Experiments on Stabilization of the Hanging Equilibrium of a 3D Asymmetric Rigid Pendulum

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***Abstract*— We introduced a theory for stabilization of a 3D pendulum, consisting of a rigid body that is supported at a frictionless pivot, in a 2005 ACC paper. One of the controllers proposed in that paper, based on angular velocity feedback only, asymptotically stabilizes the hanging equilibrium of the pendulum. This paper continues this line of research, providing a description of an experimental setup and a sample of experimental results illustrating the closed loop properties of the 3D pendulum. Experimental results are discussed and compared with the theory presented in the previous paper.**

I. INTRODUCTION

Pendulum models have provided a rich source of examples that have motivated and illustrated many recent developments in nonlinear dynamics and in nonlinear control [1], [2]. Much of the published research treats 1D planar pendulum models or 2D spherical pendulum models or some multi-body version of these. In a recent paper [3], we summarized much of this published research, emphasizing papers that treat control issues. In addition, we introduced a new 3D pendulum model that, surprisingly, seems not to have been studied in the prior literature. Another overview of pendulum control problems was given in [2], which also provides motivation for the importance of such control problems. This paper provides experimental verification of stabilization results for a 3D asymmetric rigid pendulum presented in [4].

In [4], we studied stabilization problems for a 3D asymmetric rigid pendulum defined in terms of the reduced attitude. The reduced attitude is the attitude of the pendulum, modulo rotations about the vertical. In other words, two attitudes have identical reduced attitudes if they differ only by a rotation about the vertical. A 3D rigid pendulum is supported at a pivot. The pivot is assumed to be frictionless and inertially fixed. The rigid body is asymmetric and the location of the center of mass is distinct from the location of the pivot. Forces that arise from uniform and constant gravity act on the rigid body. Three independent control moments are assumed to act on the rigid body.

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In this paper, experiments verify the stabilization theory presented in [4]. We follow the development and notation introduced in [3]. In particular, the formulation of the model depends on construction of a Euclidean coordinate frame fixed to the rigid body with origin at the pivot and an inertial Euclidean coordinate frame with origin at the pivot. Without loss of generality, we assume that the inertial coordinate frame is selected so that the first two axes lie in a horizontal plane and the "positive" third axis points down. The relevant mathematical model is expressed in terms of the angular velocity vector and the reduced attitude vector of the rigid body. The reduced attitude vector is a unit vector in the direction of gravity, expressed in the body fixed coordinate frame. The control problem treated in this paper is asymptotic stabilization of an equilibrium of the 3D pendulum defined by zero angular velocity and a reduced attitude vector that corresponds to the hanging equilibrium configuration.

The Triaxial Attitude Control Testbed (TACT) has been described in detail in [5] with mathematical models given in [6]. This paper includes a description of the specific experimental setup of the TACT, the measurement system, and the attitude estimation algorithm. Experimental results for closed-loop responses are presented. The discussion ties these results to the theory presented in [4].

The main contribution of this paper is its summary and experimental verification of results for asymptotic stabilization of the hanging equilibrium of a 3D asymmetric rigid pendulum. These results provide near-global asymptotic stabilization in a direct way using a single nonlinear controller.

II. SUMMARY OF STABILIZATION RESULTS

In this section, we review the control model introduced in [4], assuming full control actuation. We present appropriate definitions as background for the control experiments. In the following sections, we present a description of the experimental testbed, and we discuss experimental results for asymptotic stabilization of the hanging equilibrium, based on feedback of the angular velocity.

As shown in [3], the control model for the fully actuated 3D asymmetric rigid pendulum is given by

$$\begin{cases} J\dot{\omega} = J\omega \times \omega + mg\rho \times \Gamma + u, \\ \dot{\Gamma} = \Gamma \times \omega, \end{cases} \tag{1}$$

where, $\omega \in \mathbb{R}^3$ represents the angular velocity of the rigid body, $\Gamma \in S^2$ represents the reduced attitude vector, and $u \in \mathbb{R}^3$ is the control input. The reduced attitude vector is a unit vector in the direction of gravity in the body-fixed frame. The reduced attitude is the complete attitude, modulo rotations about the vertical. Here J is the inertia matrix of the rigid 3D pendulum, m is its total mass, ρ is the vector from the pivot to the center of mass in the body-fixed coordinate frame, and g is the constant acceleration of gravity. The hanging equilibrium of the uncontrolled pendulum is denoted by $\Gamma_h = \frac{\rho}{\|\rho\|}$.

