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Adaptive Decentralized Control with Nonminimum-Phase Closed-Loop Channel Zeros

Syed Aseem Ul Islam, Yousaf Rahman, and Dennis S. Bernstein

Abstract— **We apply retrospective cost adaptive control (RCAC) to a two-channel decentralized disturbance rejection problem. It is shown that the closed-loop channel zeros for each subcontroller consist of the plant zeros and poles of the remaining subcontroller. The nonminimum-phase (NMP) closed-loop channel zeros are included in the modeling information required by RCAC. Two adaptation schemes are presented. In one-controller-at-a-time adaptation, one subcontroller is adapted with the other subcontroller fixed at zero. The first subcontroller is then fixed while the second subcontroller is adapted taking into account the NMP closed-loop channel zeros. We also consider concurrent adaptation, where both controllers are updated at the same time. Finally, we apply this technique to decentralized control of the position and shape of a 2DOF lumped flexible body.**

I. Introduction

Decentralized control is both theoretically deep and immensely practical [1]–[7]. The monograph [8] and survey papers [9]–[12] reflect intense longstanding interest in the theoretical challenges of decentralized control.

From a practical point of view, many MIMO control systems are deliberately implemented in a decentralized architecture, where SISO subcontrollers are designed and implemented separately to achieve distinct goals, such as loops that enhance damping, follow position commands, and regulate performance variables such as temperature and pressure. In these applications, it is typically the case that the coupling among loops is either weak or can be accounted for by a modest amount of iteration and retuning. The use of decentralized control in these situations vastly simplifies verification and validation by separating the operation and impact of each subcontroller. The use of a fully coupled MIMO control system may be either unnecessary in terms of achievable performance, risky in terms of unintended loop interaction, or challenging in terms of online diagnostics for fault detection.

A more challenging situation arises in the case where the coupling between control channels is strong, but centralized control is not feasible. For example, communication constraints may render it impossible for measurements at a given location to be disseminated in real time to other locations and likewise for the dissemination of control signals. Aside from communication constraints, the sheer dimensionality of a large-scale model may render it impossible to design a fully centralized controller based on the overall model. In this case, the model must be decomposed into localized subsystems with individual subcontrollers designed for each subsystem.

The theoretical challenges of decentralized control include the possibility of fixed modes [13]–[15]. In this case, poles that can be assigned by centralized control are unmovable with a decentralized architecture. A deeper issue concerns the ability of one subcontroller to "communicate" with another subcontroller through the plant; this is the essence of the Witsenhausen conjecture, where one subcontroller has limited control authority and another subcontroller has noisy measurements [16].

The present paper approaches the decentralized control problem using retrospective cost adaptive control (RCAC) [17]–[19]. Since RCAC uses limited modeling information, it does not require a detailed model of the overall system. RCAC does require, however, modeling information involving nonminimum-phase (NMP) zeros of the plant. In decentralized control, the effective plant for each subcontroller consists of the actual plant and all other subcontrollers that are in feedback with the actual plant. The goal of the present paper is thus to apply RCAC to a decentralized control problem, where the required modeling information for the adaptation of each subcontroller is determined by the remaining subcontrollers in feedback with the actual plant. This approach goes beyond [20], which did not account for NMP zeros in the effective plant relative to each subcontroller.

To explore the effect of NMP zeros on the effective plant for each subcontroller, we consider a two-channel broadband disturbance rejection problem with a single measurement and two actuators. Although the plant is controllable by each subcontroller separately, the authority of each actuator is limited in terms of its effect on the plant modes. This means that the subcontrollers must work together to reject the disturbance over the full plant bandwidth. We use RCAC to adapt the first subcontroller separately. We then fix the first subcontroller and adapt the second subcontroller, which interacts with the effective plant consisting of the actual plant and first subcontroller. Since the first subcontroller converges to a fixed-gain unstable controller, RCAC must account for the NMP zero arising due to the unstable controller pole. By accounting for this NMP zero, the second subcontroller adapts and successfully assists the first subcontroller in rejecting the disturbance. Without accounting for the NMP zero, the adaptation of the second subcontroller fails, as expected. We also consider decentralized control of the position and shape of a 2DOF lumped flexible body. This example is more challenging since it is open-loop unstable.

