DURABLE GOODS AND CONFORMITY

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ABSTRACT

This paper argues that there should be less product variety in markets for durable goods compared to markets for nondurable goods. In durable goods markets there are incentives to purchase goods that reflect the preferences of the average consumer. Put differently, buyers *conform* to the average preference. Thus the distribution of durable goods available in markets tends to be compressed relative to the actual diversity of consumers' tastes. The reason for conformity in these markets is natural: durables (for example houses) are often traded and as a result, demand for these goods is influenced by resale concerns. In equilibrium, conformity increases with durability, patience, and the likelihood of trade. Surprisingly, we show that there is often too much product diversity – or too little conformity – in markets for durable goods.

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> "*Conformity, in terms of size, condition & features, tends to support your home's market value more than anything else.*" [1]

1. Introduction

Durable goods survive for long periods of time and are often possessed by many different people over the life of the good. Because many durables change hands from time-to-time, the efficient provision of these goods should to some extent reflect the preferences of many potential owners. Indeed, if durables are exchanged very frequently, they should have features that cater to the average preference in the market rather than the preferences of a single individual. As a result, there should be less product variety in durable goods markets – the distribution of varieties available for purchase should be compressed relative to the underlying distribution of preferences. Put differently, there should be *conformity* in markets for durable goods.

Market forces provide incentives for conformity in the provision of durable goods. These incentives typically arise through resale concerns. Because durables are often traded in secondary markets, resale concerns influence the original purchase decisions and thus encourage conformity. In contrast, nondurable goods are consumed by a single person and thus there is no incentive to conform to the average preference.

For example, new houses often have features and styles that differ only superficially from one house to the next. New housing developments are often derided because they consist merely of "cookie-cutter" houses or "McMansions." These houses are virtually the same – most have cathedral ceilings, walk-in closets, built-in jacuzzis, mud rooms, and so forth. Of course many of these features are desirable but it seems unlikely that preferences are so aligned as to justify such a homogeneous mix of products. The apparent homogeneity among new houses may instead be an efficient market reaction. Rather than catering to individual tastes, builders conform to the average taste anticipating the eventual resale of the house.

Conformity also likely arises for durables other than housing. For example, since used cars are frequently traded, there may be pressure to conform in automobile purchases. A new car buyer might purchase a car with an automatic

[1] Quoted from Accurate Appraisals. See http://www.accurate-appraisal.com/faq.htm.

transmission even if he prefers manual transmission because of resale concerns. The used car market, because it is localized and fragmented, creates an incentive to conform to the average preference. Conformity may also arise in markets for business capital. In this case, firms face a tradeoff between having capital equipment that fits their specific production purposes and having capital that has a high resale value. Because capital goods that are valued by many other firms can be easily resold, firms have some incentive to conform by employing capital that reflects the needs of other firms. As it does in our model, the incentive to conform increase with the durability of the capital goods.

We analyze these issues with a matching model in which agents buy and sell a long-lived durable good that must be resold from time-to-time. Although the specific function of the durable is not important for the analysis, we refer to the durable as a house. There are two types of houses and agents differ according to their preferences over the two types. Frictions in the resale market imply that agents are not perfectly matched with others who have the same preferences. Thus, if someone buys an unusual house, he runs the risk that he will not be able to sell it if he needs to move. Resale concerns can be so strong that the individual chooses to purchase a good that he dislikes relative to other available goods. When this occurs we say that the individual is conforming to the market.

In equilibrium, there is a tendency to conform to the average preference. Rather than being a knife-edge phenomenon, conformity is the typical outcome in markets for durable goods. Because resale concerns increase with durability and the incidence of trade, there is greater conformity in markets for long-lived durables and for people who trade frequently. While agents do not care directly about the preferences or actions of other agents, in equilibrium they act as though they do. Because they buy and sell a common set of goods, durability and trade endogenously align the preferences of the agents.