As presented in [3], the hanging equilibrium of the uncontrolled pendulum is locally stable in the sense of Lyapunov. A simple controller is developed in [4] that makes the hanging equilibrium asymptotically stable. The controller is based on the observation that the control model given by equation (1) is input-output passive if the angular velocity is taken as the output. The total energy is the storage function. Since the total energy, $\frac{1}{2}\ \omega^{\mathrm{T}} J\omega - mg\rho^{\mathrm{T}}\Gamma$, has a minimum at the hanging equilibrium $(0, \Gamma_h)$, a control law based on angular velocity feedback is natural.

Let $\Psi : \mathbb{R}^3 \mapsto \mathbb{R}^3$ be a smooth function such that

$$\epsilon_1 \|x\|^2 \le x^{\mathrm{T}} \Psi(x) \le \epsilon_2 \|x\|^2, \quad \forall x \in \mathbb{R}^3, \tag{2}$$

where $\epsilon_2 \ge \epsilon_1 > 0$. We propose a class of controllers, referred to as damping injection controllers, given by

$$u = -\Psi(\omega), \tag{3}$$

where $\Psi(\cdot)$ satisfies (2). It was shown in [4] that the above family of controllers, which requires only angular velocity feedback, renders the hanging equilibrium of a 3D asymmetric pendulum asymptotically stable. For any $\epsilon > 0$, a guaranteed domain of attraction $\mathcal{H}$ is given by

$$\mathcal{H} = \left\{ (\omega, \Gamma) \in \mathbb{R}^3 \times S^2 : \frac{1}{2}\omega^{\mathrm{T}} J\omega \right.$$
$$\left. + \frac{1}{2} mg\|\rho\| \left(\Gamma - \Gamma_h\right)^{\mathrm{T}} \left(\Gamma - \Gamma_h\right) \le 2mg\|\rho\| - \epsilon \right\}. \tag{4}$$

The controller (3) is traditional in that it exploits passivity to inject damping. However, the controller cannot provide global asymptotic stability due to a fundamental topological limitation that arises from the fact that the configuration space S^2 is compact. The set described in (4) provides a conservative domain of attraction. As shown in [4], the maximal domain of attraction is in fact $\mathbb{R}^3 \times S^2$, except for a set of Lebesgue measure zero. Note that it does not contain the inverted reduced equilibrium $(0, \Gamma_i)$, where $\Gamma_i = \frac{-\rho}{\|\rho\|}$.

III. Triaxial Attitude Control Testbed (TACT)

The experimental facility, located in the Attitude Dynamics and Control Laboratory at the University of Michigan, is referred to as the Triaxial Attitude Control Testbed (TACT). The TACT consists of a rotational platform, supported by a triaxial air bearing, which allows nearly unrestricted three degrees of rotational motion. Control actuators and instrumentation are mounted to the platform. Computer and communications systems are available for data acquisition, test operations, and post-test processing. The physical features of the TACT have been described in [5]. A picture of the TACT is provided in Figure 1.

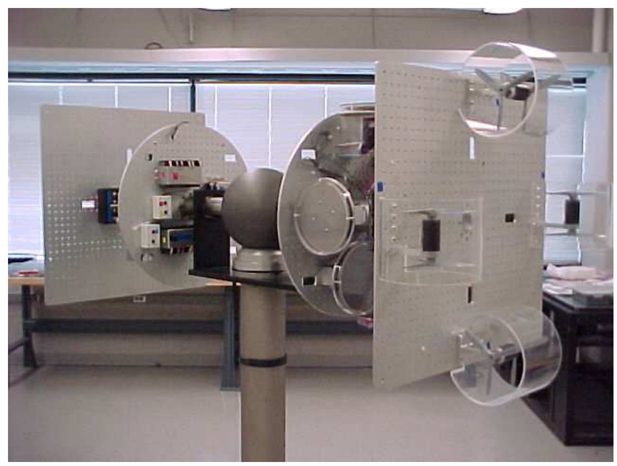

Fig. 1. Triaxial Attitude Control Testbed (TACT)

Fan thrusters are rigidly attached to the platform and provide control actuation for the TACT. These thrusters operate on ±24 VDC and are open-loop speed controlled. A total of four thrusters are mounted to a square outer plate on the TACT, equidistant from the roll axis. Two thrusters provide direct actuation about the yaw axis, while the other two provide actuation about the pitch axis. Combining one from each group provides actuation about the roll axis. Each fan motor is driven by a current-regulated PWM amplifier. Electric power is supplied from a series of 12-V lead acid batteries. Details of the thrusters' construction and specifications can be found in [5]. A closer picture of the fan thrusters on the TACT is provided in Figure 2.

