Matrix Mathematics

Errata and Addenda for the First Edition

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May 12, 2010

This document contains an updated list of errata for the first edition of *Matrix Mathematics*. Please email me if you discover additional errors, and I will include them in future updates.

All of these errors are corrected in the second edition, which is also greatly expanded relative to the first edition.

- Page 32, line 6: change "either $\mathbb{F} = \mathbb{C}$" to "either $\mathbb{F} = \mathbb{R}$"
- Page 52, Fact 2.10.15: $iii)$ is true with "$\subseteq$" replaced with "=", $iv)$ is true with "$\leq$" replaced with "=", and the last identity in v) can be omitted
- Page 83, Definition 3.1.3: delete "$l \triangleq \min\{n, m\}$" and replace "$l$" in iii) with "$\min\{n, m\}$"
- Page 94, Fact 3.5.22: delete "$A + A^*$", delete "and $A - A^*$", and replace "are skew" with "is skew". Append "Now assume that $n = m$. Then, $A + A^*$ is Hermitian, and $A - A^*$ is skew Hermitian."
- Page 96, Fact 3.5.26: replace "$\mathbb{R}^3$" by "$\mathbb{R}^{3\times 3}$"
- Page 101, Fact 3.7.22: replace "$(s - \jmath)(s + \jmath)$" with "$(s - \jmath)/(s + \jmath)$"
- Page 101, Fact 3.7.23: replace "$B \triangleq (I - A)(I + A)^{-1}$" with "$B = \lambda(I - A)(I + A)^{-1}$"
- Page 107, Fact 3.8.23, condition iii): change "For all" to "For all nonzero"
- Page 115, Fact 3.12.8: replace J_n by J_{2n}
- Page 128, Definition 4.3.4: replace $P \in \mathbb{F}^{n\times n}[s]$ by $P \in \mathbb{F}^{n\times m}[s]$
- Page 137, line 4: replace the second instance of λ_1 with λ_2
- Page 141, Fact 4.8.2: replace "$\mathbb{F}^n$" with "$\mathbb{F}^n[s]$"
- Page 151, Fact 4.9.16: replace I_n by I_{2n}

- Page 151: Divide the left hand side of the last displayed equation by $\|A^k x_0\|$
- Page 156, Fact 4.11.1: In statement *ii*), replace "nonnegative" by "nonzero nonnegative"
- Page 172: Replace "are $B_i \stackrel{\triangle}{=} [\lambda_i]$ for all $i = 1, \ldots, r$ and $\hat{B}_i \stackrel{\triangle}{=} \left[\begin{smallmatrix} \nu_i & \omega_i \\ -\omega_i & \nu_i \end{smallmatrix}\right]$ for all $i = 1, \ldots, l$." with "satisfy $B_i \stackrel{\triangle}{=} [\lambda_i]$ for all $i = 1, \ldots, r$ and $\mathrm{spec}(\hat{B}_i) = \{\nu_i + \jmath\omega_i, \nu_i - \jmath\omega_i\}$ for all $i = 1, \ldots, l$." Also, replace "[367, p. 152]" with "[367, p. 82]".
- Page 181, Proposition 5.5.25: delete statement $xviii)$
- Page 192, Fact 5.9.16: replace "A, B" with "$A, B \in$"
- Page 200, Fact 5.10.25: replace "$\sqrt{ac}$" by "$2\sqrt{ac}$".
- Page 201, Fact 5.11.4: In the last displayed equation change "λ_{n-i}" to "λ_{n-i+1}"
- Page 214, Fact 5.13.7: change the second instance of "normal" to "circulant"
- Page 217, Fact 5.14.13: change x_i^* to x_i^{T} twice
- Page 221, Fact 5.14.35: delete "and assume that A is skew symmetric."
- Page 226, Proposition 6.1.7: In *viii*), change $A^{\mathrm{L}} \in \mathbb{F}^{m \times m}$ to $A^{\mathrm{L}} \in \mathbb{F}^{m \times n}$
- Page 226, last line: change $x \in \mathbb{F}^n$ to $x \in \mathbb{F}^m$
- Page 236, Fact 6.4.16: change $B \in \mathbb{F}^{m \times m}$ to $B \in \mathbb{F}^{m \times n}$
- Page 240, Fact 6.4.29: change $D \stackrel{\triangle}{=} B^+C^+C$ to $D \stackrel{\triangle}{=} B^*C^+B$
- Page 240, Fact 6.4.30: change $(1 + b^*A^+b)$ to $(1 + b^*A^+b)^{-1}$
- Page 248, Definition 7.1.2: change "of A" to "of A and B".
- Page 256, Fact 7.4.24: in xii) change $P_{n,l}$ to $P_{l,n}$.
- Page 257, Fact 7.5.6: change to

