2013 American Control Conference (ACC)
Washington, DC, USA, June 17-19, 2013

Adaptive Spacecraft Attitude Control with Reaction Wheel Actuation

Gerardo Cruz[1] and Dennis S. Bernstein[2]

***Abstract*— We apply retrospective cost adaptive control (RCAC) to spacecraft attitude control using reaction-wheel actuators. The controller is applied to the motion-to-rest (M2R) maneuver, where the spacecraft has an arbitrary initial attitude and angular-velocity and the objective is to bring the body to rest at a specified inertial attitude. First, we show that RCAC can complete the M2R maneuver using full information of the mass properties of the spacecraft. Then, we remove the spacecraft and reaction-wheel inertia information from the controller and examine the closed-loop settling time for off-nominal spacecraft inertias. Thus, the controller uses limited knowledge of the system. Finally, the algorithm is tested in the presence of disturbances, unknown actuator misalignment, and noise and bias in the angular-velocity measurement.**

I. INTRODUCTION

Spacecraft attitude control is a nonlinear problem involving the special orthogonal group SO(3) of 3×3 rotation matrices. Thus, adaptive control is useful when a sufficiently accurate model of the spacecraft is not available for fixed-gain controller synthesis. We take a different approach to the problem of attitude control for reaction-wheel-actuated spacecraft motivated by [1], which uses linear adaptive controllers based on parameterizations of the equations of motion.

Retrospective cost adaptive control (RCAC) is a multi-input, multi-output direct adaptive controller that requires limited modeling information. The algorithm has been applied to nonlinear plants such as multiple linkages [5], [6]. In this paper we apply RCAC to spacecraft attitude control. We extend the thruster control approach in [7] by utilizing reaction wheels with momentum-storage capabilities as actuators. The momentum stored in the wheels introduces additional nonlinearities in Euler's equation as shown in [8]. These nonlinearities present an additional challenge for the linear compensator produced by RCAC. The goal of this work is to control the spacecraft without knowledge of the spacecraft or reaction-wheel inertias. Specifically, we evaluate the closed-loop performance for motion-to-rest (M2R) maneuvers. For these maneuvers the spacecraft has an arbitrary initial attitude and angular-velocity, and the objective is to bring the spacecraft to rest at a specified inertial attitude. We examine the relation between the spacecraft inertia, the reaction-wheel inertia, and the closed-loop settling time. Furthermore, we investigate the effect of inertially constant disturbance torques as well as noise and bias on the angular-velocity measurement.

[1]Graduate Student, Department of Aerospace Engineering, University of Michigan, email: gecruz@umich.edu

[2]Professor, Department of Aerospace Engineering, University of Michigan, email: dsbaero@umich.edu

II. NONLINEAR EQUATIONS OF MOTION

For the spacecraft model, we consider a single rigid body controlled by torque actuators, in this case reaction wheels, which provide on-board momentum storage. We consider only the rotational motion of the spacecraft and not the translational motion of the spacecraft's center of mass. We assume that a body-fixed frame is defined for the spacecraft, whose origin is chosen to be the center of mass, and that an inertial frame is specified for determining the attitude of the spacecraft. The equations of motion for a spacecraft actuated by reaction wheels are given by Euler's equation, Poisson's equation, and the reaction-wheel dynamics, which are given, respectively, by

$$J\dot{\omega} = (J\omega - B_{\rm sc}\nu) \times \omega + B_{\rm sc}u + z_{\rm dist}, \tag{1}$$

$$\dot{R} = R\omega^{\times}, \tag{2}$$

$$\dot{\nu}_i = R_{W_i} e_{W_i} u_i, \text{ for } i = 1, 2, \dots n_w, \tag{3}$$

where $\omega \in \mathbb{R}^3$ is the angular-velocity of the spacecraft frame with respect to the inertial frame resolved in the spacecraft frame, $J \in \mathbb{R}^{3\times3}$ is the constant, positive-definite inertia matrix of the spacecraft, the columns of $B_{\rm sc} \in \mathbb{R}^{3\times n_w}$ are given by

$$b_i = -\alpha_i R_{W_i} e_{W_i}, \text{ for } i = 1, 2, \dots n_w, \tag{4}$$

$\nu \in \mathbb{R}^{n_w}$ is a vector composed of the angular-rate of each reaction-wheel with respect to the spacecraft frame, $u \in \mathbb{R}^{n_w}$ is a vector composed of the control input to each wheel, which has units of angular acceleration, and $z_{\rm dist} \in \mathbb{R}^3$ represents all internal and external disturbance torques applied to the spacecraft aside from control torques. Furthermore, $R \in \mathbb{R}^{3\times3}$ is the proper orthogonal matrix (that is, the rotation matrix) that transforms the components of a vector resolved in the spacecraft frame into the components of the same vector resolved in the inertial frame and $\omega^{\times}$ is the skew-symmetric cross-product matrix of ω. Finally, α_i is the positive moment of inertia about the rotation axis of the ith reaction-wheel, $R_{W_i} \in \mathbb{R}^{3\times3}$ is the rotation matrix that transforms the components of a vector resolved in the ith reaction-wheel frame into the components of the same vector resolved in the spacecraft body frame, and $e_{W_i} \in \mathbb{R}^3$ is the unit vector that represents the direction of the ith reaction-wheel's spin axis resolved in the ith wheel frame. For convenience, in (1), (2), and (3) we omit the argument t, recognizing that ω, R, ν, u, and $z_{\rm dist}$ are time-varying quantities.