A triaxial accelerometer is mounted to the TACT and aligned with the body-fixed axes. Before each test, the accelerometer triad must be calibrated to read 1-g on each individual axis when aligned with the downward vertical. To do this, the TACT is rotated about each axis independently

Fig. 2. Fan Thruster Assembly on TACT

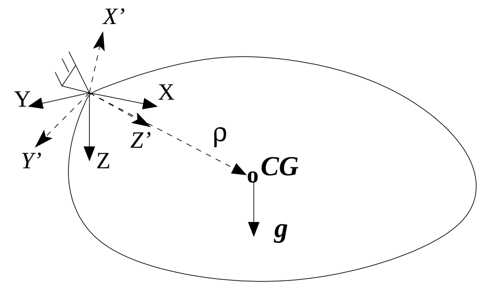

Fig. 3. Schematic of a Rigid Pendulum

while taking continuous readings from the accelerometers. The accelerometer axis is defined to be aligned with the downward vertical when a maximum occurs in the data. Bias and scaling factors are then adjusted such that each accelerometer outputs 1-g at this maximum point.

We are able to estimate the reduced attitude using accelerometers and gyros fixed to the platform. We compute the acceleration due to gravity; this leads directly to the reduced attitude. Since the accelerometer triad is not attached at the pivot point, the accelerometers measure not only the acceleration of gravity, but also angular and centripetal acceleration terms. Centripetal acceleration can be determined indirectly by using the knowledge of the angular velocity. We thus use a triaxial gyro to measure angular velocity and to compensate for the effects of centripetal acceleration. The angular acceleration term is ignored in the computation of the reduced attitude, but preliminary calculations suggest that its contribution is quite small.

An embedded processor is used for real-time on-board processing. This processor is based on a 486 chipset with 4 GB solid state hard disk and three Multi-Q I/O boards allowing 24 A/D channels, 24 D/A channels, and 24 encoder channels. A host PC, which is detached from the TACT, communicates with the testbed through a wireless ethernet connection. This communication is required for experiment monitoring and data acquisition. Software controllers are developed on the host PC using Simulink and then uploaded to the on-board processor for experimentation.

The TACT allows us to study a wide variety of pendulum problems. Since the system's center of gravity is not located at the pivot point, the TACT behaves like a 3D pendulum. A schematic of a 3D rigid pendulum is provided in Figure 3.

IV. TACT Closed Loop Stabilization Experiments

In this section we summarize experimental protocols and results that demonstrate the controller given in (3) asymptotically stabilizes the hanging equilibrium based on angular velocity feedback alone. The controller used for this purpose was chosen such that $\Psi(\omega) = -P\omega$, where $P = 10I_3$. This choice of equal gains in the three feedback channels does not represent a particularly effective choice in terms of closed-loop performance. However, our objective in the experiments is to demonstrate closed-loop stabilization; we did not seek to tune the gains to achieve closed-loop performance.

A brief description of the experimental procedure is presented. For each of the stabilization experiments, the controller is first developed on the host PC and then uploaded to the on-board processor via wireless ethernet. Real-time plots are created on the host PC for experiment monitoring. The pendulum is released from its initial conditions, as described below for each case, and the controller is activated to stabilize the pendulum to the hanging equilibrium. Important data is continuously transferred over the wireless ethernet to the host PC for experiment monitoring and post-test processing.

The aim of these experiments is to verify and support the theory presented in [4]. With that in mind, we report three experiments that encompass several different initial angular velocities and pendulum configurations. The first two experiments verify that when starting inside the domain of attraction (4), the pendulum remains inside that set and it is attracted to the hanging equilibrium; initial conditions were chosen to start within the domain of attraction. The third experiment verifies that even when starting outside the domain of attraction, the pendulum eventually enters the domain of attraction and is attracted to the hanging equilibrium. These initial conditions were chosen to excite all degrees of freedom of the system.