  Let $A \in \mathbb{F}^{n \times n}$, let $B \in \mathbb{F}^{m \times m}$, assume that A is positive definite, and define $p(s) \stackrel{\triangle}{=} \det(I - sA)$, and let $\mathrm{mroots}(p) = \{\lambda_1, \ldots, \lambda_n\}_{\mathrm{m}}$. Then,

$$\det(A \oplus B) = (\det A)^m \prod_{i=1}^{n} \det(\lambda_i B + I).$$

- Page 257, Fact 7.5.7: change to

  Let $A, C \in \mathbb{F}^{n \times n}$, let $B, D \in \mathbb{F}^{m \times m}$, assume that A is positive definite, assume that C is positive semidefinite, define $p(s) \stackrel{\triangle}{=} \det(C - sA)$, and let $\mathrm{mroots}(p) = \{\lambda_1, \ldots, \lambda_n\}_{\mathrm{m}}$. Then,

$$\det(A \otimes B + C \otimes D) = (\det A)^m \prod_{i=1}^{n} \det(\lambda_i D + B).$$

- Page 257, Fact 7.5.8: change to

  Let $A, D \in \mathbb{F}^{n\times n}$, let $C, B \in \mathbb{F}^{m\times m}$, assume that $\operatorname{rank} C = 1$, and assume that A is nonsingular. Then,

  $$\det(A \otimes B + C \otimes D) = (\det A)^m (\det B)^{n-1} \det\big[B + \big(\operatorname{tr} CA^{-1}\big)D\big].$$

- Page 276, Corollary 8.4.15: replace "$p > 1$ or $p < n$" with "$p \in (1, n)$".

- Page 284, Proposition 8.5.15: in $xxvii$) change "tr" to "-tr" and append "$r \in (0, 1)$"

- Page 294, Fact 8.7.37: change "upper" to "lower"

- Page 297: Delete Fact 8.8.17

- Page 297, Fact 8.8.22: change [278] to [282]

- Page 301, Fact 8.9.3 and Fact 8.9.4: replace "$A, B, C \in \mathbb{F}^{n\times m}$" by "$A \in \mathbb{F}^{n\times n}, B \in \mathbb{F}^{n\times m}$, and $C \in \mathbb{F}^{m\times m}$"

- Page 310, Fact 8.10.25: replace "$A^{p/2}B^pA^{p/2}$" with "$A \log(A^{p/2}B^pA^{p/2})$" twice

- Page 311, Fact 8.11.3: Replace the second "real" with "positive" and the third "real" with "nonnegative"

- Page 313, Fact 8.11.19: Replace $[\det(I - A^*B)]^2$ with $|\det(I - A^*B)|^2$

- Page 315, Fact 8.11.26: replace $\det(C - B^*C^+B)$ with $\det(C - B^*A^{-1}B)$.

- Page 316, Fact 8.11.31: Reverse the determinant inequality.

- Page 319, Fact 8.12.14: Replace "$x^{\mathrm{T}}Ax \geq 0$ for all" by "$x^{\mathrm{T}}Ax > 0$ for all nonzero" and replace "$x^{\mathrm{T}}Ax \leq 0$ for all" by "$x^{\mathrm{T}}Ax < 0$ for all nonzero"

- Page 323, Fact 8.14.1: append to the first sentence "and let $\operatorname{mspec}(A) = \{\lambda_1, \lambda_2\}_{rmms}$"

- Page 326, Fact 8.14.12: Replace $\sigma_{\max}^{1/2}(A^{1/2}BA^{1/2})$ with $\sigma_{\max}(A^{1/2}B^{1/2})$

- Page 326, Fact 8.15.10: Delete the first result in *ix*)

- Page 327, Fact 8.14.14: replace "$\mathbf{H}^n \mapsto \mathbb{R}$" by "$\mathbf{H}^n \mapsto \mathbf{H}^n$" twice

- Page 327, Fact 8.14.19: The inequality should read

  $$\sigma_{\max}^2(A_{12}) \leq \sigma_{\min}(A_{11})\sigma_{\min}(A_{22}),$$

  which is sufficient but not necessary.

