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# Adaptive Road-Following Preview Control Using Radius of Curvature Data

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***Abstract*— We consider a tracking problem for a car moving at a constant speed on a piecewise-circular road with known radius-of-curvature. Retrospective-cost-based adaptive control is applied under various off-nominal conditions, including unknown bank and inclination angles. A preview estimate of the time-to-departure, based on radius-of-curvature information, is used to define a performance variable.**

## I. Introduction

Each year in the U.S., approximately 40% of speeding-related fatalities (about 5,000) and 20% of non-speeding related fatalities (about 5,000) occur on a curved portion of the road [1]. Fatality on a curve can be due to run-off-the-road crashes, crashing into moving or nonmoving objects, rollovers, and multiple event accidents. Causes of these accidents include the driver's panic actions, speeding, the demand of fast reaction time, the driver's inability to adapt to sudden changes, and road conditions.

Accidents on curves can potentially be reduced through electronic controls that can exploit the handling capability of the vehicle. Knowledge of the road geometry (curvature, bank, and inclination) is essential, as is knowledge of the road conditions (coefficient of friction). With this information, the goal is to determine and maintain a safe path over which electronic controls can guide the vehicle. Research along these lines is reported in [2, 3].

A challenging issue within this context is the extent to which the control system can augment or override decisions of the driver. These issues are important for future implementation but are outside the scope of the present paper.

In the present paper we consider a tracking problem for a car moving at a constant speed. We assume that radius of curvature information is available at each point along a road that is piecewise straight and circular. We assume that the only control input is the steering angle. In addition, we assume that the road friction is sufficient to avoid skidding. Under these assumptions, we adopt an adaptive control approach that incorporates both feedback and feedforward preview inputs. The feedback control assumes that the displacement and velocity of the car from the center of its lane are known; however, neither yaw rate nor roll angle are assumed to be known.

The adaptive control method that we apply is based on retrospective cost optimization (RCO) as discussed in [4, 5]. RCO-based adaptive control is a sampled-data adaptive control technique that requires knowledge of the relative degree, the first nonzero Markov parameter, and the nonminimum-phase zeros of the system; otherwise, RCO requires no matching assumptions on the command or the disturbance. The required modeling information is provided by system identification methods; no additional knowledge about car parameters such as cornering stiffness or moment of inertia is required. RCO-based adaptive control can use multiple measurements, which can represent both feedback and feedforward signals. We take advantage of this flexibility by including a preview estimate of time-to-departure, which is based on current and future radius-of-curvature information.

Various approaches to steering control that incorporate road preview are reported in [6–8]. Preview control is combined with frequency-shaped linear quadratic (FSLQ) control theory in [6]. In [7], the road curvature is modeled as low-pass-filtered Gaussian white noise, and time-invariant LQR is extended to preview control. Finally, [8] applies the optimal preview control algorithm presented in [9] to automatic lane-tracking control.

Since RCO-based adaptive control requires only limited modeling information, we implement this controller in simulation using only data obtained from the simulation platform. In particular, we perform Markov parameter identification using the CarSim simulation environment [10], and then implement the RCO-based adaptive control law within the CarSim environment. No other vehicle modeling is required for this implementation. We consider various off-nominal conditions, including unknown bank and inclination angles.

## II. Problem Setup

We consider the problem of having a car track a specified road while moving at a constant speed. We assume that the radius-of-curvature at each point along the road is known in advance. This information facilitates the use of preview control within a feedforward control setting. However, the bank and inclination of the road are unknown. For simplicity, the road is *piecewise circular*, which means that it consists of segments that are either straight or arcs of circles. We assume that the road is free of bumps and the ambient wind is zero.

The only available control input is assumed to be the front wheel steering. The speed of the car is maintained at a given constant value without explicit reference to throttle or braking commands. For feedback control we assume that the lateral displacement of the car from the center of lane and its derivative are known.

