat{\theta}_{r,d,\ell})\|_2^2} \tag{26}$$

to be the norm of the residual of the data window of size $w+1$ starting at time step k. Expressions (24), (25), and (26) measure the accuracy of the transmissibility from $y_{\rm i}$ to $y_{\rm o}$ for the estimate $\hat{\theta}_{r,d,\ell}$ of $\theta_{r,d}$. The identification data set used to obtain $\hat{\theta}_{r,d,\ell}$ is different from the validation data set used to compute (24), (25), and (26).

For subspace identification, we delay the pseudo output d steps with respect to the pseudo input and we use the Matlab command `n4sid(data,n)` to obtain a noncausal state space model of order n. Let $\hat{\theta}_{{\rm ss},n,d,\ell} \triangleq (\hat{A}_{n,d,\ell}, \hat{B}_{n,d,\ell}, \hat{C}_{n,d,\ell}, \hat{D}_{n,d,\ell})$ be the identified state space model of order n using ℓ samples and d steps of delay applied for the pseudo output with respect to the pseudo input. Then, the identified transmissibility operator $\mathcal{T}(\mathbf{q}^{-1}, \hat{\theta}_{{\rm ss},n,d,\ell})$ can be represented by the noncausal state space model

$$x(k+1) = \hat{A}_{n,d,\ell}x(k) + \hat{B}_{n,d,\ell}y_{\rm i}(k+d), \quad (27)$$

$$y_{\rm o}(k|\hat{\theta}_{{\rm ss},n,d,\ell}) = \hat{C}_{n,d,\ell}x(k) + \hat{D}_{n,d,\ell}y_{\rm i}(k+d). \quad (28)$$

Therefore, we can write

$$\mathcal{T}(\mathbf{q}^{-1}, \hat{\theta}_{{\rm ss},n,d,\ell}) = \mathbf{q}^{d}(\hat{C}_{n,d,\ell}(\mathbf{q}I - \hat{A}_{n,d,\ell})^{-1}\hat{B}_{n,d,\ell} + \hat{D}_{n,d,\ell}), \quad (29)$$

and the state space model output can be written as

$$y_{\rm o}(k|\hat{\theta}_{{\rm ss},n,d,\ell}) = \mathcal{T}(\mathbf{q}^{-1}, \hat{\theta}_{{\rm ss},n,d,\ell})y_{\rm i}(k). \quad (30)$$

The residual of the identified transmissibility $\mathcal{T}(\mathbf{q}^{-1}, \hat{\theta}_{{\rm ss},n,d,\ell})$ at time k is defined to be the one-step prediction error

$$\begin{aligned} e(k|\hat{\theta}_{{\rm ss},n,d,\ell}) &\triangleq y_{\rm o}(k) - y_{\rm o}(k|\hat{\theta}_{{\rm ss},n,d,\ell}) \\ &= y_{\rm o}(k) - \mathcal{T}(\mathbf{q}^{-1}, \hat{\theta}_{{\rm ss},n,d,\ell})y_{\rm i}(k). \end{aligned} \quad (31)$$

The accuracy of $\hat{\theta}_{{\rm ss},n,d,\ell}$ is measured by the performance metric

$$V(\hat{\theta}_{{\rm ss},n,d,\ell}, r, \ell) \triangleq \frac{1}{\ell - d - r + 1}\sum_{k=r}^{\ell-d} \|e(k|\hat{\theta}_{{\rm ss},n,d,\ell})\|_2, \quad (32)$$

where r is as in (25) in order to perform a fair comparison between (25) and (32).

Constructing a meaningful transmissibility operator requires knowledge of the number m of independent external excitation signals acting on the system. Since m may be unknown, we estimate m using the following procedure. Let $\hat{m} \in \{1, \ldots, p-1\}$ and $\hat{p} \in \{1, \ldots, p-\hat{m}\}$. We identify a transmissibility operator with $\hat{m}$ pseudo inputs and $\hat{p}$ pseudo outputs using the methods discussed above. For each identified transmissibility operator we compute the residual using (24) or (31) and the norm of the residual using (25) or (32). The estimated number of independent external excitation signals is the value of $\hat{m}$ at which a sharp drop occurs in the norm of the residual. If a sharp drop is not obvious, then the estimated number of external excitation signals is the smallest value of $\hat{m}$ for which no sizable improvement is obtained for larger values of $\hat{m}$. Redundant sensors can then be removed or retained for possible benefit in terms of the accuracy of the identified transmissibility operators. This method will be illustrated in the next section.

