Game Theory and The Nash Equilibrium

Dr. Charles Severance - University of Michigan
School of Information / Informatics
www.dr-chuck.com

Game Theory

Game theory attempts to mathematically capture behavior in strategic situations, in which an individual's success in making choices depends on the choices of others. Traditional applications of game theory attempt to find equilibria in these games. In an equilibrium each player of the game has adopted a strategy that they are unlikely to change.

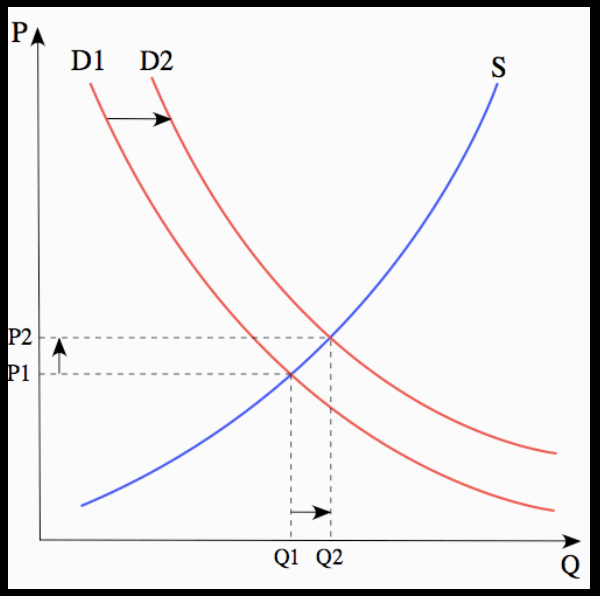
http://en.wikipedia.org/wiki/Game_theory

PJC Teen MP3 Player and Online Music Survey - Table 1	Spring-09	Fall-08	Spring-08	Fall-07	Spring-07
Do You Own An MP3 Player?		87%	87%	80%	78%
What MP3 Player Do You Own?					
Apple iPod (any variation)		84%	86%	82%	82%
Zune		3%	3%	2%	--
Sony		3%	2%	4%	4%
Dell		0%	0%	0%	2%
iRiver		0%	0%	1%	2%
Sandisk		2%	3%	4%	2%
Creative		2%	1%	2%	3%
Other		6%	5%	6%	5%
Plan To Buy MP3 Player In Next 12 Months?		34%	28%	47%	42%
What MP3 Player Planning To Buy In Next 12 Months?					
Apple iPod (any variation)		79%	80%	78%	73%
Zune		15%	13%	13%	--
Sony		3%	1%	4%	11%
Dell		0%	0%	0%	0%
iRiver		0%	3%	1%	5%
Sandisk		0%	1%	1%	2%
Creative		0%	1%	1%	2%
Other		3%	0%	2%	7%

http://www.appleinsider.com/articles/09/04/08/apple_near_saturation_point_for_ipod_itunes_use_by_teens.html

Economics - Old School

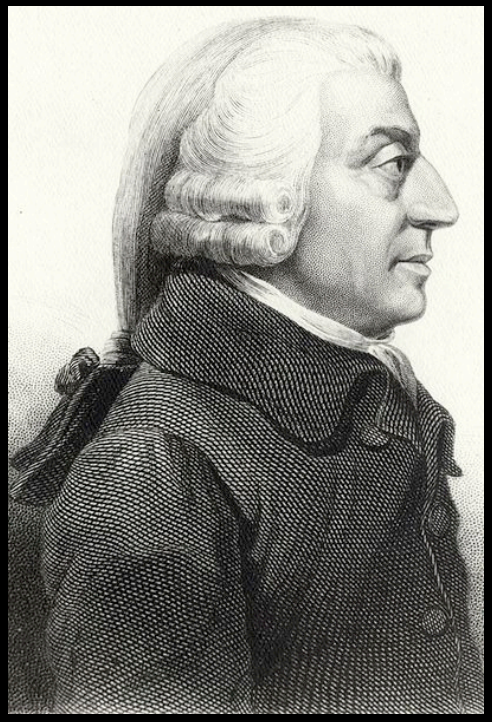
- Law of Supply and Demand
- Demand goes up, prices go up, supply goes up
- It just works!



http://en.wikipedia.org/wiki/Supply_and_demand

Adam Smith

- "In competition, individual ambition serves the common good"
- "The Butcher, the Baker, and the Brewer provide goods and services to each other out of self-interest; the unplanned result of this division of labor is a better standard of living for all three."
- Wealth of Nations (1776)