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We model the problem as the linear discrete-time system

$$x(k+1) = Ax(k) + Bu(k) + D_1 w(k), \tag{1}$$
$$y(k) = Cx(k) + D_2 w(k), \tag{2}$$
$$z(k) = E_1 x(k) + E_0 w(k), \tag{3}$$

where $x(k) \in \mathbb{R}^n, y(k) \in \mathbb{R}^{l_y}, z(k) \in \mathbb{R}^{l_z}, u(k) \in \mathbb{R}^{l_u}$ and $k \geq 0$. The input $u$ denotes the steering angle, while the exogenous signal $w$ represents the curvature, bank angle, and inclination angle along the road. Our goal is to minimize the performance vector $z(k)$, which consists of the displacement $h$ from the center of the lane, and its derivative $\dot{h}$. As described in [4], adaptive algorithm requires specific, limited modeling information relating to (1)-(3).

Let c denote the center of mass of the car, $O_{\mathrm{A}}$ denote a point on the center of the lane, $\mathrm{F_A}$ be a road-fixed frame, and $\mathrm{F_B}$ be a car-fixed frame, as shown in Figure 1. Let $\vec{r}_{\mathrm{c}/O_{\mathrm{A}}}$ denote the position of c relative to $O_{\mathrm{A}}$, and $\vec{v}_{\mathrm{c}/O_{\mathrm{A}}/\mathrm{A}}$ denote the velocity with respect to $\mathrm{F_A}$. We resolve these vectors as $\vec{r}_{\mathrm{c}/O_{\mathrm{A}}}\Big|_{\mathrm{A}} = \begin{bmatrix} h \\ d \end{bmatrix}$, $\vec{v}_{\mathrm{c}/O_{\mathrm{A}}/\mathrm{A}}\Big|_{\mathrm{A}} = \begin{bmatrix} \dot{h} \\ \dot{d} \end{bmatrix}$, $\vec{v}_{\mathrm{c}/O_{\mathrm{A}}/\mathrm{A}}\Big|_{\mathrm{B}} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$. The speed of the car $V_{\mathrm{car}}$ is then given by

$$V_{\mathrm{car}} = \sqrt{v_x^2 + v_y^2} = \sqrt{\dot{h}^2 + \dot{d}^2} = \frac{v_x}{\cos(\beta)}, \tag{4}$$

where $\beta$ is the sideslip angle.

We assume that measurements of $h$ and $\dot{h}$ are available, so that $y(k) = z(k) = \begin{bmatrix} h(k) & \dot{h}(k) \end{bmatrix}^{\mathrm{T}}$. When we use preview, we assume that measurements of $v_x$ and $\beta$ are available as well as knowledge of the radius-of-curvature and road width at each point on the road surface. We then use this data to extrapolate and thus estimate the time-to-departure $T_{\mathrm{dep}}$, and define the preview variable $\xi_{T_{\mathrm{dep}}}$, which is further discussed in Section III. The performance vector is then extended to $z(k) = \begin{bmatrix} h(k) & \dot{h}(k) & \xi_{T_{\mathrm{dep}}}(k) \end{bmatrix}^{\mathrm{T}}$. We do not assume that additional output measurements such as yaw rate or roll angle are available.

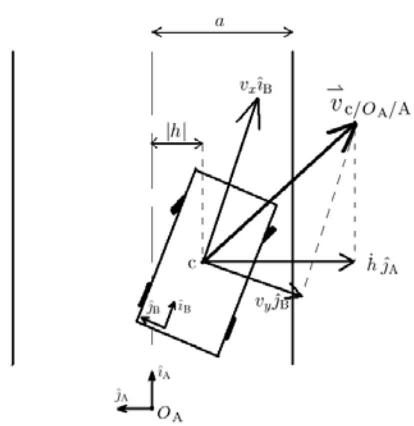

Fig. 1. Illustration of the car-road model on a straight track.

The simulation architecture is shown in Figure 2. In order to apply RCO-based adaptive control, we require specific modeling information, which we obtain from parameter estimation based on simulation. In practice, this modeling data would be obtained from road tests. For identification and implementation of RCO-based adaptive control, CarSim is interfaced with Simulink. Since all required modeling data are obtained by system identification methods, there is no need to specify the state space matrices in (1)–(3). RCO-based adaptive control is described in detail in [4].