- Page 341, Fact 8.17.5: replace "log majorizes" by "weakly log majorizes"

- Page 342, Fact 8.17.6: replace "e^t" by "t"

- Page 344, proof of Proposition 9.1.2: replace the second "$|y| \leq\leq |x|$" by "$|x| \leq\leq |y|$"

- Page 345, Proposition 9.1.5: replace "$1 \leq p < q \leq \infty$" by '$1 < p < q < \infty$"

- Page 348, line 1: replace "$1 \leq p \leq q$" by "$1 \leq p \leq q \leq \infty$"

- Page 353, line 2: replace "$A, B \in \mathbb{F}^{n\times m}$" by "$A \in \mathbb{F}^{n\times m}$ and $B \in \mathbb{F}^{m\times l}$"
- Page 375, Fact 9.9.8: replace "$\mathbb{F}^{n\times n}$" with "$\mathbb{F}^{m\times m}$", and replace "$\|A\|\|B\|$" with "$\|\langle A\rangle\|\|\langle B\rangle\|$"
- Page 385, Fact 9.10.4, line 3: Replace AB by A^*B
- Page 387, Fact 9.10.8: replace "a_{jj}" and "a_{ii}" with "$A_{(j,j)}$" and "$A_{(i,i)}$", and replace "Fact 9.10.8" with "Fact 11.17.2"
- Page 388, Fact 9.11.1: replace $x \in \mathbb{F}^n$ by $\mathbb{F}^m$
- Page 389, Fact 9.11.9: replace "entries of A" with "components" twice
- Page 421, paragraph after Fact 11.1.4: delete "v) and vi) of" and append to the same sentence "and v) of Proposition 11.5.8, respectively."
- Page 433, Proposition 11.5.4: replace final "$\mathcal{S}$" by "$\mathcal{S}_0$". Same change in Proof.
- Page 434, Proposition 11.5.8: in xi) replace "sp" and "Sp" by "$\mathrm{sp}_{\mathbb{R}}$" and "$\mathrm{Sp}_{\mathbb{R}}$", respectively.
- Page 444. Delete Proposition 11.8.5 and its proof.
- Page 449, Fact 11.10.7: replace "$\mathbb{F}$" by "$\mathbb{R}$" and replace "unitary" with "orthogonal with $\det A = 1$"
- Page 450, Fact 11.10.9: replace
$$\phi \triangleq \cos^{-1}\bigl(x^{\mathrm{T}}y\bigr).$$
with
$$\phi \triangleq \cos^{-1}\biggl(\frac{x^{\mathrm{T}}y}{\|x\|_2\|y\|_2}\biggr).$$
Furthermore, in the fourth statement, replace $z \triangleq \nu \times y$ with $z \triangleq \|y\|_2^{-1}\nu \times y$.
- Page 454, Fact 11.11.14: replace "all $k \in \mathbb{P}$" with "every nonzero integer k"
- Page 454, Fact 11.11.15: replace "orthogonal" with "unitary" and replace "symmetric" with "Hermitian"
- Page 455, Fact 11.12.3: Replace $\mathrm{Dexp}\bigl(e^{tA};B\bigr)$ with $\mathrm{Dexp}(A;B)$
- Page 458, Fact 11.12.18: replace "xii) of Proposition 11.4.7" with "$xiii$) of Proposition 11.5.5"
- Page 466, Fact 11.15.10: replace "positive eigenvalue" with "nonnegative eigenvalue"
- Page 468, Fact 11.17.10: replace "$\beta > \mathrm{spabs}(A)$" with "$\beta \in (\mathrm{spabs}(A), 0)$" and replace "satisfy" with "be positive definite and satisfy"
- Page 471, Fact 11.17.25: delete 0 in the upper right hand corner
- Page 492, line 5: change "Definition 4.7.2" to "Definition 4.7.3"
- Page 501, (12.6.8): replace "(A, C)" with "(A, B)"
- Page 510, line -8: replace "(A, C)" with "(A, B)"
- Page 519, line 2: replace "Proposition 12.9.3" by "Proposition 12.9.11"

- Page 528, equation (12.13.3), change $-BD^{-1}$ to BD^{-1}
- Page 539, Proposition 12.16.13, ii): replace "q" with "p" twice
- Page 541, line -2: replace "a stabilizing solution" with "the stabilizing solution"