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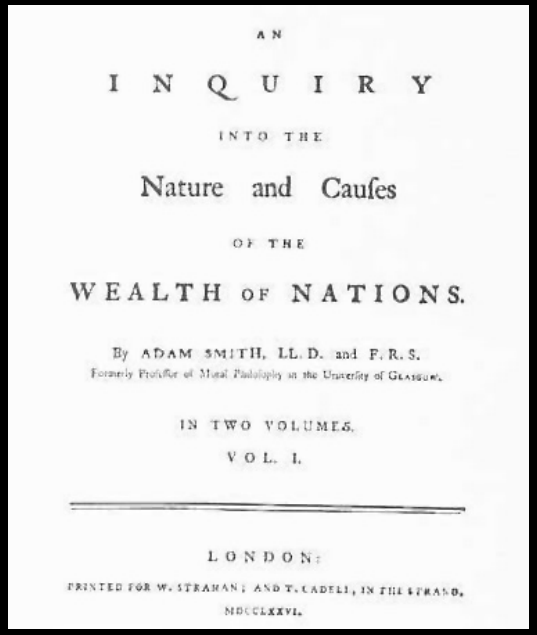
Wealth of Nations

An Inquiry into the Nature and Causes of the Wealth of Nations is the magnum opus of the Scottish economist Adam Smith. It is a clearly written account of economics at the dawn of the Industrial Revolution - advocating a free market economy as more productive and more beneficial to society.



AN

INQUIRY

INTO THE

Nature and Cauſes

OF THE

WEALTH OF NATIONS.

By ADAM SMITH, LL.D. and F.R.S.

Formerly Profeſſor of Moral Philoſophy in the Univerſity of GLASGOW.

IN TWO VOLUMES.

VOL. I.

LONDON:

PRINTED FOR W. STRAHAN; AND T. CADELL, IN THE STRAND.

MDCCLXXVI.

http://en.wikipedia.org/wiki/The_Wealth_of_Nations

John Forbes Nash

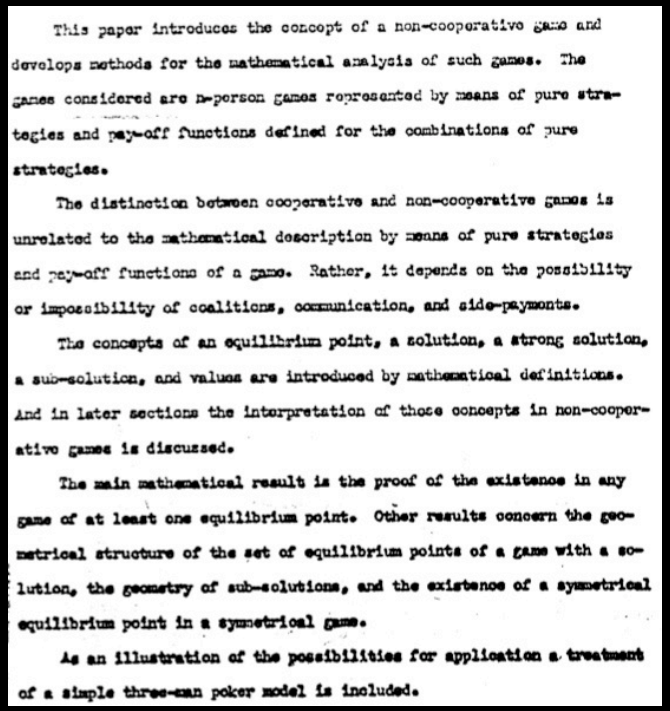
- Received his Phd. Mathematics at Princeton in 1950 at 22 years old
- Mathematics faculty at MIT - 1951 - 1958
- Schizophrenia 1959 - 1995
- Nobel Prize in Economic Sciences - 1994

http://en.wikipedia.org/wiki/John_Forbes_Nash

John Nash - Thesis

Abstract

This paper introduces the concept of a non-cooperative game and develops methods for the mathematical analysis of such games. ...The concepts of an equilibrium point, a solution, a strong solution, a sub-solution, and values are introduced by mathmatical definitions.



This paper introduces the concept of a non-cooperative game and develops methods for the mathematical analysis of such games. The games considered are n-person games represented by means of pure strategies and pay-off functions defined for the combinations of pure strategies.

The distinction between cooperative and non-cooperative games is unrelated to the mathematical description by means of pure strategies and pay-off functions of a game. Rather, it depends on the possibility or impossibility of coalitions, communication, and side-payments.

The concepts of an equilibrium point, a solution, a strong solution, a sub-solution, and values are introduced by mathematical definitions. And in later sections the interpretation of these concepts in non-cooperative games is discussed.