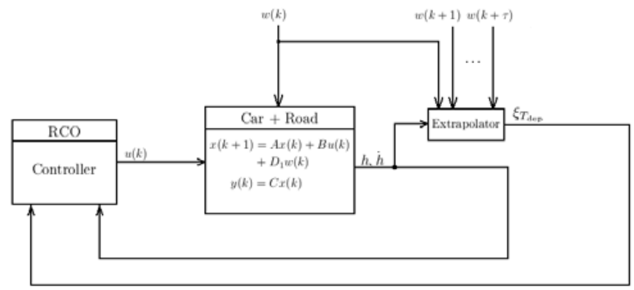


Fig. 2. Block diagram of the control architecture. The retrospective cost optimization and extrapolation logic are handled by Matlab and Simulink, while the car-road model and the road database are provided by Carsim.

## III. Definition of the Preview Variable

In this section, we construct the preview variable $\xi_{T_{\mathrm{dep}}}$, which requires an estimate of $T_{\mathrm{dep}}$. The speed $V_{\mathrm{car}}$, the radius-of-curvature $\rho$, and width of the track $2a$ are assumed to be known and constant, as shown in Figure 3.

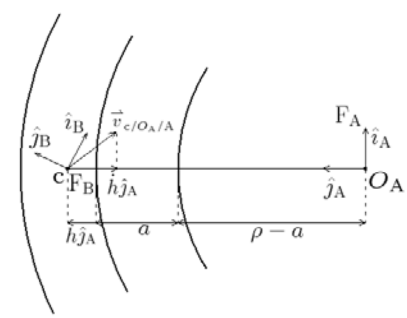

Fig. 3. Illustration of the variables used to estimate $T_{\mathrm{dep}}$ on a curve with constant radius of curvature and road width.

We define the estimated preview tracking error $h_{\mathrm{est}}(k,T)$ by

$$h_{\mathrm{est}}(k,T) \triangleq ((\rho + h(k))^2 + (V_{\mathrm{car}}T)^2 - 2(\rho + h(k))V_{\mathrm{car}}T\cos(\frac{\pi}{2} + \arcsin\frac{\dot{h}(k)}{V_{\mathrm{car}}}))^{-1} - \rho \tag{5}$$

where $T$ is the preview period. For a straight road, $h_{\mathrm{est}}(k,T)$ is given by

$$\lim_{\rho\to\infty} h_{\mathrm{est}}(k,T) = \dot{h}(k)T + h(k).$$

Next, $\xi_T(k)$ is defined by

$$\xi_T(k) \triangleq h_{\mathrm{est}}(k,T)f(T), \tag{6}$$

where $f(T)$ is a monotonically decreasing positive nonnegative function for $T > 0$, and $\lim_{T\to\infty} f(T) = 0$. We choose $f(T) = e^{-T^2}$. We then estimate the time-to-departure $T_{\mathrm{dep}}$ as the minimum positive $T_{\mathrm{dep}}$ that satisfies

$$|h_{\mathrm{est}}(k, T_{\mathrm{dep}})| = a.$$

Finally, we obtain the preview variable $\xi_{T_{\rm dep}}(k)$ by setting $T = T_{\rm dep}$ in (6), so that

$$\xi_{T_{\rm dep}}(k) = h_{\rm est}(k, T_{\rm dep})f(T_{\rm dep}). \tag{7}$$

Since $f(T_{\rm dep})$ is monotonically decreasing and nonnegative for all $T_{\rm dep}$, minimizing $|\xi_{T_{\rm dep}}(k)|$ maximizes $T_{\rm dep}$. Note that $T_{\rm dep} \to \infty$ as $\xi_{T_{\rm dep}}(k) \to 0$.

We now demonstrate the extrapolation of $T_{\rm dep}$ under the assumption that $\rho$ is constant. Suppose the vehicle is tracking the centerline of the curved track shown in Figure 3 with the present tracking error $h$ and its derivative $\dot{h}$. Depending on $\rho$, $a$, and the direction of $\vec{v}_{\rm c/O_A/A}$, the vehicle leaves the road from either the inner or outer edge. It can be shown that the car departs from the inner edge of the road if both

$$\dot{h} < 0 \tag{8}$$

and

$$0 < \cos^{-1}\left(\frac{|\dot{h}|}{V_{\rm car}}\right) \leq \sin^{-1}\left(\frac{\rho - a}{\rho + h}\right). \tag{9}$$

Otherwise, the vehicle leaves from the outer edge. Note that, if $\vec{v}_{\rm c/O_A/A}$ is constant and $\rho < \infty$, then the vehicle always leaves the track in finite time.