The equilibrium level of conformity is often not socially optimal. Surprisingly, the model suggests that there is typically *too little* conformity in durable goods markets. There are two reasons for this inefficiency. First, by conforming, agents reduce search costs. If they have a house that few others want, they will have difficulty selling it if they need to move. Of course the original builder has an incentive to conform to reduce the severity of these search costs. However, the search costs affect both buyers and sellers. Because the original owner only internalizes his own search costs, he has too little incentive to conform. Second, even if the house is sold, it is possible that the house will not be an ideal match for the new owner. The seller typically does not fully internalize the social costs incurred when the buyer settles for a house that is not ideal for his needs. Because he will likely be matched with someone who has typical preferences, by conforming, the original owner would reduce the number of mismatches. Yet, unless the seller captures all of the surplus from trade, he does not fully internalize this cost and again, there is too little incentive to conform.

The rest of the paper is organized as follows: Section 2 presents the model, describes optimal behavior, and defines and characterizes the equilibrium. Section 3 presents the main results of the paper. We show that conformity increases with durability and the incidence of trade. Section 3 also considers the welfare implications of conformity. Section 4 discusses the results and considers several possible extensions. Section 5 discusses related literature. Section 6 concludes.

2. Model

We consider a continuous-time matching model in which agents can own one of two types of a durable good. Although as we discussed above, our analysis holds for many durable goods markets, we take the good in our model to be a house. We denote the two types of houses as type a and b. The houses could differ along many dimensions. For example, type a houses could be "traditional" houses while type b could be "modern" houses. Alternatively, type a could be a two-story house with a large yard while type b might be a one-story house with a small yard. Every agent must have a house in every period.

We normalize the utility functions so that all consumers get a flow utility of 1 from living in the type a house. Consumers have different tastes for the type b house. Specifically, each consumer has an individual taste parameter z which quantifies their preference for type b houses. For a consumer with taste parameter $z \in \mathbb{R}$ the flow utility from living in a b house is $1 + z$. Thus, the flow utility for an agent with a given z and a given house $x \in \{a, b\}$ is

$$u_z(x) = \begin{cases} 1 & \text{if } x = a \\ 1 + z & \text{if } x = b \end{cases}.$$

We assume that z is distributed over the population according to a distribution function F. The support of F is a nonempty subset of $\mathbb{R}$.

From time to time agents switch houses. Agents may switch their house for one of two reasons. First, the house may "die." We think of this as capturing normal depreciation but it may also include extreme idiosyncratic events such as fires, etc. When this occurs, the agent must build a new house. We refer to this event as the "build shock." We assume that the build shock obeys a Poisson process with an exogenous arrival rate δ. An agent who gets the build shock decides which type of house to build and incurs a building cost c. Since we want to focus on heterogeneity in tastes, we assume the building cost is the same for all agents and for either type of house.

Second, the agent may be required to trade his house which we refer to as the "trade shock." Agents who get the trade shock must move out of their house and into a new house. We allow agents to differ in the likelihood of receiving the trade shock. Thus, some agents move often while others do not. Heterogeneity in the likelihood of trade allows us to distinguish between individual trade hazards and aggregate trade hazards. We assume that for each agent, the trade shock obeys a Poisson process with an exogenous arrival rate γ. The arrival rate is distributed over the agents according to the distribution G. The support of G is restricted to the interval $[0, \infty]$. Here $\gamma = 0$ corresponds to an individual who never needs to trade and $\gamma = \infty$ implies that the individual trades continuously. Individual trade hazards (γ) and preferences (z) are independent.

To better motivate the trade shock, we imagine that each agent lives and works in one of two cities of equal size. Agents who get the trade shock have to move from one city to the other. When this happens the agent first has an opportunity to trade his house. The agent is matched randomly with a trading partner who is moving in the opposite direction. If both agents agree to trade, they simply exchange houses, otherwise the trade is rejected. If the trade fails, the agents are forced to scrap their old houses and build new houses of their choice. Let π be the difference between the build cost c and the scrap value of their old house. It is important to emphasize that agents are not trading because their preferences over houses change. They trade simply because they have to move from one city to the other. Thus, in the trade state, some agents will exchange houses of the same type (e.g., an a for an a) as well as houses of different types.