The main mathematical result is the proof of the existence in any game of at least one equilibrium point. Other results concern the geometrical structure of the set of equilibrium points of a game with a solution, the geometry of sub-solutions, and the existence of a symmetrical equilibrium point in a symmetrical game.

As an illustration of the possibilities for application a treatment of a simple three-man poker model is included.

http://www.princeton.edu/mudd/news/faq/topics/Non-Cooperative_Games_Nash.pdf

A Beautiful Mind (film)

- Based on John Forbes Nash
- Post WWII - Math "Won the war"
- Broad-based investment in maintaining the US intellectual lead
- Mathemeticians were valued, recruited, brilliant, arrogant, and quirky

John Nash - Thesis

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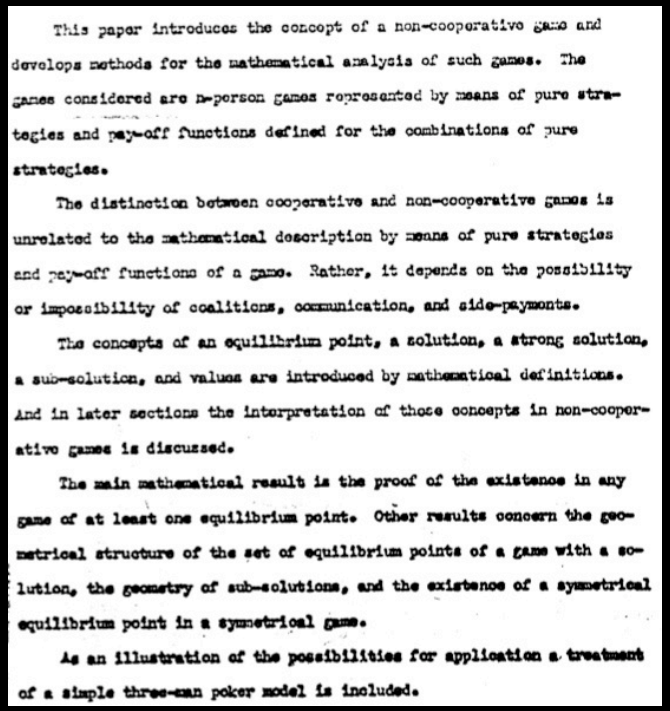

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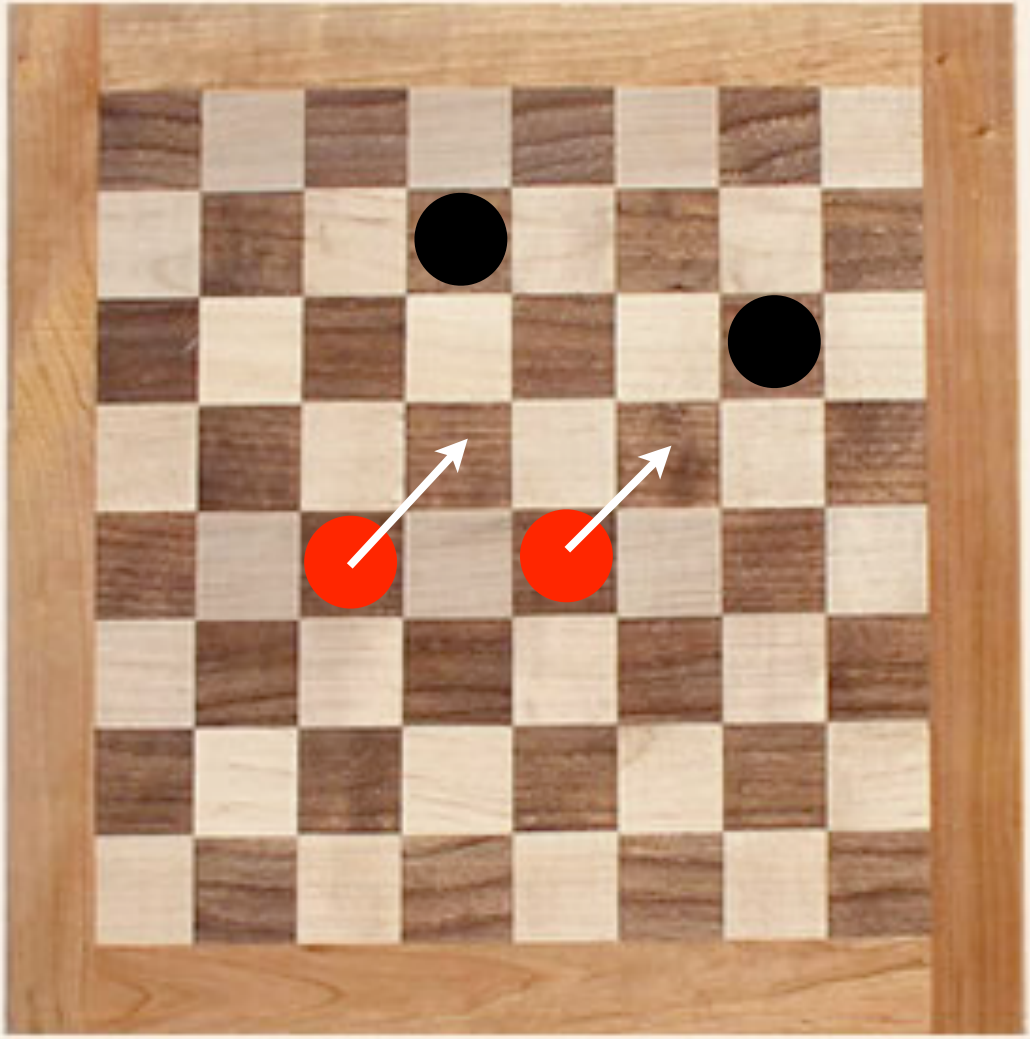
Game Theory - Rational Play

