Damage Localization for Structural Health Monitoring Using Retrospective Cost Model Refinement

Anthony M. D'Amato[**]

University of Michigan, Ann Arbor, Michigan, 48109 , United States of America

Sunil. L. Kukreja [†]

NASA Dryden Flight Research Center, Edwards, California, 93523, United States of America

Dennis S. Bernstein [‡]

University of Michigan, Ann Arbor, Michigan, 48109, United States of America

Model refinement is directly applicable to health monitoring, where the goal is to determine changes in a system that may reflect damage. As data become available, an initial model for the undamaged system is updated; the updated model is then compared with the original model, and changes in the model are analyzed to deduce potential damage. We propose that a model that is physically representative of a structure, such as a finite element model, can be refined using empirical data. Furthermore, we propose that by constraining the refined model to a physical representation that damage in the system can be localized. We demonstrate the idea and algorithm on mass-spring-damper systems.

Nomenclature

M_i	Initial System Model
M_d	Damaged System Model
M_Δ	Correction Model
M	Mass Matrix
K	Stiffness Matrix
C	Damping Matrix

I. Introduction

Structural health monitoring (SHM) techniques are used to detect and diagnose various types of damage in civil and aeroelastic structures. Among the many applications of these techniques, commercial and government organizations have an interest in maintaining aging aircraft. Moreover, space systems must consider global deterioration of satellites over time, due to exposure to extreme temperatures, cosmic radiation, atomic oxygen, and impacts with foreign objects.

Model refinement is directly applicable to health monitoring, where the goal is to determine changes in a system that may reflect damage. As data become available, an initial model for the undamaged system is updated; the updated model is then compared with the original model, and changes in the model are analyzed to deduce potential damage. If the model is based on spatial discretization, then the updated model may facilitate damage localization. Model refinement is variously known as model correction, empirical correction, model refinement, or model updating, and relevant literature includes.[1–12]

[*]NASA GRSP Fellow, Department of Aerospace Engineering, 1320 Beal, and AIAA student member.
[†]Engineer, DFRC-RC, and AIAA member.
[‡]Professor, Department of Aerospace Engineering, 1320 Beal Avenue, and AIAA member.

The proposed project focuses on the case in which the initial model may include components that are acknowledged to have some uncertainty. For example, the initial model represents some nominal condition, where the true model has been damaged. The goal of model refinement is then to use data to improve the accuracy of the initial model by identifying the unknown subsystem.

The unknown physics of a subsystem may range from the simplest case of an unknown parameter (such as a diffusion constant), to a multivariable spatially dependent static mapping (such as a damping or stiffness matrix). The novel idea of this research is to constrain the identified subsystem to physically meaningful realizations. We hypothesis that by constraining these realizations, specific types of system changes can be quantified, such as reduction in stiffness or damping, and therefore in a multi-degree-of-freedom model the damage might also be localized.

II. Problem Formulation

Consider the mass-spring-damper structure shown in Figure 1. We choose this structure to represent a simple one dimensional finite element model. The goal of this research will be to detect local changes in stiffness and damping using only empirical data, namely, knowledge of the driving force u, and output data, specifically, position and/or velocity information.

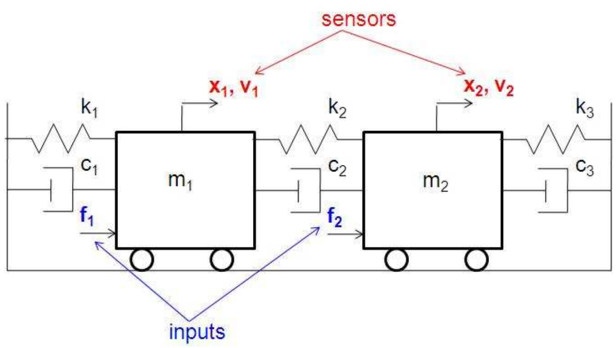


Figure 1. Mass-spring-damper structure. This structure is used to represent a 1 dimensional finite element model structure.