If (8) and (9) both hold, then $T_{\rm dep}$ is given by the minimum positive solution of

$$V_{\rm car}^2 T^2 - 2(\rho + h)|\dot{h}|T + (\rho + h)^2 - (\rho - a)^2 = 0.$$

Otherwise, $T_{\rm dep}$ is the positive solution of

$$V_{\rm car}^2 T^2 - 2(\rho + h)\cos(\Psi(\dot{h}, V_{\rm car}))V_{\rm car}T + (\rho + h)^2 - (\rho + a)^2 = 0,$$

where

$$\Psi(\dot{h}, V_{\rm car}) = \frac{\pi}{2} + \arcsin\frac{\dot{h}}{V_{\rm car}}.$$

## IV. Markov Parameter Identification

We estimate $H_i$ offline through least square identification in conjunction with a $\mu$-Markov model structure [11], where $H_i$ represents the $i$th Markov parameter from $u$ to $y = \left[\begin{array}{cc} h & \dot{h} \end{array}\right]^{\rm T}$. For identification, we apply a white noise steering input to the vehicle moving at 90 km/h along a straight road for 100 sec. We sample the input and outputs $h$ and $\dot{h}$ with a sample interval $T_{\rm s}$ of 0.01 sec, yielding 10001 samples for each signal. We then apply least squares $\mu$-Markov identification to the sampled signals to obtain estimates of $H_i$, each of which is a $2 \times 1$ matrix. Various identification methods are compared in [11].

Next, we estimate the Markov parameters for $\xi_{T_{\rm dep}}$. Let $H_{h,i}$ denote the estimate of the $i$th Markov parameter for $h$. Then, the estimate of the $i$th Markov parameter for $\xi_{T_{\rm dep}}$ is

$$H_{\xi_{T_{\rm dep}},i} = H_{h,\tau+i}f(T_{\rm dep}), \tag{10}$$

where $\tau \triangleq \lfloor \frac{T_{\rm dep}}{T_{\rm s}} \rfloor$. Therefore, the Markov parameters for $\xi_{T_{\rm dep}}$ are estimated by shifting $H_{h,i}$ back in time by $T_{\rm dep}$ seconds and scaling by $f(T_{\rm dep})$.

The estimates of $H_{h,i}$ and $H_{\dot{h},i}$ are illustrated in Figures 4 and 5. Note that $H_{h,i}$ is almost linear, particularly for $i \geq 100$. Therefore, we approximate $H_{h,\tau+i}$ by the least squares line fit to reduce implementation complexity.

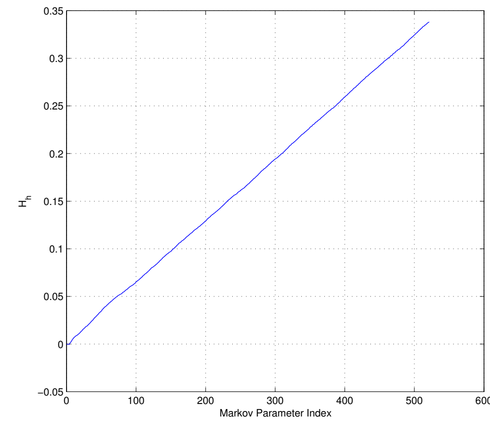


Fig. 4. Markov parameter estimates for $H_{h,i}$, obtained through $\mu$-Markov least-squares estimation.

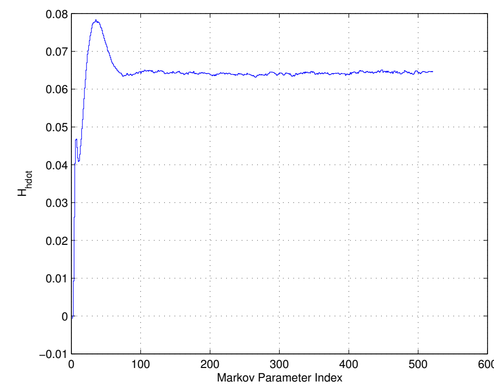


Fig. 5. Markov parameter estimates for $H_{\dot{h},i}$, obtained through $\mu$-Markov least-squares estimation.