- We assume players want to win - to maximize their return
- What is the "rational strategy" given what we believe about the other player
- Playing chess or checkers

Which is the rational move for Red??

Is our opponent always rational?

Have you ever played a non-rational opponent? What did you do? What were your goals?



Prisoner's Dilemma

	Prisoner B Stays Silent	Prisoner B Betrays
Prisoner A Stays Silent	Each serves 6 months	Prisoner A: 10 years Prisoner B: goes free
Prisoner A Betrays	Prisoner A: goes free Prisoner B: 10 years	Each serves 5 years

Example PD payoff matrix

	Cooperate	Defect
Cooperate	3, 3	0, 5
Defect	5, 0	1, 1

	Cooperate	Defect
Cooperate	win-win	lose much-win much
Defect	win much-lose much	lose-lose

The mathematical formulation is shown with numbers where "more is better".

Payoff Matrix - Prisoner's Dilemma

- Two Players (Red / Blue)
- Players know all the rules of the game
- The only thing they do not know is how their opponent will play
- Each entry gives the payoff for the players

Example PD payoff matrix

	Cooperate	Defect
Cooperate	3, 3	0, 5
Defect	5, 0	1, 1

Higher is better

http://en.wikipedia.org/wiki/Prisoners_dilemma

Analyzing the Game

- Cooperative Approaches
  - Pareto Optimal
  - Social Welfare Maximizer
- Non-Cooperative / Rational Play
  - Best Response / Dominant Strategy
  - Nash Equilibrium

Pareto Optimal

- A solution is Pareto Optimal if no one can be better off without making someone else worse off
- The best win-win
- Often not "rational" for either player

Example PD payoff matrix

	Cooperate	Defect
Cooperate	3, 3	0, 5
Defect	5, 0	1, 1

Social Welfare Maximizer

- The pair of strategies where the sum of the outcomes is maximized is called the "social welfare maximizer"
- "Its logical. The needs of the many outweigh... the needs of the few... or the one."

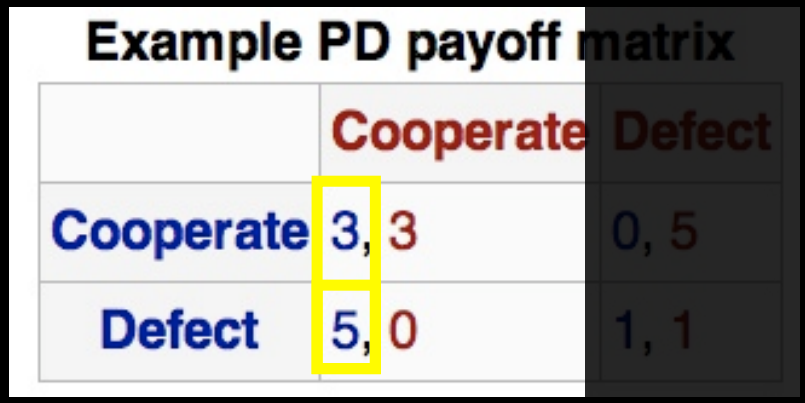
Example PD payoff matrix

	Cooperate	Defect
Cooperate	3, 3	0, 5
Defect	5, 0	1, 1

http://www.youtube.com/watch?v=hFyl4GxBzEw

Best Response

- Once you know what your opponent's strategy, the Best Response is what you should do to maximize payoff
- If we know Red will Cooperate - we should "Defect" as it means we will maximize
- Because we are rational, we don't care about their payoff