V. Application to Fault Detection for an Acoustic System

In order to investigate the ability of transmissibility operators to detect changes in the dynamics of an acoustic system, we consider the experimental setup shown in Figure 1. The setup consists of a drum with two speakers w_1 and w_2 and four microphones mic_1–mic_4. Each speaker is an actuator, and each microphone is a sensor that measures the acoustic response at its location. Two plastic rectangles are placed inside the drum, and these can be removed during operation to emulate changes to the system. All actuator signals are generated using MATLAB and sent to the speakers through a data acquisition card. The sampling rate is 1000 Hz.

Let u_1 and u_2 be the measurements of the signals of the speakers w_1 and w_2, respectively, and let y_1–y_4 be the measurements obtained by the sensors mic_1–mic_4, respectively. For $i = 1, 2, 3$, let $Y_i \triangleq [y_1 \ \ldots \ y_i]^{\rm T} \in \mathbb{R}^i$ and let $\mathcal{T}_i$ be the transmissibility whose pseudo input is Y_i and whose pseudo output is y_4. We assume that data is available for $1 \le k \le 30,000$.

We delay the pseudo output d steps with respect to the pseudo input and perform the identification using the pseudo input and the delayed pseudo output. We use LS, PEM, and IV with a noncausal FIR model with $r = 24$, $d = 25$, and the first $\ell = 10,000$ samples to obtain the identified transmissibilities $\mathcal{T}_i(\mathbf{q}^{-1}, \hat{\theta}_{r,d,\ell})$ of $\mathcal{T}_i(\mathbf{p})$ for $i = 1, 2, 3$. The least squares estimate is obtained using (22), where the PEM and IV estimates are obtained using the Matlab commands `pem(data,'nb',(r+d+1)*ones(1,i),'nc',r+d+1)` and `iv4(data,'nb',(r+d+1)*ones(1,i))`, respectively, where `i` is the number of pseudo inputs.

For subspace identification, we delay the pseudo output d steps with respect to the pseudo input, and we use the Matlab command `n4sid(data,n)` to obtain the identified

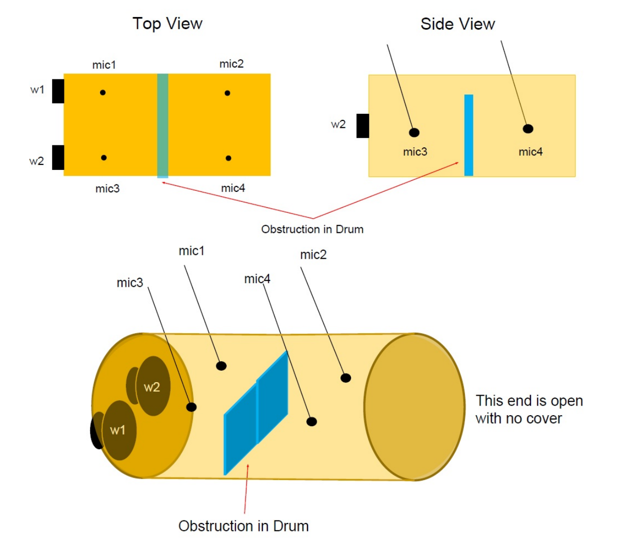

Fig. 1. Experimental setup. The setup consists of a drum with two speakers w_1 and w_2 and four microphones mic_1–mic_4. Each speaker is an actuator and each microphone is a sensor measures the acoustic response at its location. Two plastic rectangles are placed inside the drum (shown in blue) and can be removed during operation to emulate changes to the system.

noncausal state space model of order n. Since an n^{th}-order IIR model has $2n+2$ parameters and an n^{th}-order FIR model has n parameters, we compare an IIR model of order n to an FIR model of order $2n+2$. Since the order of the noncausal FIR model we use is $r+d+1$, for subspace identification we use the model order $n = (r+d-1)/2 = 24$.