The equations of motion are given by

$$M\ddot{x} + C\dot{x} + Kx = F, \tag{1}$$

where

$$x = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix},\ M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix},\ C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix},$$
$$K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix},\ F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} u. \tag{2}$$

The mass-spring-damper system can be represented in state space form as

$$A = \begin{bmatrix} 0_{2\times 2} & I_{2\times 2} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0_{2\times 1} & M^{-1}F \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = 0, \tag{3}$$

where the continuous time transfer function is

$$M_{\text{continous}} = C[Is - A]^{-1}B. \tag{4}$$

In practice (4) is discretized, thus all models prestned are discrete-time realizations. For the purpose of this research we consider the damaged system to have a discrete time model M_d and an initial, or undamaged model M_i.

Consider the adaptive model refinement scheme shown in Figure 2, where M_i is modified using M_Δ to match M_d.

We seek to identify the feedback term M_Δ shown in Figure 2 using a given initial model M_i. The objective is to identify M_Δ such that the resulting closed-loop model is equivalent to the damaged system.

$$M_d = \frac{M_i}{1 - M_i M_\Delta} \tag{5}$$

As shown in Figure 2, we use an adaptive feedback model structure in order to identify M_Δ. To achieve model matching, we minimize the performance variable z in the presence of the identification signal w. In particular, we use the retrospective correction optimization (RCO) adaptive control algorithm given in.[14] The only signal available to the controller is the plant output y. This problem setup is a minor variation of the approach used in.[9]

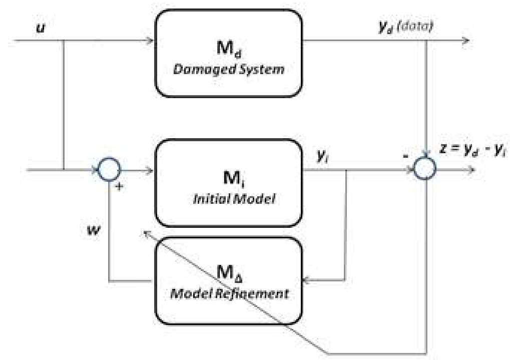


Figure 2. Adaptive Model Refinement Scheme. This adaptive model refinement setup uses a linear fractional transformation M_Δ, to refine the initial model M_i. The goal is to reduce the performance variable z to zero, which means that the closed loop initial model and model refinement block have accurately approximated the damaged system.

III. Retrospective Cost Optimization

We now review the RCO adaptive control algorithm and show how it is used to identify linear time-invariant dynamic systems. A detailed discussion of RCO is presented in.[14]

RCO depends on several parameters that are selected *a priori*. Specifically, n_c is the estimated controller order, $p \geq 1$ is the data window size, and μ is the number of Markov parameters.

The adaptive update law is based on a quadratic cost function, which involves a time-varying weighting parameter $\alpha(k) > 0$, referred to as the *learning rate* since it affects the convergence speed of the adaptive control algorithm.

We use an exactly proper time-series controller of order n_c such that the control $w(k)$ is given by

$$w(k) = \sum_{i=1}^{n_c} M_i(k) w(k-i) + \sum_{i=0}^{n_c} N_i(k) \hat{v}(k-i), \tag{6}$$

where $M_i \in \mathbb{R}^{l_w \times l_w}$, $i = 1, \ldots, n_c$, and $N_i \in \mathbb{R}^{l_w \times l_v}$, $i = 0, \ldots, n_c$, are given by an adaptive update law. The control can be expressed as

$$w(k) = \theta(k) \psi(k), \tag{7}$$

where

$$\theta(k) \triangleq \begin{bmatrix} N_0(k) & \cdots & N_{n_c}(k) & M_1(k) & \cdots & M_{n_c}(k) \end{bmatrix} \tag{8}$$

is the *controller parameter block matrix* and the *regressor vector* $\psi(k)$ is given by

$$\psi(k) \triangleq \begin{bmatrix} \hat{v}(k) \\ \vdots \\ \hat{v}(k-n_\mathrm{c}) \\ w(k-1) \\ \vdots \\ w(k-n_\mathrm{c}) \end{bmatrix} \in \mathbb{R}^{n_\mathrm{c}l_w+(n_\mathrm{c}+1)l_v}.$$

For positive integers p and μ, we define the *extended performance vector* $Z(k)$ and the *extended control vector* $W(k)$ by

$$Z(k) \triangleq \begin{bmatrix} z(k) \\ \vdots \\ z(k-p+1) \end{bmatrix}, \quad W(k) \triangleq \begin{bmatrix} w(k) \\ \vdots \\ w(k-p_\mathrm{c}+1) \end{bmatrix}, \tag{9}$$

where $p_\mathrm{c} \triangleq \mu + p$.