A. *Error Dynamics*

The objective of the attitude control problem is to determine control inputs such that the spacecraft attitude given by R follows a commanded attitude trajectory given by a possibly time-varying C^1 rotation matrix $R_\mathrm{d}(t)$. For $t \ge 0$, $R_\mathrm{d}(t)$ is given by

$$\dot{R}_\mathrm{d}(t) = R_\mathrm{d}(t)\omega_\mathrm{d}(t)^\times, \quad (5)$$
$$R_\mathrm{d}(0) = R_{\mathrm{d}0}, \quad (6)$$

where ω_d is the desired, possibly time-varying angular velocity of the spacecraft relative to the inertial frame. The attitude error, that is, the rotation between $R(t)$ and $R_\mathrm{d}(t)$, is given by

$$\tilde{R} \triangleq R_\mathrm{d}^\mathrm{T} R, \quad (7)$$

which satisfies Poisson's equation

$$\dot{\tilde{R}} = \tilde{R}\tilde{\omega}^\times, \quad (8)$$

where the angular-velocity error $\tilde{\omega}$ is defined by

$$\tilde{\omega} \triangleq \omega - \tilde{R}^\mathrm{T}\omega_\mathrm{d}. \quad (9)$$

We rewrite (1) in terms of the angular-velocity error as

$$\begin{aligned} J\dot{\tilde{\omega}} =& [J(\tilde{\omega} + \tilde{R}^\mathrm{T}\omega_\mathrm{d}) + B_\mathrm{sc}\nu] \times (\tilde{\omega} + \tilde{R}^\mathrm{T}\omega_\mathrm{d}) \\ &+ J(\tilde{\omega} \times \tilde{R}^\mathrm{T}\omega_\mathrm{d} - \tilde{R}^\mathrm{T}\dot{\omega}_\mathrm{d}) + B_\mathrm{sc}u + z_\mathrm{dist}. \end{aligned} \quad (10)$$

III. The RCAC Algorithm

RCAC is a discrete-time controller that minimizes the command-following error corresponding to the performance variable z. The algorithm does not require detailed plant information; instead, RCAC uses knowledge of the Markov parameters of the transfer function G_{zu} which relates the input u to the performance z.

Consider the MIMO discrete-time system

$$x(k) = Ax(k-1) + Bu(k-1), \quad (11)$$
$$y_0(k) = E_1 x(k), \quad (12)$$
$$z(k) = y_0(k) - E_0 r(k), \quad (13)$$

where $x(k) \in \mathbb{R}^{l_x}$, $y_0(k) \in \mathbb{R}^{l_y}$, $z(k) \in \mathbb{R}^{l_z}$, $u(k) \in \mathbb{R}^{l_u}$, $r(k) \in \mathbb{R}^{l_r}$, and $k \ge 0$. For $i \ge 1$, define the Markov parameter of G_{zu}

$$\mathcal{H}_i \triangleq E_1 A^{i-1} B. \quad (14)$$

Let n be a positive integer. Then, for all $k \ge n$,

$$x(k) = A^n x(k-n) + \sum_{i=1}^{n} A^{i-1} B u(k-i) \quad (15)$$

thus

$$z(k) = E_1 A^n x(k-n) + HU(k-1) - E_0 r(k), \quad (16)$$

where

$$H \triangleq \begin{bmatrix} \mathcal{H}_1 & \cdots & \mathcal{H}_n \end{bmatrix} \in \mathbb{R}^{l_z \times n l_u}$$

and

$$U(k-1) \triangleq \begin{bmatrix} u(k-1) \\ \vdots \\ u(k-n) \end{bmatrix} \in \mathbb{R}^{n l_u}.$$