Syed Aseem Ul Islam, Yousaf Rahman, and D. S. Bernstein are with the Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI, USA. {aseemisl, yousafrm, dsbaero}@umich.edu

II. Standard Problem

Consider the standard problem consisting of the discrete-time, linear time-invariant system

$$x(k+1) = Ax(k) + Bu(k) + D_1 w(k), \quad (1)$$
$$y(k) = Cx(k) + D_0 u(k) + D_2 w(k), \quad (2)$$
$$z(k) = E_1 x(k) + E_2 u(k) + E_0 w(k), \quad (3)$$

where $x(k) \in \mathbb{R}^n$ is the state, $y(k) \in \mathbb{R}^{l_y}$ is the measurement, $u(k) \in \mathbb{R}^{l_u}$ is the control input, $w(k) \in \mathbb{R}^{l_w}$ is the exogenous input, and $z(k) \in \mathbb{R}^{l_z}$ is the performance variable. The system (1)–(3) may represent a continuous-time, linear time-invariant system sampled at a fixed rate. The goal is to develop a feedback or feedforward controller that operates on y to minimize z in the presence of the exogenous signal w. For the servo problem, the components of w can represent either a command signal r to be followed, an external disturbance d to be rejected, or sensor noise v that corrupts the measurement, depending on the choice of D_1, D_2, and E_0. Depending on the application, components of w may or may not be measured, and, for feedforward control, the measured components of w can be included in y by suitable choice of C and D_2. For fixed-gain control, z need not be measured, whereas, for adaptive control, z is assumed to be measured.

Using the forward shift operator $\mathbf{q}$, we can rewrite (1)–(3) as

$$y(k) = G_{yw}(\mathbf{q})w(k) + G_{yu}(\mathbf{q})u(k), \quad (4)$$
$$z(k) = G_{zw}(\mathbf{q})w(k) + G_{zu}(\mathbf{q})u(k), \quad (5)$$

where

$$G_{yw}(\mathbf{q}) \triangleq D^{-1}(\mathbf{q})N_{yw}(\mathbf{q}) = C(\mathbf{q}I - A)^{-1}D_1 + D_2, \quad (6)$$
$$G_{yu}(\mathbf{q}) \triangleq D^{-1}(\mathbf{q})N_{yu}(\mathbf{q}) = C(\mathbf{q}I - A)^{-1}B + D_0. \quad (7)$$
$$G_{zw}(\mathbf{q}) \triangleq D^{-1}(\mathbf{q})N_{zw}(\mathbf{q}) = E_1(\mathbf{q}I - A)^{-1}D_1 + E_0, \quad (8)$$
$$G_{zu}(\mathbf{q}) \triangleq D^{-1}(\mathbf{q})N_{zu}(\mathbf{q}) = E_1(\mathbf{q}I - A)^{-1}B + E_2, \quad (9)$$

The discrete-time, linear time-invariant controller has the form

$$u(k) = G_{\rm c}(\mathbf{q})y(k). \quad (10)$$

Note that $\mathbf{q}$ is a time-domain operator that accounts for initial conditions. For pole-zero analysis, $\mathbf{q}$ can be replaced by the Z-transform complex variable $\mathbf{z}$, in which case (4), (5), and (10) do not account for the initial conditions. Figure 1 illustrates (4)–(10).

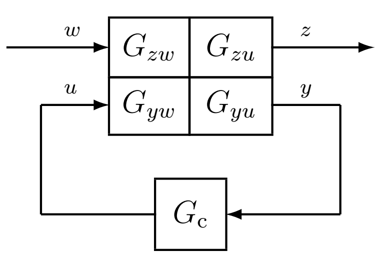


Fig. 1: Standard problem.

The closed-loop transfer function from the exogenous signal w to the performance variable z is given by

$$\widetilde{G}_{zw} \triangleq G_{zw} + G_{zu}G_{\rm c}(I - G_{yu}G_{\rm c})^{-1}G_{yw}. \quad (11)$$

We refer to the poles of $\widetilde{G}_{zw}$ as the closed-loop poles, and the transmission zeros of $\widetilde{G}_{zw}$ as the closed-loop zeros. In the case where u, w, y, and z are scalar signals, $\widetilde{G}_{zw}$ can be written as

$$\widetilde{G}_{zw} = \frac{N_{zu}N_{yw}N_{\rm c} + N_{zw}(DD_{\rm c} - N_{yu}N_{\rm c})}{D(DD_{\rm c} - N_{yu}N_{\rm c})}, \quad (12)$$

where $G_{\rm c} = \frac{N_{\rm c}}{D_{\rm c}}$.