## V. Controller Parameter Tuning

In this section, we investigate by simulations the least amount of Markov parameter estimates $\mu$ that is required in order to achieve closed-loop stability. We also present simulation results with various values of $n_{\rm c}$ and $\mu$, and compare the transient and tracking performances.

We consider the track shown in Figure 6. We define the output and performance vectors $y = z \triangleq \left[\begin{array}{ccc} h & \dot{h} & \xi_{T_{\rm dep}} \end{array}\right]^{\rm T}$. We take the adaptive controller order $n_c = 1$, learning rate $\alpha(k) = 2000$, and $R_1 = {\rm diag}(20, 20, 1)$ [4].

We first set $\mu = 1$. We conclude by simulation that the vehicle cannot follow the track when $\mu = 1$.

Now, we choose $\mu = 2$ and keep the other parameters constant. Figure 7 shows that the performance variables do not diverge, although the tracking error is large.

To obtain better tracking, we now vary $n_c$ and $\mu$. Figure 8 shows that regardless of $\mu$, we get poor transients as we increase $n_c$. Furthermore, $n_c$ does not affect the tracking error significantly for $\mu = 10, 15$ and $20$. Therefore, we conclude that $n_c = 1$ yields the best transient performance. On the other hand, increasing $\mu$ by keeping $n_c$ constant leads to worse transient behavior, but improved tracking error. For $n_c = 1$, using $\mu = 15$ yields the best performance, although decreasing $\mu$ to 10 yields similar results.

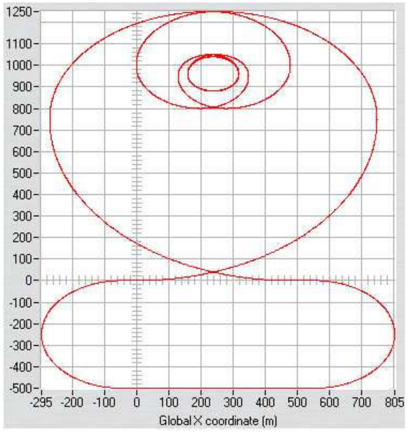


Fig. 6. Spiral loop track. Starting from the origin, the track spirals inward first, then outward. After two 180-degree curves, the track ends at the origin.

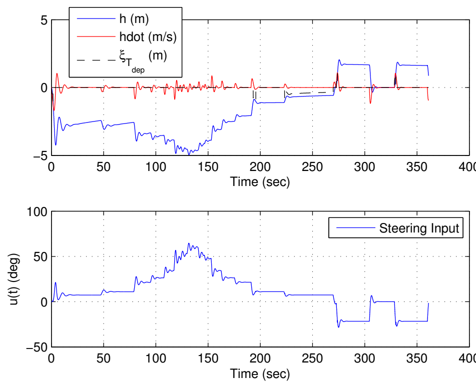


Fig. 7. Input and output plots with $\mu = 2$ on spiral track.

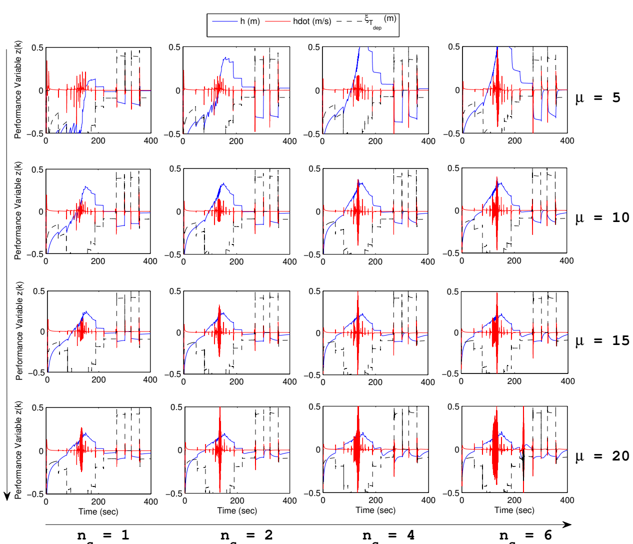


Fig. 8. Simulation results obtained by varying the values of $n_c$ and $\mu$ and keeping the remaining parameters constant. Each plot in a given row corresponds to the same $\mu$, and each plot in a given column corresponds to the same $n_c$.