From (7), it follows that the extended control vector $W(k)$ can be written as

$$W(k) \triangleq \sum_{i=1}^{p_\mathrm{c}} L_i \theta(k-i+1)\psi(k-i+1), \tag{10}$$

where

$$L_i \triangleq \begin{bmatrix} 0_{(i-1)l_w \times l_w} \\ I_{l_w} \\ 0_{(p_\mathrm{c}-i)l_w \times l_w} \end{bmatrix} \in \mathbb{R}^{p_\mathrm{c}l_w \times l_w}. \tag{11}$$

We define the *surrogate performance vector* $\hat{Z}(\hat{\theta}(k), k)$ by

$$\hat{Z}(\hat{\theta}(k), k) \triangleq Z(k) - \bar{B}_{zw}\left(W(k) - \hat{W}(k)\right), \tag{12}$$

where

$$\hat{W}(k) \triangleq \sum_{i=1}^{p_\mathrm{c}} L_i \hat{\theta}(k)\psi(k-i+1), \tag{13}$$

and $\hat{\theta}(k) \in \mathbb{R}^{l_w \times [n_\mathrm{c}l_w+(n_\mathrm{c}+1)l_v]}$ is the *surrogate controller parameter block matrix.* The block-Toeplitz *surrogate control matrix* $\bar{B}_{zw}$ is given by

$$\bar{B}_{zw} \triangleq \begin{bmatrix} 0_{l_z\times l_w} & \cdots & 0_{l_z\times l_w} & H_d & \cdots & H_\mu & 0_{l_z\times l_w} & \cdots & 0_{l_z\times l_w} \\ 0_{l_z\times l_w} & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0_{l_z\times l_w} & \vdots \\ 0_{l_z\times l_w} & \cdots & 0_{l_z\times l_w} & 0_{l_z\times l_w} & \cdots & 0_{l_z\times l_w} & H_d & \cdots & H_\mu \end{bmatrix}.$$

where the *relative degree* d is the smallest positive integer i such that the ith Markov parameter H_i of $\hat{\mathcal{L}}_\mathrm{m} \triangleq C_0 A_0^{i-1} B_0$ is nonzero. The leading zeros in the first row of $\bar{B}_{zw}$ account for the nonzero relative degree d. The algorithm places no constraints on either the value of d or the rank of H_d or $\bar{B}_{zw}$.

Furthermore, we define

$$D(k) \triangleq \sum_{i=1}^{n_\mathrm{c}+\mu-1} \psi^\mathrm{T}(k-i+1) \otimes L_i, \tag{14}$$

$$f(k) \triangleq Z(k) - \bar{B}_{zw}W(k). \tag{15}$$

$$\tag{16}$$

We now consider the cost function

$$J(\hat{\theta}, k) \triangleq \hat{Z}^{\mathrm{T}}(\hat{\theta}, k) R_1(k) \hat{Z}(\hat{\theta}, k) + \mathrm{tr}\left[R_2(k)\left(\hat{\theta} - \theta(k)\right)^{\mathrm{T}} R_3(k)\left(\hat{\theta} - \theta(k)\right)\right], \tag{17}$$

where $R_1(k) \triangleq I_{pl_z}$, $R_2(k) \triangleq \alpha(k) I_{n_c(l_w+l_v)}$, and $R_3(k) \triangleq I_{l_w}$. and $\alpha(k)$ the positive scalar is the learning rate.