III. Decentralized Control

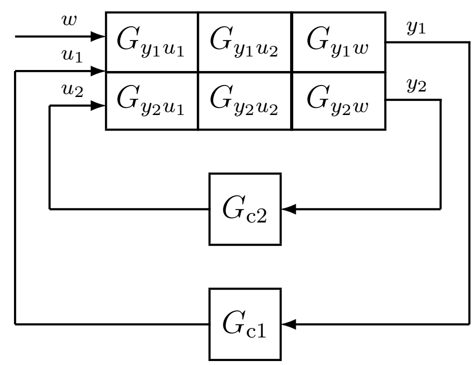


Fig. 2: Standard problem with two subcontrollers.

To determine the effective plant relative to each subcontroller, we consider a decentralized controller consisting of two SISO subcontrollers $G_{\rm c1}$ and $G_{\rm c2}$, as shown in Figure 2. Ignoring the exogenous input w, the control inputs u_1, u_2 and outputs y_1, y_2 are related as

$$y_1 = G_{y_1u_1}u_1 + G_{y_1u_2}u_2, \quad (13)$$
$$y_2 = G_{y_2u_1}u_1 + G_{y_2u_2}u_2, \quad (14)$$
$$u_1 = G_{\rm c1}y_1, \quad (15)$$
$$u_2 = G_{\rm c2}y_2. \quad (16)$$

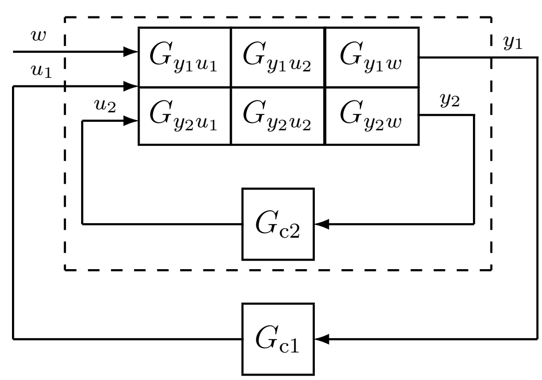


Fig. 3: Effective plant relative to $G_{\rm c1}$.

Substituting (16) into (14) yields

$$y_2 = G_{y_2u_1}u_1 + G_{y_2u_2}G_{\rm c2}y_2, \quad (17)$$
$$y_2 = \frac{G_{y_2u_1}}{1 - G_{y_2u_2}G_{\rm c2}}u_1. \quad (18)$$

Next, substituting (18) into (16) yields

$$u_2 \;=\; G_{\rm c2}\frac{G_{y_2u_1}}{1-G_{y_2u_2}G_{\rm c2}}u_1. \tag{19}$$

Finally, substituting (19) into (13) yields

$$y_1 \;=\; \frac{G_{y_1u_1}(1-G_{y_2u_2}G_{\rm c2})+G_{y_1u_2}G_{\rm c2}G_{y_2u_1}}{1-G_{y_2u_2}G_{\rm c2}}u_1. \tag{20}$$

We define the notation

$$G_{y_1u_1}\triangleq\frac{N_{y_1u_1}}{D},\quad G_{y_1u_2}\triangleq\frac{N_{y_1u_2}}{D},\quad G_{y_2u_1}\triangleq\frac{N_{y_2u_1}}{D},$$
$$G_{y_2u_2}\triangleq\frac{N_{y_2u_2}}{D},\quad G_{\rm c1}\triangleq\frac{N_{\rm c1}}{D_{\rm c1}},\quad G_{\rm c2}\triangleq\frac{N_{\rm c2}}{D_{\rm c2}}. \tag{21}$$

Using the definitions (21) yields

$$y_1 \;=\; \frac{D_{\rm c2}DN_{y_1u_1}+N_{\rm c2}(N_{y_1u_2}N_{y_2u_1}-N_{y_1u_1}N_{y_2u_2})}{D_{\rm c2}D^2-N_{\rm c2}N_{y_2u_2}D}u_1. \tag{22}$$

Let $i=1,2$, and $j=3-i$. Then (22) can be written as

$$y_i \;=\; \frac{D_{{\rm c}j}DN_{y_iu_i}+N_{{\rm c}j}(N_{y_iu_j}N_{y_ju_i}-N_{y_iu_i}N_{y_ju_j})}{D_{{\rm c}j}D^2-N_{{\rm c}j}N_{y_ju_j}D}u_i. \tag{23}$$