We divide the matrices H and $U(k-1)$ into known and unknown parts so that

$$HU(k-1) = H''U''(k-1) + H'U'(k-1). \quad (17)$$

where $H' \in \mathbb{R}^{l_z \times l_h l_u}$ and $H'' \in \mathbb{R}^{l_z \times l_u(n-l_h)}$ are the known and unknown Markov parameters respectively and $U' \in \mathbb{R}^{l_h l_u}$ and $U'' \in \mathbb{R}^{(n-l_h)l_u}$ are the corresponding control inputs. Next, we collect the unknown parts of the system into

$$\mathcal{S}(k) \triangleq E_1 A^n x(k-n) - E_0 r(k) + H''U''(k-1). \quad (18)$$

and rewrite (16) using (17) as

$$z(k) = \mathcal{S}(k) + H'U'(k-1). \quad (19)$$

A. Retrospective Cost

We replace the controls in U' in (19) with the retrospective controls $\hat{U}'$ and define the *retrospective performance*,

$$\hat{z}(k) \triangleq \mathcal{S}(k) + H'\hat{U}'(k-1), \quad (20)$$

we substract (19) from (20) and solve for $\hat{z}(k)$ which yields

$$\hat{z}(k) = z(k) - H'U'(k-1) + H'\hat{U}'(k-1). \quad (21)$$

Then, we define the *retrospective cost function*

$$J_z(\hat{U}'(k-1), k) \triangleq \hat{z}^\mathrm{T}(k) R(k) \hat{z}(k), \quad (22)$$

where $R(k) \in \mathbb{R}^{l_z \times l_z}$ is a positive-definite performance weighting. The optimized control values $\hat{U}'(k-1)$ are then used to compute the next control input $u(k)$. Given a full column rank H', the global minimizer for the quadratic cost function in (22) is given by

$$\begin{aligned} &\hat{U}'(k-1) = \\ &\left[H'^\mathrm{T} R(k) H'\right]^{-1} H'^\mathrm{T} R(k) \left[H'U'(k-1) - z(k)\right]. \end{aligned} \quad (23)$$

B. Controller Construction

We design a strictly proper time-series controller of order n_c which can be written as

$$u(k) = \theta(k)\phi(k-1), \quad (24)$$

where

$$\theta(k) \triangleq [M_1(k) \cdots M_{n_\mathrm{c}}(k) \; N_1(k) \cdots N_{n_\mathrm{c}}(k)] \in \mathbb{R}^{l_u \times n_\mathrm{c}(l_u + l_z)}, \quad (25)$$

for $M_i(k) \in \mathbb{R}^{l_u \times l_u}$, $N_i(k) \in \mathbb{R}^{l_u \times l_z}$, and

$$\phi(k-1) \triangleq \begin{bmatrix} u(k-1) \\ \vdots \\ u(k-n_\mathrm{c}) \\ z(k-1) \\ \vdots \\ z(k-n_\mathrm{c}) \end{bmatrix} \in \mathbb{R}^{n_\mathrm{c}(l_u + l_z)}. \quad (26)$$

Next, we define the *Recursive Least Squares* cumulative cost function

$$J_{\mathrm{R}}(\theta(k)) \triangleq \left[(\theta(k)-\theta(0))\, P^{-1}(0)\, (\theta(k)-\theta(0))\right]^{\mathrm{T}} + \sum_{i=1}^{k} \|\phi^{\mathrm{T}}(i-2)\theta^{\mathrm{T}}(k) - \hat{u}^{\mathrm{T}}(i-1)\|^2, \quad (27)$$

where $P(0) \in \mathbb{R}^{n_c(l_u+l_z)\times n_c(l_u+l_z)}$ is the initial covariance matrix. The minimizer for (27) is given by

$$\theta^{\mathrm{T}}(k) \triangleq \theta^{\mathrm{T}}(k-1) + \frac{P(k-1)\phi(k-2)[\theta(k-1)\phi(k-2) - \hat{u}(k-1)]^{\mathrm{T}}}{[1+\phi^{\mathrm{T}}(k-2)P(k-1)\phi(k-2)]}. \quad (28)$$

where the error covariance $P(k)$ is updated by

$$P(k) \triangleq P(k-1) - \frac{P(k-1)\phi(k-2)\phi^{\mathrm{T}}(k-2)P(k-1)}{[1+\phi^{\mathrm{T}}(k-2)P(k-1)\phi(k-2)]}. \quad (29)$$

IV. RCAC Parameters for Attitude Control

RCAC requires a vector performance variable, thus, the attitude-error $\tilde{R}$ given by Poisson's equation (8) cannot be used directly. As in [7], we reformulate the attitude-error dynamics using the vector parameter S presented in [9].