Each experiment involves large perturbations from the hanging equilibrium. Time response plots of both the angular velocity vector and the reduced attitude vector are provided for each case. The TACT was set up so that the reduced hanging equilibrium is given by $\Gamma_h = \frac{\rho}{\|\rho\|} = (0, 0, 1)^{\mathrm{T}}$ in the body fixed coordinate frame.

For the first experiment, the 3D pendulum was released with an initial attitude $\Gamma = (0.35, 0.90, 0.25)^{\mathrm{T}}$ and zero angular velocity. The controller was activated to stabilize the pendulum to the hanging equilibrium. As seen from experimental closed loop responses shown in Figures 4 and 5, the angular velocity converges to zero and $\Gamma(t)$ converges to $\Gamma_h = (0, 0, 1)^{\mathrm{T}}$ as $t \to \infty$.

For the second experiment, the 3D pendulum was released from the hanging equilibrium position with angular velocity $\omega = (0, 0, 40)^{\mathrm{T}}$ deg/s. The controller was activated to return the pendulum to the hanging equilibrium. As seen from experimental closed loop responses shown in Figures 6 and 7, the angular velocity converges to zero and $\Gamma(t)$ converges to $\Gamma_h = (0, 0, 1)^{\mathrm{T}}$ as $t \to \infty$.

For the third experiment, the 3D pendulum was released from an initial attitude $\Gamma = (-0.25, -0.20, 0.95)^{\mathrm{T}}$ and angular velocity $\omega = (-30, -5, -15)^{\mathrm{T}}$ deg/s. The controller was activated to stabilize the pendulum to the hanging equilibrium. As seen from experimental closed loop responses shown in Figures 8 and 9, the angular velocity converges to zero and $\Gamma(t)$ converges to $\Gamma_h = (0, 0, 1)^{\mathrm{T}}$ as $t \to \infty$.

It can be seen that a residual damped oscillation remains in the pitch axis angular velocity for each of the experiments. Increasing the feedback control gain for that axis is expected to compensate for the oscillation. It is also noted that measurements during the initial transient period are especially noisy. This can be attributed to both gyro noise and an attitude estimation algorithm that ignores angular acceleration, which can add to noisy measurements during the initial transient period. To minimize estimation errors, gyro biases were adjusted before each experiment since any additional offset in these sensors would show up in the attitude estimation algorithm through the angular velocity terms.

All three experiments verify the theory of asymptotic stabilization presented in [4]. Initial conditions chosen both inside and outside the domain of attraction all exhibit rapid convergence to the hanging equilibrium. Comparing the results, it is seen that initial perturbations in the yaw angular velocity leads to slow pitch angle convergence; the nonlinear coupling is small in this case. Also, initial perturbations that excite all degrees of freedom lead to relatively faster convergence; the nonlinear coupling is large in this case.

V. Concluding Remarks

In this paper we present experimental results that verify the stabilization of the hanging equilibrium for the asymmetric 3D rigid pendulum. We discuss the experimental setup, the measurement system and the attitude estimation algorithm that are crucial to the experiments. We study a controller, based on feedback of angular velocity only, that asymptotically stabilizes the hanging equilibrium. Experimental results are provided to illustrate the closed loop properties of the 3D pendulum system.

We have demonstrated the simplicity of the control implementation for the feedback controller. The simple damping injection controller has proven to asymptotically stabilize the hanging equilibrium of the 3D pendulum. This paper justifies the importance of theory as a guide for new nonlinear control experiments. These experiments are an invaluable tool that allow us to asses the practical value of theoretical control results, identify new control problems and new applications, and establish the need for new theoretical results.

The experiments reported herein describe our first experiments carried out on reduced attitude stabilization. These initial closed-loop experiments have taught us several important lessons. The robustness of the TACT and associated instrumentation is crucial for experimental accuracy and repeatability. Angular velocity estimates, obtained from gyros, are sufficiently accurate for our purposes, but additional attention needs to be given to estimation of the reduced attitude. As with many physical systems such as the TACT, integration and implementation of complex hardware and software packages requires much care and tuning to provide results in an accurate and meaningful manner. These lessons should prove valuable as we continue our experimental research on stabilization of the 3D pendulum.