If $y_i=y_j$, then

$$G_{y_iu_j} \;=\; G_{y_ju_i},\quad G_{y_iu_j} \;=\; G_{y_ju_j}, \tag{24}$$

and thus (23) reduces to

$$y_i \;=\; \frac{D_{{\rm c}j}N_{y_iu_i}}{D_{{\rm c}j}D-N_{{\rm c}j}N_{y_iu_j}}u_i. \tag{25}$$

If $u_i=u_j$, then

$$G_{y_iu_i} \;=\; G_{y_iu_j},\quad G_{y_ju_i} \;=\; G_{y_ju_j}, \tag{26}$$

and thus (23) reduces to

$$y_i \;=\; \frac{D_{{\rm c}j}N_{y_iu_i}}{D_{{\rm c}j}D-N_{{\rm c}j}N_{y_ju_i}}u_i. \tag{27}$$

It follows from (25) and (27) that the effective plant relative to the subcontroller $G_{{\rm c}i}$ is NMP if and only if either $G_{y_iu_i}$ is NMP or $G_{{\rm c}j}$ is unstable. This observation has implications for the choice of the target model used by RCAC to adapt $G_{{\rm c}i}$.

IV. RCAC Algorithm

A. Controller Structure

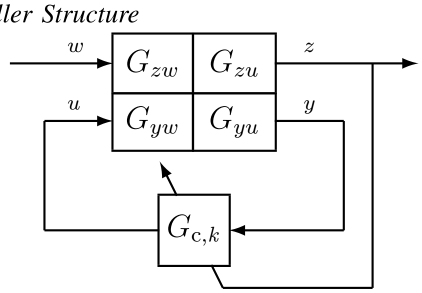


Fig. 4: Adaptive standard problem.

Define the dynamic compensator

$$u(k)=\sum_{i=1}^{n_{\rm c}}P_i(k)u(k-i)+\sum_{i=k_{\rm c}}^{n_{\rm c}}Q_i(k)y(k-i), \tag{28}$$

where $P_i(k)\in\mathbb{R}^{l_u\times l_u}$ and $Q_i(k)\in\mathbb{R}^{l_u\times l_y}$ are the controller coefficient matrices, and $k_{\rm c}\ge 0$. For controller startup, we implement (28) as

$$u(k)=\begin{cases}0, & k<k_{\rm w},\\ \Phi(k)\theta(k), & k\ge k_{\rm w},\end{cases} \tag{29}$$

where the regressor matrix $\Phi(k)$ is defined by

$$\Phi(k)\triangleq\begin{bmatrix} u(k-1)\\ \vdots\\ u(k-n_{\rm c})\\ y(k-k_{\rm c})\\ \vdots\\ y(k-n_{\rm c})\end{bmatrix}^{\rm T}\otimes I_{l_u}\in\mathbb{R}^{l_u\times l_\theta}, \tag{30}$$

$k_{\rm w}$ is an initial waiting period that allows $\Phi(k)$ to be populated with data, and the controller coefficient vector $\theta(k)$ is defined by

$$\theta(k)\triangleq{\rm vec}\left[\begin{array}{cccccc} P_1(k) & \cdots & P_{n_{\rm c}}(k) & Q_{k_{\rm c}}(k) & \cdots & Q_{n_{\rm c}}(k)\end{array}\right]^{\rm T}\in\mathbb{R}^{l_\theta}, \tag{31}$$

$l_\theta\triangleq l_u^2n_{\rm c}+l_ul_y(n_{\rm c}+1-k_{\rm c})$, "$\otimes$" is the Kronecker product, and "vec" is the column-stacking operator. Note that $k_{\rm c}=0$ allows an exactly proper controller, whereas $k_{\rm c}\ge 1$ yields a strictly proper controller of relative degree at least $k_{\rm c}$. In all examples in this paper, we use $k_{\rm c}=1$, and unless specified otherwise, we use $k_{\rm w}=n_{\rm c}$. In terms of $\mathbf{q}$, the transfer function of the controller from y to u is given by

$$G_{{\rm c},k}(\mathbf{q})=\left(\mathbf{q}^{n_{\rm c}}I_{l_u}-\mathbf{q}^{n_{\rm c}-1}P_1(k)-\cdots-P_{n_{\rm c}}(k)\right)^{-1}\cdot\left(\mathbf{q}^{n_{\rm c}-k_{\rm c}}Q_{k_{\rm c}}(k)+\cdots+Q_{n_{\rm c}}(k)\right). \tag{32}$$