For $i = 1, 2, 3$, let e_i denote the ith column of the 3×3 identity matrix, and let $A_{\mathrm{att}} = \mathrm{diag}(a_1, a_2, a_3)$ be a diagonal positive-definite matrix. Then

$$S \triangleq \sum_{1}^{3} a_i \left(\tilde{R}^{\mathrm{T}} e_i\right) \times e_i \in \mathbb{R}^3 \quad (30)$$

is a vector measure of attitude-error. Thus, we define the performance variable for the attitude control problem as

$$z \triangleq \begin{bmatrix} \tilde{\omega} \\ S \end{bmatrix} \in \mathbb{R}^6. \quad (31)$$

A. Baseline Markov Parameter

We utilize the procedure outlined in [7] to compute a baseline Markov parameter for the spacecraft system through linearization of the error dynamics in (8) and (10). We parameterize the SO(3) attitude-error $\tilde{R}$ using the vector

$$\tilde{r} \triangleq \begin{bmatrix} \tilde{r}_1 & \tilde{r}_2 & \tilde{r}_3 \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^9, \quad (32)$$

where $\tilde{r}_i \in \mathbb{R}^{1\times 3}$ is the ith row of the rotation matrix $\tilde{R}$. Then, the attitude-error dynamics in (8) become

$$\dot{\tilde{r}} = \begin{bmatrix} -\tilde{\omega}^{\times} & & \\ & -\tilde{\omega}^{\times} & \\ & & -\tilde{\omega}^{\times} \end{bmatrix} \tilde{r}. \quad (33)$$

We rewrite Euler's equation in (9) as

$$J\dot{\tilde{\omega}} = [J(\tilde{\omega} + \mathcal{D}(\omega_{\mathrm{d}})\tilde{r}) + B_{\mathrm{sc}}\nu] \times [\tilde{\omega} + \mathcal{D}(\omega_{\mathrm{d}})\tilde{r}] + J[\tilde{\omega} \times \mathcal{D}(\omega_{\mathrm{d}})\tilde{r} - \mathcal{D}(\dot{\omega}_{\mathrm{d}})\tilde{r}] + B_{\mathrm{sc}}u + z_{\mathrm{dist}}. \quad (34)$$

The operator $\mathcal{D}(x) : \mathbb{R}^3 \to \mathbb{R}^{3\times 9}$ is

$$\mathcal{D}(x) \triangleq \begin{bmatrix} x_1 I_3 & x_2 I_3 & x_3 I_3 \end{bmatrix}, \quad (35)$$

where x_i for $i = 1, 2, 3$, is the ith component of x.

We construct a state vector by stacking the angular-velocity error $\tilde{\omega}$, the attitude-error $\tilde{r}$, and the reaction-wheel angular-rates ν. We define our system using this state vector and the rewritten error dynamics in (33) and (34) combined with the reaction wheel dynamics in (3).

We linearize the system by computing a Jacobian about the equilibrium $x_{\mathrm{e}}^{\mathrm{T}} = \begin{bmatrix} \tilde{\omega}_{\mathrm{e}}^{\mathrm{T}} & \tilde{r}_{\mathrm{e}}^{\mathrm{T}} & \nu_{\mathrm{e}}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$. The derivative of (34) with respect to $\tilde{\omega}$, $\tilde{r}$, and ν are

$$\frac{\partial \dot{\tilde{\omega}}}{\partial \tilde{\omega}} = J^{-1}\Big[-(\tilde{\omega}_e + \mathcal{D}(\omega_d)\tilde{r}_e)^{\times} J + [J(\tilde{\omega}_e + \mathcal{D}(\omega_d)\tilde{r}_e) + B_{\mathrm{sc}}\nu]^{\times}\Big] - (\mathcal{D}(\omega_d)\tilde{r}_e)^{\times}, \quad (36)$$

$$\frac{\partial \dot{\tilde{\omega}}}{\partial \tilde{r}} = J^{-1}\Big[-(\tilde{\omega}_e + \mathcal{D}(\omega_d)\tilde{r}_e)^{\times} J\mathcal{D}(\omega_d) + [J(\tilde{\omega}_e + \mathcal{D}(\omega_d)\tilde{r}_e) + B_{\mathrm{sc}}\nu]^{\times}\mathcal{D}(\omega_d)\Big] + \tilde{\omega}_e^{\times}\mathcal{D}(\omega_d) - \mathcal{D}(\dot{\omega}_d), \quad (37)$$

$$\frac{\partial \dot{\tilde{\omega}}}{\partial \nu} = -J^{-1}[\tilde{\omega}_e + \mathcal{D}(\omega_d)\tilde{r}_e]^{\times} B_{\mathrm{sc}}. \quad (38)$$

Next, we write the derivative for each row $\tilde{r}_i$ of $\tilde{R}$ as

$$\dot{\tilde{r}}_i = (\tilde{r}_i \times \tilde{\omega})^{\mathrm{T}}. \quad (39)$$