Suppose that the system shown in Figure 1 is operating under healthy conditions, and suppose that w_1 is driven with a realization of bandlimited white noise with a bandwidth of 500 Hz and w_2 is not operating. Figure 2 shows the norm of the residual for the identified transmissibility operators with one, two, and three pseudo inputs obtained using LS, PEM, IV, and subspace methods. Figure 2 shows that the identified transmissibility operator using least squares with three pseudo inputs gives the least norm of the residual. Moreover, note from Figure 2 that the improved residual obtained by using a second and a third pseudo input are not significant, which correctly suggests that the number of independent external excitation signals acting on the systems is one. Moreover, the improved residual obtained by using a second and a third pseudo input show the benefit of sensor redundancy. Figure 3 shows y_4 and the computed one-step prediction $\hat{y}_4 \triangleq \mathcal{T}_3(\mathbf{q}^{-1}, \hat{\theta}_{r,d,\ell})[y_1\ y_2\ y_3]^{\mathrm{T}}$, for $15,000 \leq k \leq 15,300$, that is, for $t \in [15, 15.3]$ sec, where $\hat{\theta}_{r,d,\ell}$ is the least squares estimate.

Next, suppose that the system shown in Figure 1 is operating under healthy conditions, and suppose that both w_1 and w_2 are driven with realizations of bandlimited white noise with bandwidth of 500 Hz. Figure 4 shows the norm of the residual for the identified transmissibility operators with one, two, and three pseudo inputs obtained using LS, PEM, IV, and subspace methods. Figure 4 shows that the identified transmissibility operator using least squares with three pseudo inputs gives the least norm of the residual. Moreover, Figure 4 shows that the identified transmissibility operators with two pseudo inputs yield significantly lower residual than the identified transmissibility operators with one pseudo input. However, the improved residual obtained by using a third pseudo input is not significant, which correctly suggests that the number of independent external excitation signals acting on the systems is two. Moreover, the improved residual obtained by using a third pseudo input shows the benefit of sensor redundancy. Figure 5 shows y_4 and the computed one-step prediction $\hat{y}_4 \triangleq \mathcal{T}_3(\mathbf{q}^{-1}, \hat{\theta}_{r,d,\ell})[y_1\ y_2\ y_3]^{\mathrm{T}}$ for $15,000 \leq k \leq 15,300$, that is, for $t \in [15, 15.3]$ sec, where $\hat{\theta}_{r,d,\ell}$ is the least squares estimate.

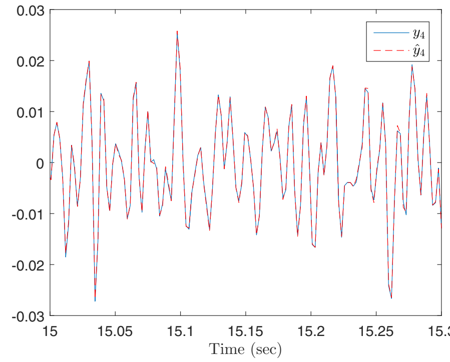

Fig. 3. For the acoustic system shown in Figure 1 operating under healthy conditions, w_1 is driven with realizations of bandlimited white noise with bandwidth of 500 Hz and w_2 is not operating. This plot shows the measurements of y_4 and the computed one-step prediction $\hat{y}_4$.

Next, suppose that the two speakers are operating simultaneously and suppose that $u_1(t) = \sin(100\pi t)$ and $u_2(t) = \sin(120\pi t)$. We use LS with a noncausal FIR model with $r = 24$, $d = 25$, and the first $\ell = 5,000$ samples to obtain the identified transmissibilities $\mathcal{T}_i(\mathbf{q}^{-1}, \hat{\theta}_{r,d,\ell})$ of $\mathcal{T}_i(\mathbf{p})$ for $i = 1, 2, 3$. At approximately $t = 10$ sec and $t = 21$ sec the first and second plastic rectangles are removed, respectively. Data for $5,000 < k \leq 30,000$ is used for validation. Figure 6 shows $E(k|\hat{\theta}_{r,d,\ell}, w)$ for $\mathcal{T}_i(\mathbf{q}^{-1}, \hat{\theta}_{r,d,\ell})$, where $w = 1000$ steps and $i = 1, 2, 3$. Note from Figure 6 the changes in $E(k|\hat{\theta}_{r,d,\ell}, w)$ at approximately $t = 10$ sec and at $t = 21$ sec due to the change in the dynamics of the drum.