If y and u are scalar signals, then $G_{\rm c}$ is SISO and (32) can be written as

$$G_{{\rm c},k}(\mathbf{q})=\frac{\mathbf{q}^{n_{\rm c}-k_{\rm c}}Q_{k_{\rm c}}(k)+\cdots+Q_{n_{\rm c}}(k)}{\mathbf{q}^{n_{\rm c}}-\mathbf{q}^{n_{\rm c}-1}P_1(k)-\cdots-P_{n_{\rm c}}(k)}. \tag{33}$$

B. Retrospective Performance Variable

As shown in [21], we define the retrospective performance variable as

$$\hat z(k,\hat\theta)\triangleq z(k-1)+G_{\rm f}(\mathbf{q})[\Phi(k-1)\hat\theta-u(k-1)], \tag{34}$$

where $\hat\theta\in\mathbb{R}^{l_\theta}$. The rationale underlying (34) is to replace the past controls with a retrospectively optimized control $\Phi(k)\hat\theta$. The optimal controller coefficient vector $\hat\theta_{\rm opt}(k)$, which is obtained by retrospective optimization below, yields the updated controller with coefficients $\theta(k+1)=\hat\theta_{\rm opt}(k)$. Consequently, the implemented control at step $k+1$ is given by

$$u(k+1)=\Phi(k+1)\hat\theta_{\rm opt}(k)=\Phi(k+1)\theta(k+1). \tag{35}$$

The $n_z\times n_u$ filter $G_{\rm f}$ has the form

$$G_{\rm f}\triangleq D_{\rm f}^{-1}N_{\rm f}, \tag{36}$$

where $D_{\rm f}$ is an $l_z\times l_z$ polynomial matrix with leading coefficient I_{l_u}, and $N_{\rm f}$ is an $l_z\times l_u$ polynomial matrix. It is shown in [21] that $G_{\rm f}$ acts as a *target model* for the closed loop transfer function from the virtual intercalated injection v_0 to z. By defining the filtered versions $\Phi_{\mathrm{f}}(k) \in \mathbb{R}^{l_z \times l_\theta}$ and $u_{\mathrm{f}}(k) \in \mathbb{R}^{l_z}$ of $\Phi(k)$ and $u(k)$, respectively, (34) can be written as

$$\hat{z}(k,\hat{\theta}) = z(k-1) + \Phi_{\mathrm{f}}(k-1)\hat{\theta} - u_{\mathrm{f}}(k-1), \qquad (37)$$

where

$$\Phi_{\mathrm{f}}(k) \triangleq G_{\mathrm{f}}(\mathbf{q})\Phi(k), \quad u_{\mathrm{f}}(k) \triangleq G_{\mathrm{f}}(\mathbf{q})u(k). \qquad (38)$$

Note that implementation requires $k_{\mathrm{w}} \geq \max(n_{\mathrm{c}}, n_{\mathrm{f}})$, where n_{f} is the McMillan degree of G_{f}.

C. Retrospective Cost

Using the retrospective performance variable $\hat{z}(k,\hat{\theta})$ defined by (34), we define the cumulative retrospective cost function

$$\begin{aligned} J(k,\hat{\theta}) \triangleq & \sum_{i=1}^{k} \lambda^{k-i} \hat{z}^{\mathrm{T}}(i,\hat{\theta}) R_z(i) \hat{z}(i,\hat{\theta}) \\ & + \sum_{i=1}^{k} \lambda^{k-i} (\Phi_{\mathrm{f}}(i)\hat{\theta})^{\mathrm{T}} R_u(i) \Phi_{\mathrm{f}}(i)\hat{\theta} \\ & + \lambda^k (\hat{\theta} - \theta(0))^{\mathrm{T}} R_\theta (\hat{\theta} - \theta(0)), \end{aligned} \qquad (39)$$

where $\lambda \in (0,1]$ is the forgetting factor, R_θ is positive definite, and, for all $i \geq 1$, $R_z(i)$ is positive definite and $R_u(i)$ is positive semidefinite. The performance variable and control input weighting matrices $R_z(i)$ and $R_u(i)$ are time-dependent and thus may depend on present and past values of y, z, and u. Recursive minimization of (39) is used to update the controller coefficient vector $\hat{\theta}$. The following result uses recursive least squares to obtain the minimizer of (39).

