Optimization R Us

One thing I've learned the hard way is to avoid using the word "optimal" when speaking with engineers who solve real problems. "I don't care if the controller is optimal, I just want it to work" is a typical response. If an optimal design provides 1% improvement over a design based on alternative principles, insights, or techniques, it's difficult to make a case for optimization (except in drag reduction, where a 1% improvement is huge). In addition, engineers intuitively recognize that attempts to optimize in one direction can lead to weaknesses in others. Failures of "optimized" components invite ridicule and are not soon forgotten.

Of course, optimization aficionados are quick to point out that optimization must not be done blindly. Textbook examples, in which narrow criteria are optimized under unrealistic assumptions, are not to be taken literally. The trick in optimization is to enforce constraints that must be satisfied while exploring as many options as possible within tradeoff space. Even our beloved linear-quadratic-Gaussian (LQG) theory, which optimizes a quadratic cost criterion, does not blindly optimize a performance objective but rather provides a technique for trading off performance for control effort. The theory is essentially a vehicle for exploring this aspect of the design landscape. On the other hand, classical LQG cannot account for model uncertainty, a minor deficiency corrected by a couple of decades of robust control research.

Setting up and interpreting the results of an optimization process requires insight and experience. In structural design for buildings and bridges, where the consequences of failure are life threatening, constraints represent assurances against catastrophe. Whether a design constraint is to be satisfied with a margin of 5% or 50% is a matter of engineering judgment. Optimization algorithms satisfy the given constraints but provide no insight into whether the constraints are chosen wisely.

IEEE Control Systems Magazine Editorial Board members meeting at the American Control Conference in Minneapolis in June 2006. From left: Kent Lundberg, Darren Cofer, Panagiotis Tsiotras, Zongli Lin, Daniel Rivera, Dawn Tilbury, Randy Beard, Shuzhi Sam Ge, Dennis Bernstein, and Alex Leonessa.

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With regard to constraints, two cases arise. In the first case, the solution lies on the boundary of the constraint set, in which case the optimization algorithm considers feasible directions that lie along the boundary or point inside the constraint set. Conceptually, the existence of a solution lying on the boundary is not surprising. Since one normally expects that the best designs are obtained by using the most resources such as the most energy, the most force, or the most money. As such, I'd be surprised to find a better car at a lower price.

In the second case, the optimal solution lies in the interior of the constraint set. In this case, the optimality conditions are simpler because we search for a zero gradient (if everything is smooth) and, if the problem is nice, we need only check a small number of stationary points. Optimizers in the interior are interesting since, unlike those on the boundary, the optimality of stationary points is governed by higher order information such as second- and higher order derivatives; in optimal control, the singular cases—whose solutions can only be identified through more differentiations—are especially intriguing.

Although optimization tools are available for virtually any optimization problem, some optimization problems are much more difficult than others. Problems involving convex functions or sums of squares are known to be tractable in a precise sense, setting off a determined search for solvable problems with engineering significance. Curiously, however, LQG is effective due to its unique and computable global minimizer rather than its membership in a tractable class. The larger point is that a lot of what we do is search for solvable optimization problems, which is clearly a solution-driven research paradigm.

In fact, we work on optimization from both ends: we develop methods for solving larger and larger classes of optimization problems with better and better algorithms while simultaneously searching for relevant optimization problems with tractable solutions.

Our use of optimization is much broader than we often realize. Every application of least squares—a seemingly mundane technique that never ceases to raise interesting theoretical problems and find new applications—is optimization at work. Indeed, much of system identification can be viewed as an application of optimization in which the solution to the optimization problem—a.k.a. the *arg min*—is usually a point of physical meaning rather than merely a better design point.

Except for truly binary properties, such as whether a system is stable or not, it's almost a challenge to find a concept that has no optimization interpretation. Indeed, even the Lyapunov function cannot escape optimality when viewed as the solution to the Hamilton-Jacobi-Bellman equation, a continuum version of dynamic programming and the principle of optimality. In addition, for multi-input, multi-output plants of arbitrary dimension, LQG is often the most convenient technique for providing a stabilizing controller.

Optimization saturates what we do and drives almost every aspect of engineering. Done meaningfully and intelligently, with engineering insight, it is the most easily understood and powerful of the concepts that we work with. The challenge is to use it wisely.

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Current and former Ph.D. students of Dennis Bernstein attending the American Control Conference in Minneapolis in June 2006. From left: Jesse Hoagg, Harshad Sane (UTRC), Sanjay Bhat (IIT Mumbai), Scott Erwin (AFRL), Wassim Haddad (Georgia Tech), Mario Santillo, Harish Palanthandalam-Madapusi, Dennis Bernstein, Nalin Chaturvedi, Ashwani Padthe, Seth Lacy (AFRL), and Jaganath Chandrasekhar. Not present for the photo but attending ACC was Feng Tyan (Tamkang University).