## VI. Numerical Examples

We now illustrate the performance of RCO-based adaptive control for various road types. For preview control we extrapolate $T_{\text{dep}}$ and define $\xi_{T_{\text{dep}}}$ under the assumption that $\rho$ is constant, as shown in Section III. The only exception is the last example, where we use preview information about $\rho$ to extrapolate $T_{\text{dep}}$. It is assumed in each example that the car is moving at constant longitudinal speed $v_x = 90$ km/h.

### A. Quasi-Circular Track

We now consider a flat, quasi-circular closed track consisting of six different circular arcs with radii 100, 150, and 250 m in the horizontal plane, Preview is not used, and thus $y = z = \begin{bmatrix} h & \dot{h} \end{bmatrix}^{\mathrm{T}}$. We take $n_c = 1$, $\alpha(k) = 2000$ for all $k \geq 0$, and $R_1 = \mathrm{diag}(1, 20)$.

The closed-loop responses are shown in Figure 9. Note that the tracking error does not exceed 1.25 m, and decreases as the car repeats the track. The controller gains plotted in Figure 10 show that the algorithm adapts to different radii of curvature.

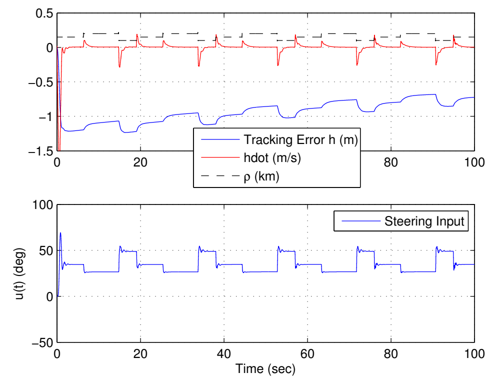


Fig. 9. Steering input, closed-loop responses, and road radius of curvature. These results are obtained for the simulation on the quasi-circular track.

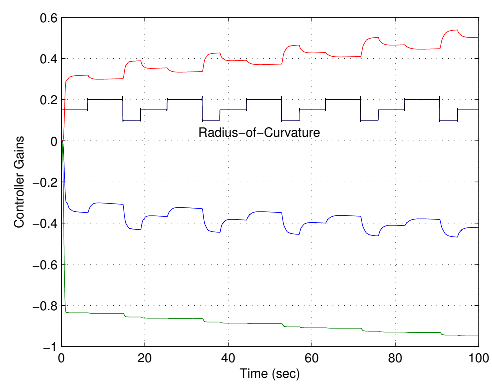


Fig. 10. These traces show the time histories of the controller gains. The components adapt to various radii of curvature. These results are obtained for the simulation on the quasi-circular track.

### B. Banked Road

We now consider the track shown in Figure 11. This track contains banked sections with bank angles specified as percentages shown in Figure 11. Colors indicate the bank direction. The simulation starts from the origin, and the car starts by moving to the right.

We first do not use preview variable, so that $y = z = \begin{bmatrix} h & \dot{h} \end{bmatrix}^{\mathrm{T}}$. We take $n_c$, $\alpha(k)$, and $R_1$ as in Section VI-A. Figure 12 shows that the car remains on the road with a maximum tracking error about 1 m. Moreover, the steering input and $\dot{h}$ response are oscillatory.

Now, preview variable is added to the performance vector, so that $z = \begin{bmatrix} h & \dot{h} & \xi_{T_{\text{dep}}} \end{bmatrix}^{\mathrm{T}}$. $T_{\text{dep}}$ is extrapolated under constant $\rho$ assumption. We take $n_c = 1$, $\alpha(k) = 2000$

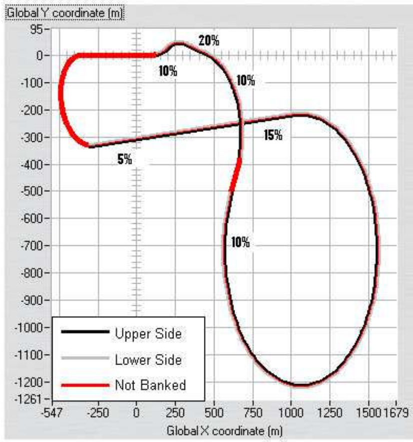


Fig. 11. This track contains banked sections. Bank angles are illustrated with percentages and colors. Black represents the higher edge, while gray represents the lower edge of the road; red means the road is not banked. Radii of curvature on this track range from 100 m to 500 m.