Proposition: Let $P(0) = R_\theta^{-1}$. Then, for all $k \geq 1$, the retrospective cost function (39) has the unique global minimizer $\theta(k+1) = \hat{\theta}_{\mathrm{opt}}(k)$, which is given by

$$\begin{aligned} \theta(k+1) = & \theta(k) - P(k)\Phi_{\mathrm{f}}^{\mathrm{T}}(k+1)\Upsilon^{-1}(k+1) \\ & \cdot \big[\Phi_{\mathrm{f}}(k+1)\theta(k) + \bar{R}(k+1)R_z(k+1)(z(k+1) \\ & - u_{\mathrm{f}}(k+1))\big], \end{aligned} \qquad (40)$$

$$\begin{aligned} P(k+1) = & \frac{1}{\lambda}P(k) \\ & - \frac{1}{\lambda}P(k)\Phi_{\mathrm{f}}^{\mathrm{T}}(k+1)\Upsilon^{-1}(k+1)\Phi_{\mathrm{f}}(k+1)P(k), \end{aligned} \qquad (41)$$

where

$$\begin{aligned} \bar{R}(k) & \triangleq (R_z(k) + R_u(k))^{-1} \\ \Upsilon(k) & \triangleq \lambda \bar{R}(k) + \Phi_{\mathrm{f}}(k)P(k-1)\Phi_{\mathrm{f}}^{\mathrm{T}}(k). \end{aligned} \qquad (42)$$

For all examples in this paper, we initialize $\theta(0) = 0$ in order to reflect the absence of additional prior modeling information, and we use $R_z(i) = I_{l_z}$.

V. Decentralized RCAC

We use RCAC in a decentralized framework for broadband disturbance rejection and command following using the architecture shown in Figure 5. The measurement and performance variable to be minimized may be different for each subcontroller.

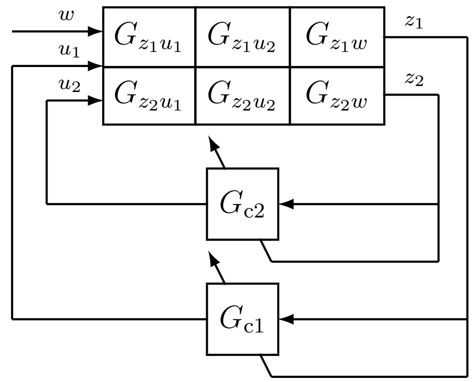


Fig. 5: Adaptive decentralized control.

We consider the following strategies for controller adaptation. One-controller-at-a-time (1CAT) is an adaptation strategy where one controller adapts while the coefficients of all other controllers are frozen. In concurrent adaptation, all controllers adapt at the same time.

To prevent G_{c1} and G_{c2} from canceling a NMP zero of the effective plant with a controller pole, the NMP zeros of the respective effective plants, given by (25) and (27), are made available to each subcontroller by making them part of N_{f}.

VI. Examples

Example 1. *Two-Mode Oscillator.* The continuous-time two-mode, undamped oscillator is given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & -0.05 \\ 0 & 0 \\ -0.05 & 1 \end{bmatrix}, \qquad (43)$$

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}^{\mathrm{T}}. \qquad (44)$$

We discretize (43)–(44) with sampling period $h = 1$ sec, and let $w \sim N(0, 0.045^2)$. We apply RCAC with 1CAT adaptation using the architecture shown in Figure 6.

Each subcontroller is provided the NMP zeros of the effective plant, given by (25). Specifically, at each step, we construct G_{f} for G_{c1} by choosing N_{f1} to be the unstable roots of $D_{\mathrm{c2}}N_{y_1u_1}$, and for G_{c2}, by choosing N_{f2} to be the unstable roots of $D_{\mathrm{c1}}N_{y_2u_2}$.

For G_{c1} we set $R_\theta = 10000$, $R_u = 0$, $n_{\mathrm{c}} = 4$, and for G_{c2} we set $R_\theta = 10$, $R_u = 0$, $n_{\mathrm{c}} = 4$. For the first 6000 steps G_{c1} is allowed to adapt while G_{c2} is fixed at zero. After 6000 steps the gains of G_{c1} are fixed,and G_{c2} is allowed to adapt until convergence is reached. Figure 7a shows closed-loop frequency response.

The same procedure is repeated with G_{c2} adapted first, followed by G_{c1}, as shown in Figure 7b. Here, for G_{c1} we set $R_\theta = 10$, $R_u = 0$, $n_{\mathrm{c}} = 4$, and for G_{c2} we set $R_\theta = 0.1$, $R_u = 0$, $n_{\mathrm{c}} = 4$.