Substituting (12) and (13) into (17), J is written as the quadratic form

$$J(\hat{\theta}, k) = c(k) + b^{\mathrm{T}} \mathrm{vec}\,\hat{\theta} + \left(\mathrm{vec}\,\hat{\theta}\right)^{\mathrm{T}} A(k) \mathrm{vec}\,\hat{\theta}, \tag{18}$$

where

$$A(k) = D^{\mathrm{T}}(k) D(k) + \alpha(k) I, \tag{19}$$

$$b(k) = 2D^{\mathrm{T}}(k) f(k) - 2\alpha(k) \mathrm{vec}\,\theta(k), \tag{20}$$

$$c(k) = f(k)^{\mathrm{T}} R_1(k) f(k) + \mathrm{tr}\left[R_2(k)\theta^{\mathrm{T}}(k) R_3(k)\theta(k)\right]. \tag{21}$$

Since $A(k)$ is positive definite, $J(\hat{\theta}, k)$ has the strict global minimizer

$$\hat{\theta}(k) = \frac{1}{2}\mathrm{vec}^{-1}(A(k)^{-1} b(k)). \tag{22}$$

The controller gain update law is

$$\theta(k+1) = \hat{\theta}(k). \tag{23}$$

The key feature of the adaptive control algorithm (7) is the surrogate performance variable $Z(k)$ based on the difference between the actual past control inputs $W(k)$ and the recomputed past control inputs based on the current control law $\hat{W}(k)$. The parameter α is chosen to be as small as possible while guaranteeing that $A(k)$ is positive definite.

IV. Preliminary Results

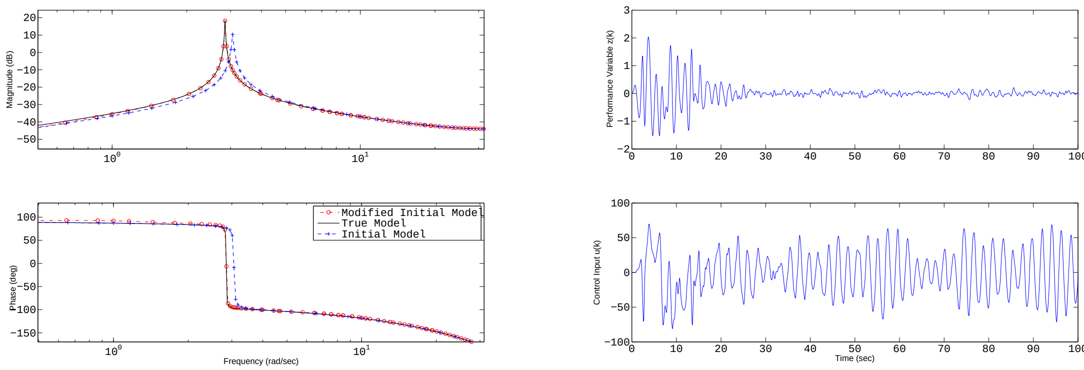
Thus far we have demonstrated on fictitious examples that changes in stiffness can be accounted for using model refinement. The next step in our proposed research is to constrain the realization of the unknown subsystem to yield the change in the global stiffness matrix. This will in turn provide localized stiffness information.

A. 1-DOF System

Our first example is a single degree of freedom case where, $m_2 = k_2 = k_3 = c_2 = c_3 = 0$. This represents the case when the total structure is represented by a single stiffness and damping value, that is, the stiffness and damping matrix are scalar. The initial model represents the nominal system, the true model represents the damaged model, has less stiffness and damping than the initial model. The modified initial model is the identified subsystem in feedback with the initial model. Figure 3(a) is a frequency response comparison of the three systems. In this case, the initial model must be corrected for changes in both stiffness and damping. Figure 6 is a comparison of the impulse responses characteristics. We note that these metrics show favorable results for correcting initial models for stiffness and damping uncertainty.