Then, the derivatives of (39) with respect to $\tilde{\omega}$, $\tilde{r}_j$, and ν are,

$$\frac{\partial \dot{\tilde{r}}_i}{\partial \tilde{\omega}} = -\tilde{r}_{e,i}^{\times}, \text{ for } i = 1, 2, 3, \quad (40)$$

$$\frac{\partial \dot{\tilde{r}}_i}{\partial \tilde{r}_j} = \left[-\tilde{\omega}^{\times}\frac{\partial \tilde{r}_i}{\partial \tilde{r}_j} + \tilde{r}_i^{\times}\frac{\partial \tilde{\omega}}{\partial \tilde{r}_j}\right]^{\mathrm{T}}, \quad (41)$$

$$\frac{\partial \dot{\tilde{r}}_i}{\partial \nu} = 0, \text{ for } i = 1, 2, 3. \quad (42)$$

where

$$\frac{\partial \tilde{r}_i}{\partial \tilde{r}_j} = \begin{cases} I_3, & i = j, \\ 0_{3\times 3}, & i \neq j, \end{cases}$$

$$\frac{\partial \tilde{\omega}}{\partial \tilde{r}_j} = \begin{cases} \mathcal{D}(\omega_d)\begin{bmatrix} I_3 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, & j = 1, \\ \mathcal{D}(\omega_d)\begin{bmatrix} 0 & I_3 & 0 \end{bmatrix}^{\mathrm{T}}, & j = 2, \\ \mathcal{D}(\omega_d)\begin{bmatrix} 0 & 0 & I_3 \end{bmatrix}^{\mathrm{T}}, & j = 3. \end{cases}$$

Finally, the Jacobian for the reaction-wheel dynamics (3) is

$$\frac{\partial \dot{\nu}}{\partial x} = 0. \quad (43)$$

We obtain the input matrix by differentiating (34), (39), and (3) with respect to the control input u which yields

$$B_{\mathrm{c}} = \begin{bmatrix} J^{-1}B_{\mathrm{sc}} \\ 0_{9\times n_w} \\ R_{W_1}e_{W_1} \\ \vdots \\ R_{W_{n_w}}e_{W_{n_w}} \end{bmatrix}. \quad (44)$$

We rewrite the attitude parameter S in terms of the attitude state $\tilde{r}$ to compute the output matrices.

$$S = -M_a R_a \tilde{r}, \quad (45)$$

where

$$M_a \triangleq \begin{bmatrix} e_1^\times & e_2^\times & e_3^\times \end{bmatrix}, R_a \triangleq \begin{bmatrix} a_1 I_3 & & \\ & a_2 I_3 & \\ & & a_3 I_3 \end{bmatrix}.$$

Thus,

$$C = \begin{bmatrix} I_3 & 0_{3\times 9} & 0_{3\times n_w} \\ 0_{3\times 3} & M_a & 0_{3\times n_w} \end{bmatrix}. \tag{46}$$

We obtain the discrete-time dynamics matrix from

$$A = e^{A_c h}, \tag{47}$$

where A_c is the Jacobian constructed from the derivatives in (36) - (43) and h is the controller time step. The discrete-time input matrix is given by

$$B = \int_0^h e^{A\tau} d\tau B_c. \tag{48}$$

The discrete-time output matrices are $E_1 = C$ and $D_d = 0$.

V. Numerical Examples

Consider a rigid body whose inertia matrix relative to its center of mass resolved in the body frame is

$$J_0 = \begin{bmatrix} 5 & -0.1 & -0.5 \\ -0.1 & 2 & 1 \\ -0.5 & 1 & 3.5 \end{bmatrix} \text{ kg-m}^2. \tag{49}$$

We use $n_w = 3$ reaction wheels whose rotational-axis moments of inertia are $\alpha_{0_i} = 0.1$ kg/m^2 for $i = 1, 2, 3$. The wheel spin axes are chosen such that $e_{W_i} = e_1$, for $i = 1, 2, 3$, are aligned orthogonally. Thus, the actuator matrix $B_{\rm sc}$ in (10) becomes

$$B_{\rm sc} = -0.1 I_3. \tag{50}$$

The spacecraft's initial motion is described by

$$\omega(0) = \frac{0.1}{\sqrt{3}} \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^{\rm T} \text{ rad/sec} \tag{51}$$

with the reaction-wheels initially at rest so that

$$\nu_i(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\rm T} \text{ rad/sec, for } i = 1, 2, 3. \tag{52}$$

The spacecraft's initial attitude is given by $R(0) = I_3$. The goal of the controller is to bring the spacecraft to rest with an attitude corresponding to an eigenaxis rotation of 40° about the body direction $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^{\rm T}$.

The controller order is set to $n_c = 3$, the recursive cost weighting $R = I$, the initial controller coefficients and the initial covariance are set to $\theta(0) = 0$ and $P(0) = 100I$, and the attitude parameter weights for S in (30) are given by $a_1 = 1, a_2 = 2$, and $a_3 = 3$. To compute the baseline Markov parameter, we compute the Jacobian about the equilibrium $\tilde{\omega}_{\rm e} = 0$, $\tilde{R}_{\rm e} = I_3$, $\nu_{\rm e} = 0$. Then, the first Markov parameter for the linearized system is

$$\mathcal{H}_1 \triangleq E_1 B = \begin{bmatrix} hJ^{-1}B_{\rm sc} \\ \frac{1}{2}h^2 M_a R_a M_a^{\rm T} J^{-1} B_{\rm sc} \end{bmatrix}. \tag{53}$$

As in [7], we saturate the control input proportionally, we limit the magnitude of the angular acceleration of each wheel to 1 rad/sec^2.