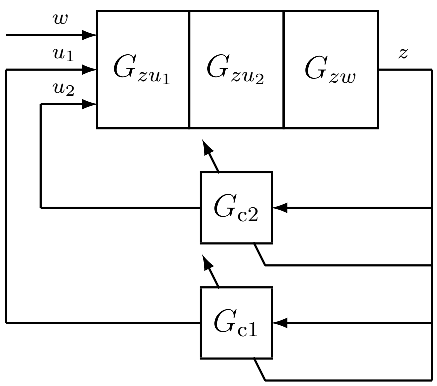
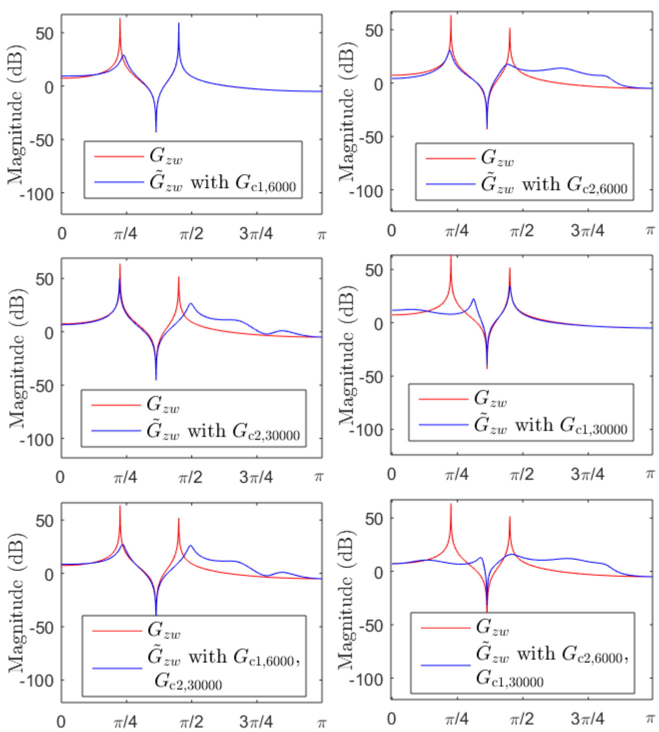


Fig. 6: Decentralized disturbance rejection of a two-mode oscillator.



(a) (b)

Fig. 7: Example 1: (a) Bode gain plots for 1CAT adaptation of G_{c1}, then G_{c2}, and (b) Bode gain plots for 1CAT adaptation of G_{c2}, then G_{c1}.

Note that, in both cases, the two subcontrollers are able to attenuate the disturbance more effectively when working together than in the case where each subcontroller is used alone. This example demonstrates an advantage of using two subcontrollers with distributed control authority over using a single subcontroller despite the fact that the subcontrollers are not interconnected. ■

Example 2. *Decentralized Position and Shape Control of a 2DOF Lumped Flexible Body.* A 2DOF lumped flexible body is modeled as two masses m_1 and m_2 connected by a spring and damper with coefficients k and b, respectively, as shown in Figure 8. G_{c1} and G_{c2} can apply forces u_1 and

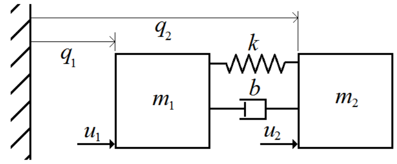


Fig. 8: 2DOF lumped flexible body.

u_2 to m_1 and m_2, respectively. This architecture is shown in Figure 9. The objective is to control the velocity of both masses as well as the separation between them using r_1 and r_2 in the presence of the disturbance $w \sim N(0, 0.09^2)$.

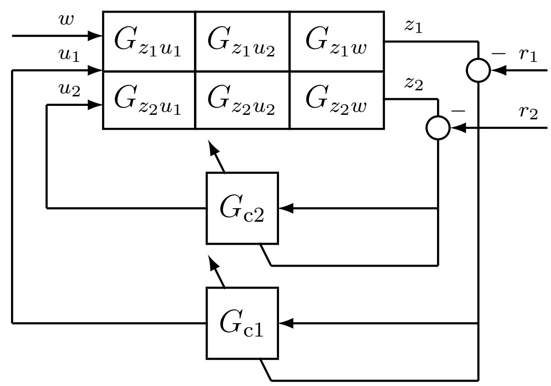


Fig. 9: Block diagram for Example 2.