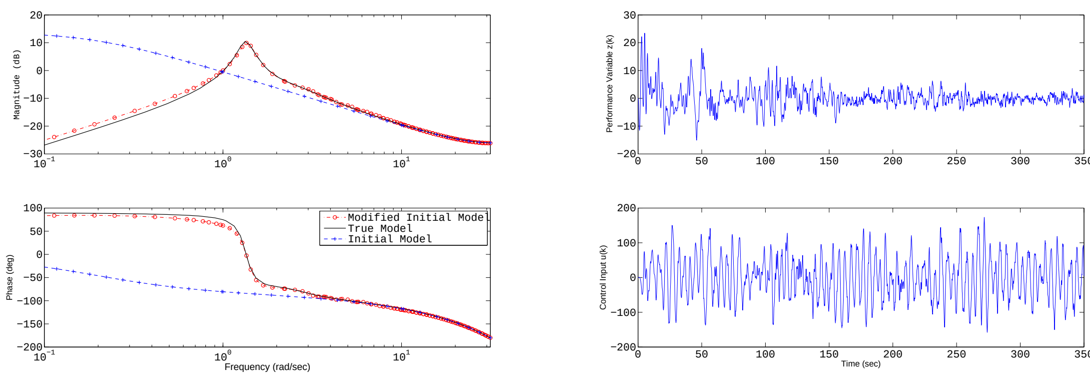
B. 2-DOF System

We now consider the full 2-DOF case. We assume that initial model contains the exact damping coefficients, but the stiffness coefficients are initialized to zero. The initial model represents the nominal system, the true model represents the damaged model, has less stiffness and damping than the initial model. The modified initial model is the identified subsystem in feedback with the initial model. From Figure 5(a), the lack of stiffness information in the initial model is evident from the frequency response. However, the model is adapted to match the true system as in the first example.



(a) Frequency Response Comparison (b) Performance Variable

Figure 3. Comparing the frequency responses of the models, we demonstrate that changes in stiffness and damping can be accounted for. Furthermore, monitoring the performance z yields insight into the performance of the RCO algorithm.



(a) Frequency Response Comparison (b) Performance Variable

Figure 5. Comparing the frequency responses of the models, we demonstrate that changes in stiffness and damping can be accounted for. Furthermore, monitoring the performance z yields insight into the performance of the RCO algorithm.

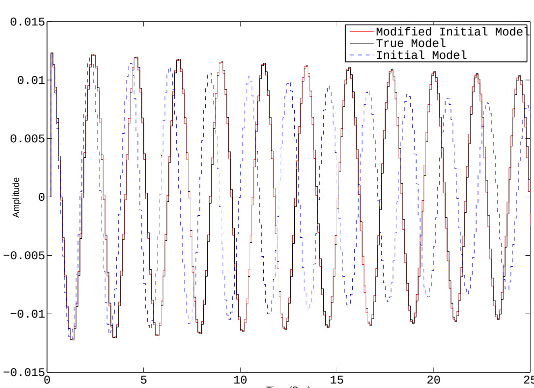
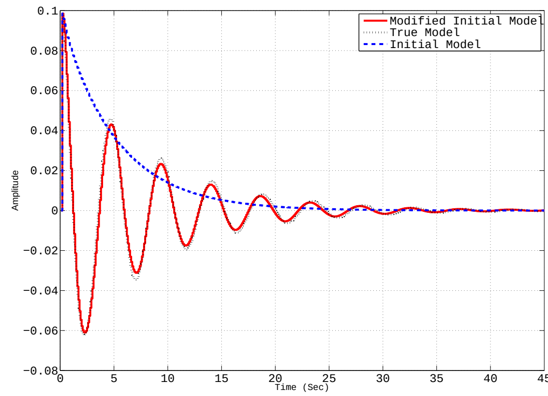


Figure 4. Impulse Response Comparison. We test the accuracy of the refined model by comparing the impulse of the true damaged model with the initial and refined initial models. The refined initial model is a good approximation of the damaged system.

Figure 6 is a comparison of the impulse responses. In this example, the three stiffness terms were initialize as zero. Since it was assumed that the damping coefficients were known, then the identified feedback term reflects the unknown global stiffness in the system.



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V. Conclusion

In previous work, model refinement was used in the scope of structural health monitoring to detect global damage in a structural member. Furthermore, it was demonstrated that system models could be updated using model refinement techniques to reflect the changes in system dynamics due to the global damage.

We now propose that using model refinement techniques with an assumed model structure, namely, a mass-spring-damper system that damage can be characterized in terms of changes in stiffness and damping and also localized spatially in the model. We propose to use a fixed initial model with retrospective cost optimization to identify a global stiffness matrix, in the presence of known mass and damping. We then compare the identified global stiffness matrix to a nominal stiffness matrix to determine the specific location of stiffness reduction in the system. Thus far we have demonstrated that non-physical realizations of the global stiffness for 2-DOF systems can be obtained. We are currently working to identify physical realizations and therefore local stiffness information for a given model.

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