Several next steps in experimentation are proposed for future research with the TACT. Feedback controllers utilizing the reduced attitude are of immediate concern. This requires the use of an on-board attitude estimation algorithm, similar to what is currently used in the post-processing procedure. By incorporating 3-axis magnetometer measurements to distinguish between rotations about the vertical, we can utilize feedback controllers that require estimation of the complete attitude. Stabilization of the inverted equilibrium has yet to be demonstrated by experimentation on the TACT, based on controller designs presented in [4]. One further assumption of this paper is full control actuation. Controllers can be developed to stabilize an equilibrium in the case of underactuation; see for example [7]. These should be studied, as well, by experiments on a TACT implementation.

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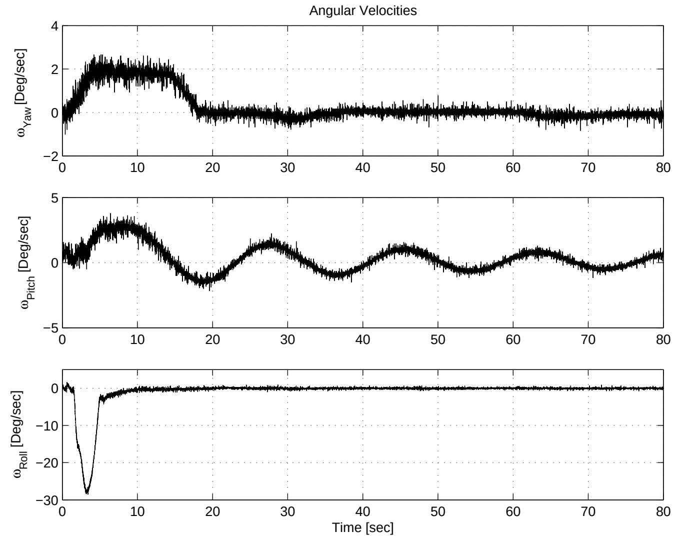

Fig. 4. CASE 1: Experimental results for the evolution of the angular velocity of the 3D pendulum.

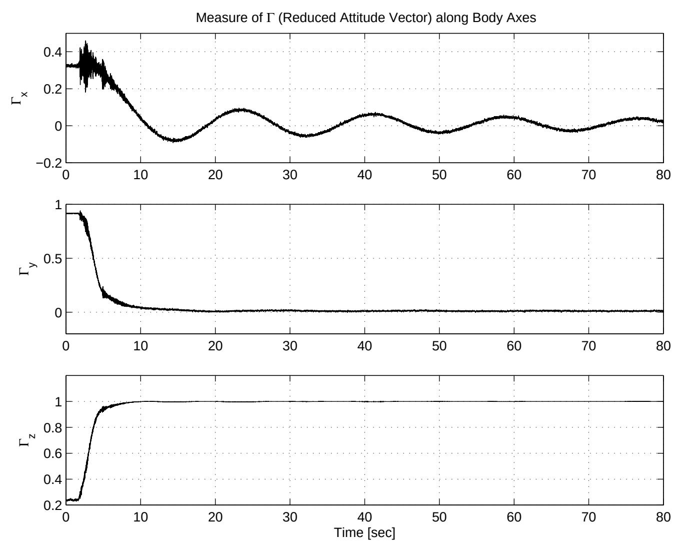

Fig. 5. CASE 1: Experimental results for the evolution of the components of the direction of gravity in the body frame.

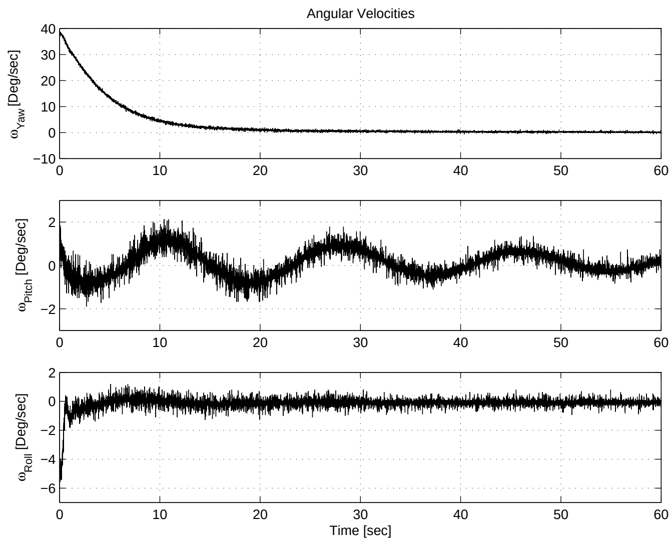

Fig. 6. CASE 2: Experimental results for the evolution of the angular velocity of the 3D pendulum.