Because π is only incurred by agents who fail to trade, we refer to π as the "trade penalty." One can alternatively interpret the trade penalty as reflecting other costs of buying and selling a house. For instance, one could think of π as the expected cost of engaging in a protracted search in an environment with the possibility of re-matching. Under this interpretation π would include the cost of renting while traders search for new houses and would also include the forgone interest on the sale price while a house waits to be sold. Sales commissions, vacancy costs, fees and the costs of renovations could also be included in the trade penalty.

If agents do not get the build shock or the trade shock they simply continue residing in their current house. Agents seek to maximize the discounted sum of flow utilities less costs. The discount rate is $r > 0$. The next section analyzes optimal behavior in this model.

2.1. Optimal Policies and Conformity

In this section we analyze the consumer's maximization problem and present our definition of conformity. In the next section we turn to equilibrium. Throughout, we confine our attention to steady state equilibria. In a steady state, the strategies of the other agents and the distribution of houses are time-invariant. We use standard dynamic programming techniques to analyze the agent's optimization problem. A policy (or strategy) for any agent consists of a decision rule in the trade state and a decision rule in the building state.

Let $V(x; z, \gamma)$ be the value of following an optimal policy for an agent with taste parameter z and trade hazard γ who currently owns a house of type $x \in \{a, b\}$. Let $B(z, \gamma)$ be the continuation value of entering the build state and let $T(x; z, \gamma)$ be the continuation value of entering the trade state when the agent has a type x house. Because we focus on the behavior of a single agent, we suppress the arguments z and γ in the following discussion. The value function satisfies

$$rV(x) = u(x) + \delta [B - V(x)] + \gamma [T(x) - V(x)]. \tag{1}$$

The continuation value of receiving the build shock is simply

$$B = \max \{V(a), V(b)\} - c. \tag{2}$$

In the trade stage, agents observe each other's houses but do not observe their taste parameters or their trade hazards. They then simultaneously choose to either accept or reject the trade. If they both accept, they swap houses. If either disagrees, the trade is rejected, both traders pay the trade penalty π and get new houses of their choice.

To compute $T(x)$, consider an agent who receives the trade shock and currently possesses a type x house. Suppose he is matched with someone with a type y house. If the trade occurs, the first agent gets the type y house and thus his payoff is simply $V(y)$. If either one rejects the trade then his payoff is $\max \{V(a), V(b)\} - \pi$. Notice that the agent's trade decision is only relevant if his trading partner accepts the trade. We assume that agents accept trades whenever $V(y) > \max \{V(a), V(b)\} - \pi$ and reject otherwise.[2] Note that neither $V(y)$ nor

[2] This rules out the trivial and uninteresting equilibrium in which agents reject every trade.

$\max\{V(a), V(b)\} - \pi$ depend on x. Thus, trade decisions are independent of the type of house the agent possesses when he enters the trade stage.

It is easy to show that if an agent builds type $x \in \{a, b\}$ then he also accepts x in trade. To see this, first note that if an agent chooses to build type x, then $V(x) = \max\{V(a), V(b)\}$. As a result, he also chooses x whenever he gets the trade shock and the trade is rejected. If he is offered x in trade, he gets $V(x)$ if he accepts and $V(x) - \pi$ if he declines. We present this observation as a Lemma. Proofs of all propositions are in the appendix.

Lemma 1. *If an agent builds type $x \in \{a, b\}$, then he also accepts x in trade.*

Because an agent's trading decisions are independent of the house he owns and also independent of his trading partner's taste parameter and trade hazard, there are only three relevant trading rules to consider: (1) accept a only; (2) accept b only; or (3) accept either a or b. We denote these trading rules simply as a, b, and ab. We refer to agents who accept only a particular type of house (either a or b but not both) as *exclusive* traders. Agents who play ab and thus accept both types are said to be *inclusive* traders.