A continuous-time model of the dynamics is given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/m_1 & -b/m_1 & k/m_1 & b/m_1 \\ 0 & 0 & 0 & 1 \\ k/m_2 & b/m_2 & -k/m_2 & -b/m_2 \end{bmatrix}, \tag{45}$$

$$B = \begin{bmatrix} 0 & 0 \\ 1/m_1 & 0 \\ 0 & 0 \\ 0 & 1/m_2 \end{bmatrix}, \tag{46}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}}, \tag{47}$$

where

$$x = \begin{bmatrix} q_1 & \dot{q}_1 & q_2 & \dot{q}_2 \end{bmatrix}^{\mathrm{T}}, \quad y = z = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \tag{48}$$

where $m_1 = 0.1$ kg, $m_2 = 0.12$ kg, $k = 0.08$ N/m, and $b = 0.05$ N-s/m. Note that A is unstable due to the rigid body mode. This model is discretized with $h = 0.01$ sec, and used with concurrent RCAC with the ramp commands

$$r_1(k) = 5hk, \quad r_2(k) = 5hk - 20. \tag{49}$$

These ramp commands specify the average velocity of the masses 5 m/s as well as their separation 20 m. Note that each subcontroller has no knowledge of the position and velocity of the other mass as well as the force applied by the other subcontroller. Also note that the performance objectives of position and shape control are not feasible using only one subcontroller.

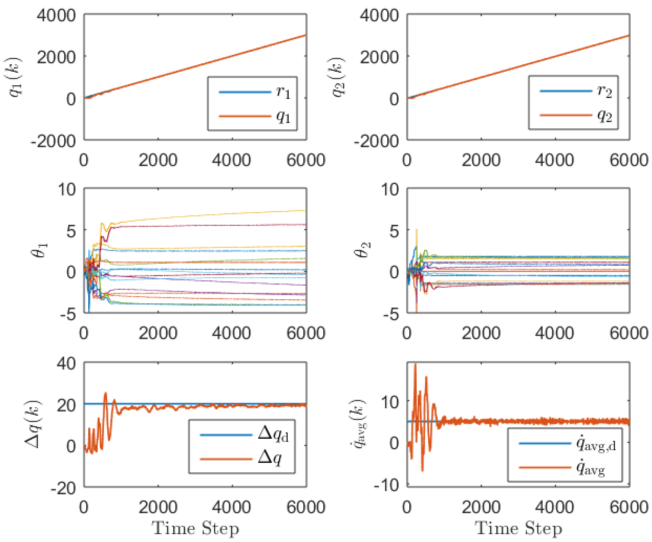


Fig. 10: Example 2: Command following for ramp position commands using decentralized RCAC with concurrent adaptation. Also shown is separation Δq and average velocity $\dot{q}_{\rm avg}$.

We set $R_\theta = 10$, $R_u = 200$, $n_{\rm c} = 8$ for both subcontrollers. For $G_{\rm c1}$ and $G_{\rm c2}$ we set $G_{\rm f}$ as the first 10 Markov parameters of the transfer functions G_{z1u1} and G_{z2u2}, respectively. Figure 7 shows the transient and asymptotic response achieved by RCAC adaptive decentralized control with concurrent adaptation. It can be observed that, in the presence of the disturbance, the subcontrollers are able to follow ramp commands, thus achieving the position and shape objectives. ■

VII. Conclusions

Adaptive decentralized control provides the means for allowing each subcontroller to adapt to the effective plant consisting of the actual plant and the remaining subcontrollers. We showed that, for two subcontrollers with either one sensor or one actuator, unstable subcontroller poles appear as nonminimum-phase (NMP) channel zeros in the effective plant. By accounting for these NMP zeros with one-controller-at-a-time (1CAT) adaptation, we used retrospective cost adaptive control (RCAC) for broadband disturbance rejection. We then considered a command-following problem involving decentralized control of a two-mass system with flexible coupling. In this case, the goal was to command the average velocity of the masses as well as their separation. Since both controllers are needed to achieve these objectives, 1CAT is not feasible. We thus used RCAC for decentralized adaptive control with concurrent controller adaptation.

The numerical experiments in this paper demonstrate the feasibility of using RCAC for adaptive decentralized control. The specific contribution of this paper in terms of NMP channels zeros is to provide a first step toward understanding the modeling requirements for adaptive decentralized control. Future research will consider the possibility of NMP channel zeros in the case of multiple sensors and actuators.

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