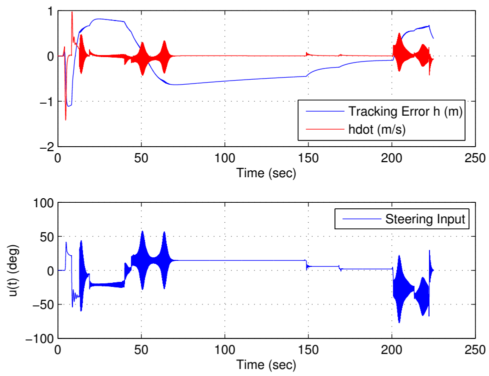


Fig. 12. Steering input and closed-loop responses for the banked road of Figure 11. Preview is not used in this simulation.

for all $k \geq 0$, and $R_1 = \mathrm{diag}(20, 20, 1)$. The closed-loop responses of $h$, $\dot{h}$, and $\xi_{T_{\mathrm{dep}}}$ are presented in Figure 13. The oscillatory behavior of $u(k)$ and $\dot{h}$ disappears, while the maximum tracking error decreases to about 0.7 m.

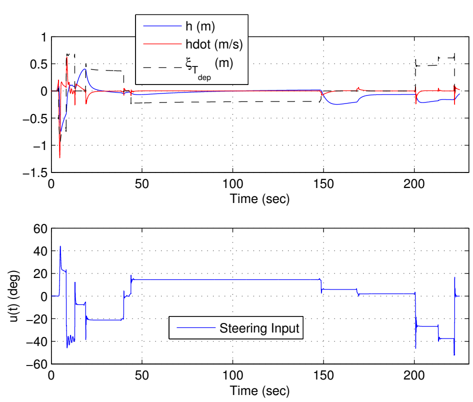


Fig. 13. Steering input and closed-loop responses for the banked road of Figure 11. Preview variable is used in this simulation.

### C. Inclined Road

Consider the track shown in Figure 14. This track has inclined sections as shown in Figure 15. The simulation starts from the origin and the car starts moving to the right. We first do not use the preview variable, and take $n_c$, $\alpha(k)$, and $R_1$ as in Section VI-A. Figure 16 shows that the car is kept on track with a maximum tracking error of about 1.2 m. We also note oscillations in $u(k)$ and $\dot{h}$.

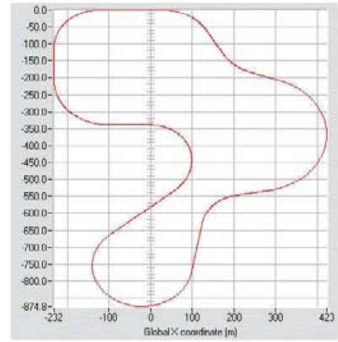


Fig. 14. Inclined road. This track contains inclined sections as shown in Figure 15. The radii of curvature on this track range from 100 m to 168 m.

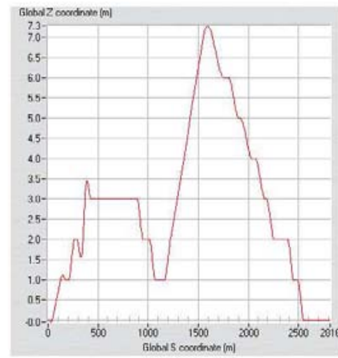


Fig. 15. Elevation in the road with respect to the distance $s$ along the road, where $s = 0$ at the origin of the inclined track shown in Figure 14.

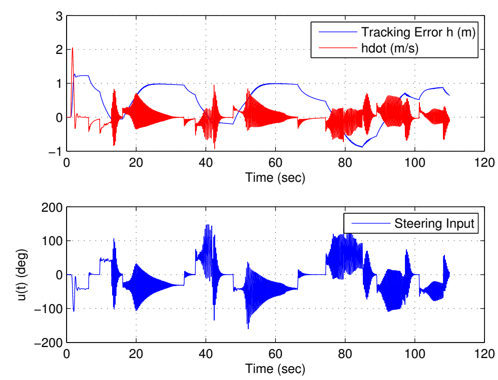


Fig. 16. These simulation results are for the inclined road of Figures 14 and 15. Preview is not used in this simulation.