A. Robustnes to Inertia Uncertainty

To evaluate the robustness of RCAC to inertia uncertainty, we examine the relationship between the Markov parameter and the spacecraft and reaction wheel inertias. We introduce two inertia-free Markov parameters $\hat{\mathcal{H}}_1$ and $\hat{\mathcal{H}}_1'$. The first inertia-free Markov parameter $\hat{\mathcal{H}}_1$, does not have information about the spacecraft inertia J, that is

$$\hat{\mathcal{H}}_1 \triangleq \begin{bmatrix} hB_{\rm sc} \\ \frac{1}{2}h^2 M_a R_a M_a^{\rm T} B_{\rm sc} \end{bmatrix}. \tag{54}$$

The second inertia-free Markov parameter, $\hat{\mathcal{H}}_1'$ does not contain information about either the spacecraft inertia J or the reaction wheel inertias α_{0_i}, thus

$$\hat{\mathcal{H}}_1' \triangleq \begin{bmatrix} h\hat{B}_{\rm sc} \\ \frac{1}{2}h^2 M_a R_a M_a^{\rm T} \hat{B}_{\rm sc} \end{bmatrix}, \tag{55}$$

where the columns of the inertia-free $\hat{B}_{\rm sc}$ are given by

$$\hat{b}_i = -R_{W_i} e_{W_i}. \tag{56}$$

Thus, the Markov parameter $\hat{\mathcal{H}}_1'$ is inertia-free but uses the actuator alignment information in $\hat{B}_{\rm sc}$.

Figure 1 compares the eigenaxis attitude error,

$$\theta_{\rm eig} \triangleq \cos^{-1}\frac{1}{2}\left(\text{tr } \tilde{R} - 1\right), \tag{57}$$

using the baseline and inertia-free Markov parameters $\mathcal{H}_1$, $\hat{\mathcal{H}}_1$, and $\hat{\mathcal{H}}_1'$. Figure 1 shows that the removal of the spacecraft inertia information affects the steady state error. When the reaction wheel inertia is also removed, the settling time approximately doubles.

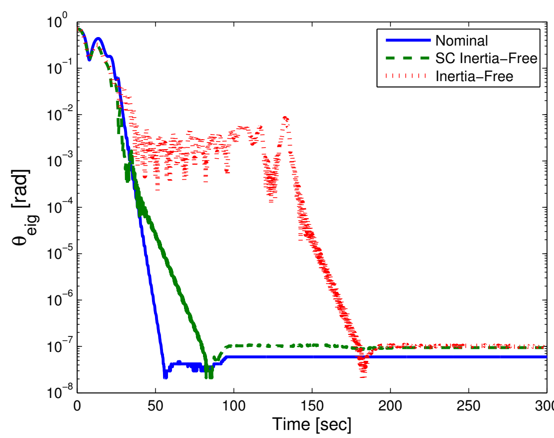

Fig. 1. Eigenaxis attitude-error for a M2R maneuver on a log scale. RCAC reaches the commanded attitude using the baseline Markov parameter $\mathcal{H}_1$, the spacecraft-inertia-free parameter $\hat{\mathcal{H}}_1$, and the inertia-free parameter $\hat{\mathcal{H}}_1'$.

Next, we vary the scaling of the inertia matrix and compare the settling time for various spacecraft inertias $J = \beta J_0$ for $\beta > 0$. To evaluate the closed-loop performance, we define the settling time $t_{\rm ss}$ as the time needed to bring the eigenaxis attitude-error within a bound $\theta_{\rm ss}$, that is,

$$\theta_{\rm eig}(t) \le \theta_{\rm ss}, \quad \text{for all } t \ge t_{\rm ss}.$$

For the following examples $\theta_{\rm ss} = 1°$ and we use the inertia-free Markov parameter $\hat{\mathcal{H}}'_1$.

Figure 2 shows that, for the Markov parameter $\hat{\mathcal{H}}'_1$, the settling time increases as the inertia of the spacecraft increases. This result is expected and is due to the decrease of control authority of the fixed-size reaction wheels. As the spacecraft inertia increases, the reaction wheels have a smaller effect on the angular acceleration $\dot{\omega}$. However, the steady state error is less than 1×10^{-7} radians and the settling time is within 200 seconds which is acceptable for many spacecraft applications.