VI. Conclusions

A transmissibility operator is a relationship between pairs or sets of sensors that is independent of the excitation signal

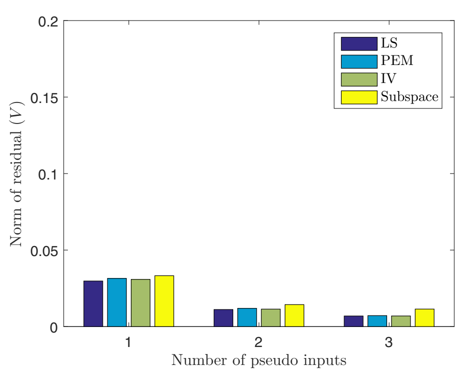

Fig. 2. Norm of the residual for the identified transmissibility operators with one, two, and three pseudo inputs for the acoustic system shown in Figure 1 operating under healthy conditions, where w_1 is driven with realizations of bandlimited white noise with bandwidth of 500 Hz. Note that the benefits produced by using a second and third pseudo inputs are not significant, which correctly suggests that the number of independent external excitation signals acting on the systems is one.

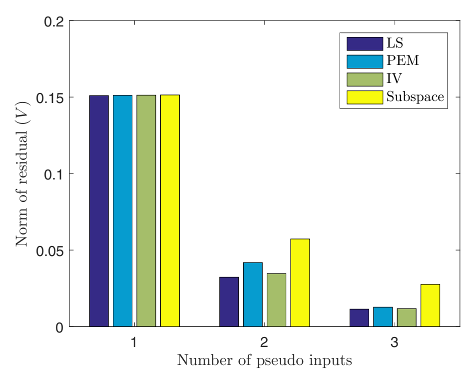

Fig. 4. Norm of the residual for the identified transmissibility operators with one, two, and three pseudo inputs for the acoustic system shown in Figure 1 operating under healthy conditions, where w_1 and w_2 are driven with realizations of bandlimited white noise with bandwidth of 500 Hz. Note that the benefit produced by using a third pseudo input is not significant, which correctly suggests that the number of independent external excitation signals acting on the systems is two.

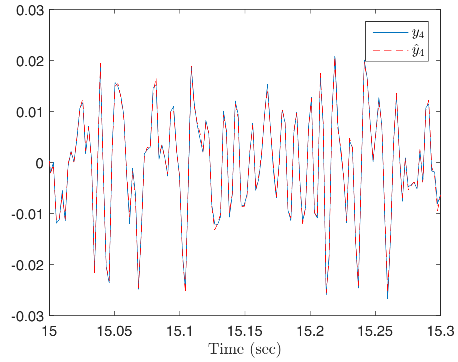

Fig. 5. For the acoustic system shown in Figure 1 operating under healthy conditions, w_1 and w_2 are driven with realizations of bandlimited white noise with bandwidth of 500 Hz. This plot shows the measurements of y_4 and the computed one-step prediction $\hat{y}_4$.

and the initial conditions of the underlying system. We showed that a transmissibility operator can be noncausal, unstable, and of unknown order and thus a noncausal FIR model can be used to identify transmissibility operators. A procedure to estimate the number of external excitation signals acting on the system was introduced. We considered an experimental setup consisting of a drum with two speakers and four microphones. Each speaker is an actuator, and each microphone is a sensor that measures the acoustic response at its location. Measurements from the four microphones were used to construct transmissibility operators, which in turn were used to detect changes in the dynamics of the system by computing the resulting one-step residual.

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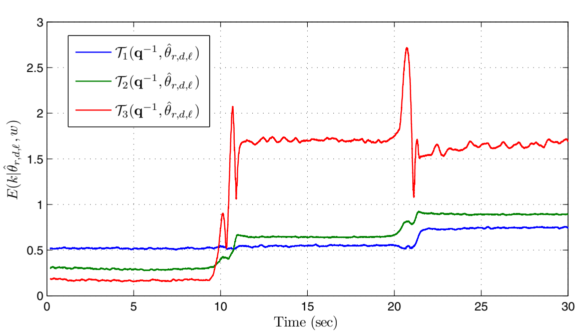

Fig. 6. For the acoustic system shown in Figure 1 with $u_1(t) = \sin(100\pi t)$ and $u_2(t) = \sin(120\pi t)$, this plot shows $E(k|\hat{\theta}_{r,d,\ell}, w)$ for $\mathcal{T}_i(\mathbf{q}^{-1}, \hat{\theta}_{r,d,\ell})$, where $w = 1000$ steps, $i = 1, 2, 3$ and at approximately $t = 10$ sec and $t = 21$ sec the first and second plastic rectangles are removed, respectively. Note the changes in $E(k|\hat{\theta}_{r,d,\ell}, w)$ at approximately $t = 10$ sec and at $t = 21$ sec due to the change in the drum dynamics.

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