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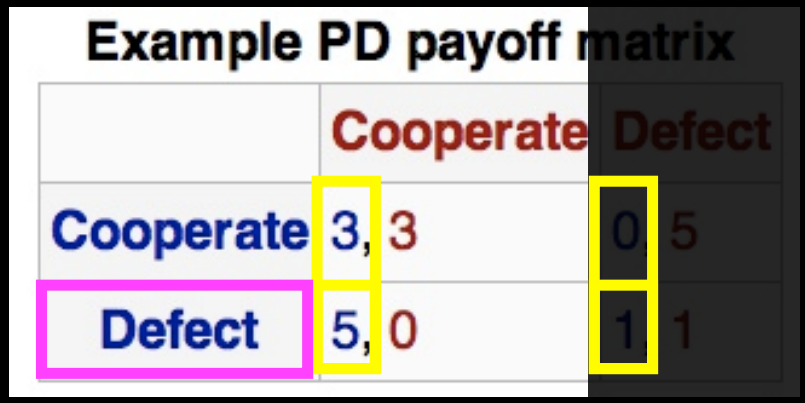
Dominant Strategy

- If you look at the sepatately at the Best Response for each of your opponent's stategies and and your Best Response is the same regardless of the strategy they choose - you have a Dominant Strategy
- It is always "rational" to Defect



Predictable Play

- If all players or all but one player have a Dominant Strategy, we can predict play, assuming rational players
- Remember that all players know all the rules - so you can compute your opponent's Dominant Strategy and even if you don't have a Dominant Strategy - you just go with your Best Response to their Dominant Strategy play - because you assume the other player is rational

No Dominant Strategies

- When more than one player does not have Dominant Strategy we still want to predict the outcome of the game
- We look for an Equilibrium - where the players naturally tend to go
- Instead of working from strategies inward to a solution - we work outward from a strategy combination - checking to see if it is an equilibrium

Nash Equilibruim

- No player can gain anything by changing their strategy
- If either player changes their mind they will be worse off
- There can be more than one Nash Equilibrium
- Pick a solution - check to see if it is a Nash Equilibrium

http://www.youtube.com/watch?v=J4n-Eg6UAw8

Nash Equilibrium for PD

- Pick Defect / Defect
- Check if Red can improve - No
- Check if Blue can improve - No
- Defect, Defect is a Nash Equilibrium
- Check the others

Example PD payoff matrix

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Higher is better

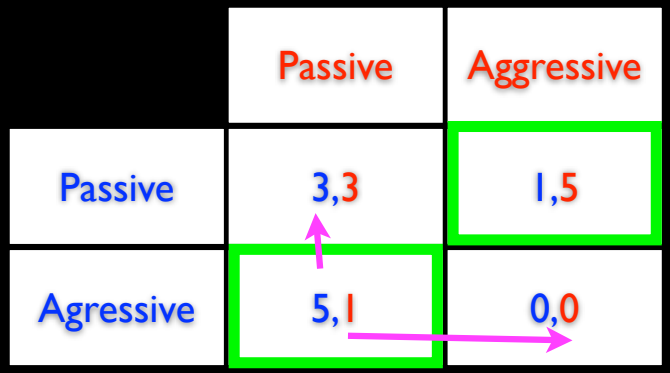
Hawk / Dove - Dominant Strategy

- Is there a Dominant Strategy?

	Passive	Aggressive
Passive	3,3	1,5
Agressive	5,1	0,0

Hawk / Dove - Nash Equilibrium

- Is there a Nash Equilibrium?
- Yes, there are two Nash Equilibria..



This is kind of sad if you think about it.

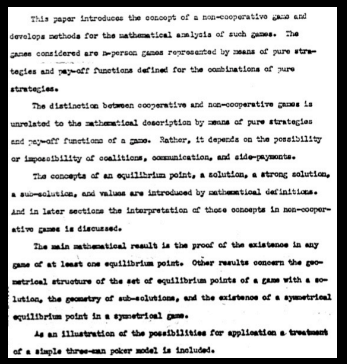
A Beautiful Mind

- The movie characterization is not *quite* accurate in describing the Nash Equilibrium
- Lets watch again for the "Artistic License"

Artistic License...

What if no one goes for the blonde, we don't get in each other's way, and we don't insult the other girls - that's the only way we win.

Nash if this is some way for you to get the blonde on your own, you can go to ...



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Questions / Comments

See Also:

A Breautiful Madness

http://www.pbs.org/wgbh/amex/nash/filmmore/pt.html

http://tinyurl.com/brilliant-madness