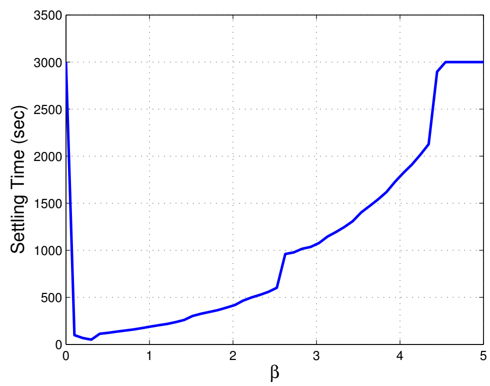

Fig. 2. M2R settling time $t_{\rm ss}$ as a function of spacecraft inertia scaling β for fixed wheel inertias α_{0_i}. As the spacecraft inertia increases, RCAC requires more time to reach the commanded attitude.

B. Frame Rotation and Actuator Misalignment

We evaluate the robustness of RCAC to unknown rotations for the M2R maneuver. First, we examine the M2R settling time as the actuator and sensor axes are rotated according to

$$J_{\rm sc} = \mathcal{R} J_0 \mathcal{R}^{\rm T}, \tag{58}$$

where J_0 is the nominal inertia matrix and $\mathcal{R}(\phi, \hat{n}) \in \mathbb{R}^{3\times3}$ is the rotation matrix given by Rodrigues's formula

$$\mathcal{R}(\phi, \hat{n}) = \cos(\phi) I_3 + [1 - \cos(\phi)]\hat{n}\hat{n}^{\rm T} + \sin(\phi)\hat{n}^{\times}, \tag{59}$$

where ϕ is the misalignment angle and $\hat{n}$ is the misalignment axis. Figure 3 shows that RCAC can complete the M2R maneuver for all possible single-axis rotations of the actuator and sensor axes.

Next, we misalign the reaction-wheels by applying an unknown rotation $\mathcal{R}$ according to (59) so that the columns of the actuator matrix $B_{\rm sc}$ become

$$b_i = -\alpha_i \mathcal{R} R_{W_i} e_{W_i}. \tag{60}$$

We vary the angle ϕ and consider the misalignment axes

$$\hat{n}_1 = e_1, \qquad \hat{n}_2 = e_2, \qquad \hat{n}_3 = e_3.$$

We use the Markov parameter $\hat{\mathcal{H}}'_1$, which does not assume knowledge of the misalignment information $\mathcal{R}$. Figure 4 shows that, as the misalignment angle increases, RCAC requires more time to settle, which reflects the lack of alignment information in the Markov parameter $\hat{\mathcal{H}}'_1$.

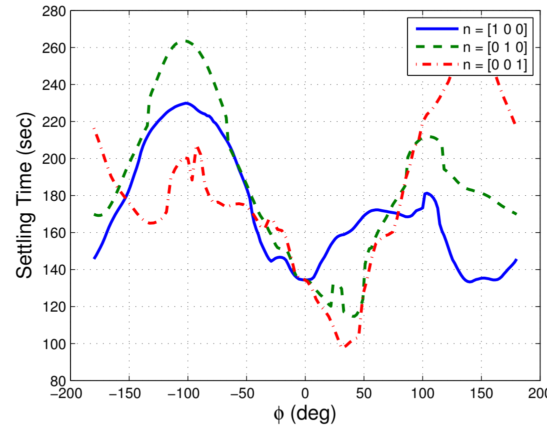

Fig. 3. Settling time $t_{\rm ss}$ as a function of the angle ϕ for different rotation axes $\hat{n}$. RCAC can complete the maneuver for all single axis rotations of the actuator and sensor axes.

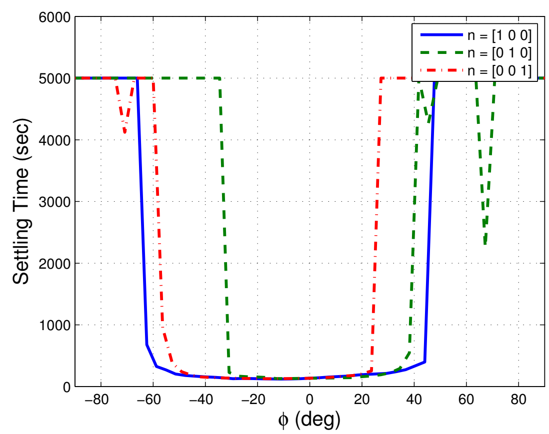

Fig. 4. Settling time $t_{\rm ss}$ as a function of the actuator misalignment angle ϕ. RCAC is robust to single-axis misalignments up to 20 degrees about all three axes.

C. Constant Disturbance Torque

We examine the performance of RCAC under the presence of a disturbance torque that is inertially constant such as solar radiation pressure. We test three different cases using the constant magnitude disturbance

$$z_{\rm dist} = R^{\rm T} d_0 \hat{n}_d, \tag{61}$$

where d_0 is the disturbance magnitude and $\hat{n}$ is the disturbance vector direction resolved in the inertial frame. Figure 5 shows the eigenaxis attitude-error for a disturbance of magnitude $d_0 = 1 \times 10^{-4}$ N-m with various inertial directions.