Preview variable is now added to the performance vector, and $T_{\mathrm{dep}}$ is estimated under the assumption that $\rho$ is constant. We take $n_c$, $\alpha(k)$ as in Section VI-A, and $R_1 = \mathrm{diag}(20, 20, 1)$. The closed-loop responses are presented in Figure 17. The transient behavior of $u(k)$ and $\dot{h}$ are improved compared to Figure 16. We also note a significant improvement in the overall tracking error. Furthermore, as shown in Figure 18, preview control drives the car on the inside of the curve unlike control without preview.

### D. Single Curve

We now consider a section of a track that consists of a straight road, followed by a curve with $\rho = 100$ m.

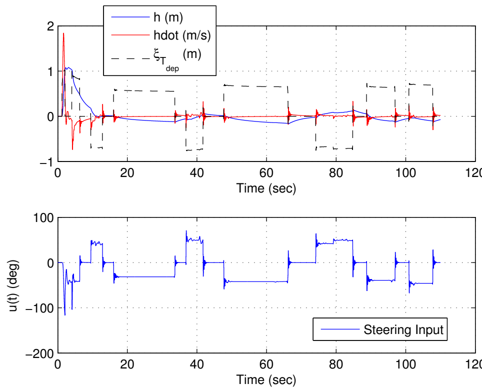


Fig. 17. These simulation results are for the inclined road of Figures 14 and 15. Preview variable is used in this simulation.

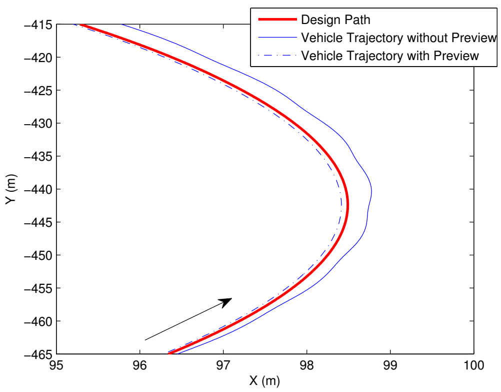


Fig. 18. Tracking on a curved section of the inclined road. The adaptive control drives the car on the inside of the curve with a smaller tracking error when we include the preview variable.

First, we do not use the preview variable, and we set the control parameters as in Section VI-A. Figure 19 shows that the control does not steer the car until the curve begins. The vehicle is driven on the outside of the curve with a maximum tracking error of about 0.5 m.

Now, we include the preview variable to the performance vector, and we extrapolate $T_{\rm dep}$ using current and preview $\rho$ information. We set the control parameters as in Section VI-C. The control starts steering to the inside of the track before the curve begins, and keeps the vehicle on the inside of the curve, as shown in Figure 19. The tracking error remains less than 0.25 m throughout the simulation.

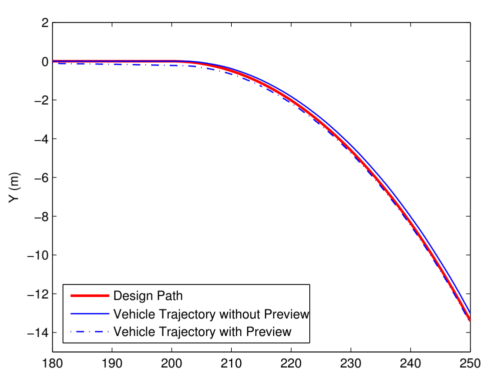


Fig. 19. Trajectories with and without preview on the single curve track.

## VII. Conclusions

We applied retrospective-cost-optimization-(RCO)-based adaptive control to the problem of track following of a car moving at a constant speed. For identification and implementation of RCO-based adaptive control we interfaced CarSim with Simulink. We then illustrated the performance of the RCO algorithm for track following on a vehicle model obtained from Carsim, under various off-nominal conditions such as unknown bank and inclination angles. We applied preview control through time-to-departure extrapolation and compared the performance with and without preview. Future work includes robustness analysis under various vehicle and road conditions, as well as control through both steering and braking.

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