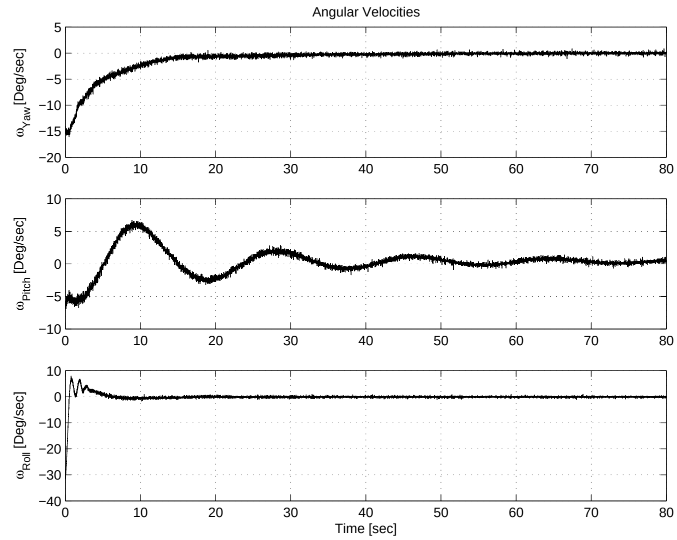

Fig. 8. CASE 3: Experimental results for the evolution of the angular velocity of the 3D pendulum.

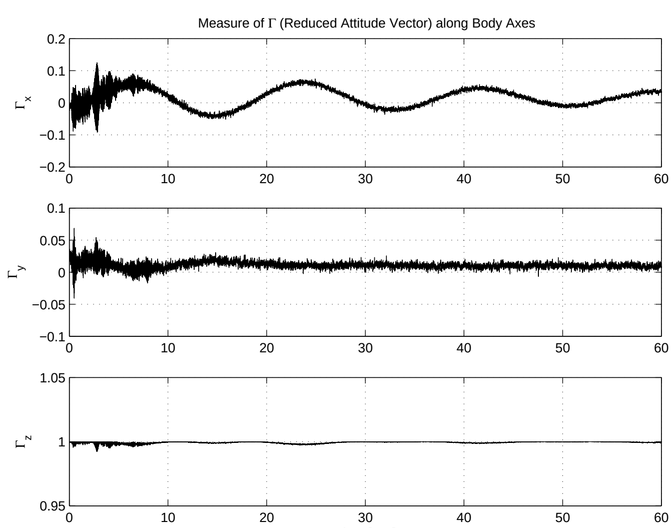

Fig. 7. CASE 2: Experimental results for the evolution of the components of the direction of gravity in the body frame.

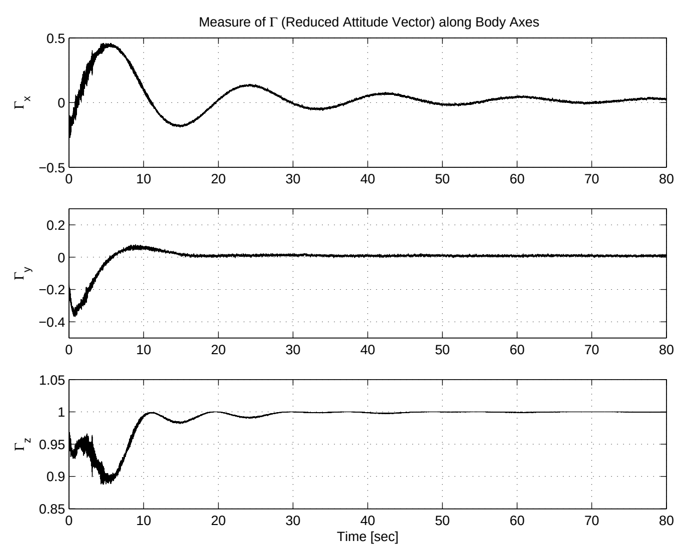

Fig. 9. CASE 3: Experimental results for the evolution of the components of the direction of gravity in the body frame.