D. Noise and Bias

We introduce noise and bias into the angular-velocity measurement and examine the impact on the settling time. Let the measured angular velocity be given by

$$y_\omega = \omega + b + \nu_\omega, \tag{62}$$

where $b \in \mathbb{R}^3$ is a constant bias and ν_ω is white Gaussian noise. Figure 6 shows the eigenaxis attitude error when the

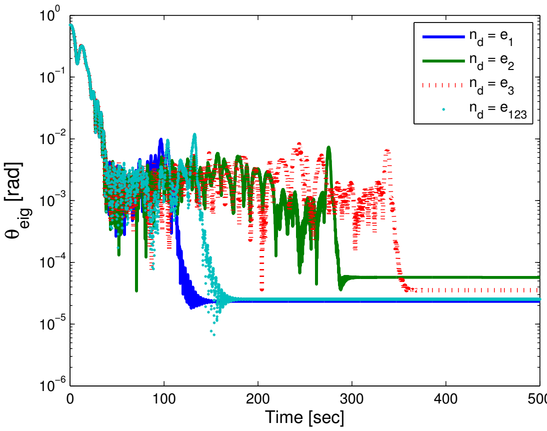

Fig. 5. Eigenaxis attitude-error on a log scale for an inertially constant disturbance torque with magnitude $d_0 = 1 \times 10^{-4}$ N-m and for different inertial directions $\hat{n}_d$; the direction $e_{123} = \frac{1}{\sqrt{3}}\,[1\ 1\ 1]^{\mathrm{T}}$. RCAC can complete the maneuver under a constant, single-axis disturbance torque using the inertia-free Markov parameter $\hat{\mathcal{H}}_1'$.

angular velocity measurement is corrupted by a constant biases $b = b_0 \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}}$ and Gaussian white noise with variance $\sigma_\omega^2 = 10^{-12}$ rad/sec. Since the angular-velocity measurement is included in the performance variable, the steady-state error increases as the bias magnitude increases.

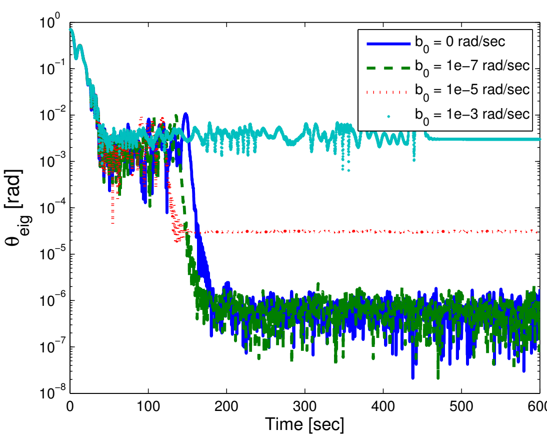

Fig. 6. Eigenaxis attitude error on a log scale for noise and bias in the angular-velocity measurement. As the bias increases, the steady-state error increases.

VI. Conclusion

RCAC was shown to achieve attitude command following for motion-to-rest maneuvers using reaction-wheel actuators. Dependence on the inertia of the spacecraft and reaction wheels was removed from the nominal Markov parameter. Thus, an inertia-free controller was utilized to achieve command following for spacecraft under various off-nominal conditions. The spacecraft and reaction wheel inertia free Markov parameter enabled the controller to reach the desired attitude with an acceptable steady state error and settling time. We demonstrated through simulations robustness to scaling of the spacecraft inertia demonstrated as well as to unknown single-axis misalignments of the reaction wheel spin axes. Furthermore, the algorithm can handle unknown inertially constant disturbances. Finally, we introduced Gaussian white noise and bias in the angular-velocity measurement and showed that the controller is stable can reach the desired attitude. Extensions of the reaction wheel case will focus on additional wheel configurations such as nonorthogonal wheel arrangements and the four wheel pyramid commonly used for redundancy. We are also interested in the problem of momentum dumping using a combination of thrusters and reaction wheels. Future applications will examine RCAC's ability to control spacecraft systems with alternative actuators such as control moment gyros and magnetic torquers. These actuators have a time-varying actuator matrix which, in the magnetic case, is singular. Also of interest are nonrigid spacecraft with time-varying inertia properties such as to actuated solar arrays, as well as spacecraft with flexible bodies such as gravity gradient booms and deployable antennas.

A mathematical proof of the algorithm's stability is still under development. The version of RCAC used in this paper can be extended to utilize a time history of control input and performances. We will examine the effect of using more measurements on the steady state error and settling time. Further work will examine other spacecraft inertias as well as thorough testing of initial conditions and set points. However, numerical simulations indicate that RCAC can effectively control the nonlinear rigid body spacecraft with reaction wheels and is robust to common off-nominal conditions.

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