Define $\lambda(y, \tau)$ as the probability of being matched with someone who has a type y house and who follows trading rule $\tau \in \{a, b, ab\}$. For example, $\lambda(a, ab)$ is the probability of meeting someone who possesses a type a house and follows trading rule ab. Agents who accept type $x \in \{a, b\}$ houses in trade either follow the exclusive trading rule x or the inclusive trading rule ab. The probability of meeting an agent who accepts a type x house in trade is therefore $\sum_{\tau \in \{x,ab\}} \sum_{y \in \{a,b\}} \lambda(y, \tau)$. We can now write the expected value of entering the trading state with a type x house as

$$\begin{aligned} T(x) &= \sum_{\tau \in \{x,ab\}} \sum_{y \in \{a,b\}} \left(\lambda(y,\tau) \max\{V(y), \max\{V(a), V(b)\} - \pi\}\right) \qquad (3) \\ &+ \left(1 - \sum_{\tau \in \{x,ab\}} \sum_{y \in \{a,b\}} \lambda(y,\tau)\right) \left(\max\{V(a), V(b)\} - \pi\right) \end{aligned}$$

Given any set of values $V(a)$ and $V(b)$, and fixed matching probabilities, equation (3) implies values $T(a)$ and $T(b)$ and equation (2) implies B. Equation (1) then implies a new set of values $\hat{V}(a), \hat{V}(b)$. It is straightforward to show that this mapping satisfies Blackwell's sufficient conditions for a contraction mapping and thus has a unique fixed point.

Lemma 2. *Given matching probabilities* $\lambda(x,\tau)$ *for* $x \in \{a,b\}$ *and* $\tau \in \{a,b,ab\}$ *with* $\sum_{\tau\in\{a,b,ab\}}\sum_{y\in\{a,b\}}\lambda(y,\tau)=1$ *there exist unique values* $V(a)$, $V(b)$, $T(a)$, $T(b)$, *and* B *satisfying (1), (2), and (3).*

We now characterize the optimal policy. An optimal policy consists of a building rule (whether to build a or b) and a trading rule (whether to accept a, b or both). There are only four relevant policies. To see this note that if an agent follows the trade rule a then by Lemma 1 he must build type a in the build stage. Similarly, if he is an exclusive b trader, he builds type b. Thus, without loss of generality we can confine our attention to the following policies: build a and follow trade rule a; build a and follow trade rule ab; build b and follow trade rule b or build b and follow trade rule ab. We write these policies compactly as (x,τ) where $x \in \{a,b\}$ and $\tau \in \{a,b,ab\}$.

Agents with different taste parameters z and different trade hazards γ choose different policies. Intuitively, for a fixed γ, agents with sufficiently negative z's (who strongly dislike type b houses) build type a houses and accept only type a houses in trade. Similarly, agents with sufficiently positive z's (who have strong preferences for b houses) build type b and accept only type b in trade. Agents with intermediate values of z do not have strong preferences for either type and thus accept either a or b in trade. The following proposition formalizes this intuition.

Proposition 1. *Given non-negative* $\lambda(a,a)$, $\lambda(b,b)$, $\lambda(a,ab)$, *and* $\lambda(b,ab)$ *summing to 1, define* $z_1(\gamma)$, $z_2(\gamma)$, *and* $z_3(\gamma)$ *as follows:*

$$z_1(\gamma) = -\pi\{r+\delta+\gamma[1-\lambda(a,a)]\}$$

$$z_2(\gamma) = \gamma\pi[\lambda(a,a)-\lambda(b,b)]$$

$$z_3(\gamma) = \pi\{r+\delta+\gamma[1-\lambda(b,b)]\}$$

Then, for an agent with taste parameter z *and trade hazard* γ, *if* $z \leq z_1(\gamma)$ *then* (a,a) *is an optimal strategy; if* $z_1(\gamma) \leq z \leq z_2(\gamma)$, (a,ab) *is an optimal strategy; if* $z_2(\gamma) \leq z \leq z_3(\gamma)$, (b,ab) *is an optimal strategy and if* $z_3(\gamma) \leq z$, (b,b) *is an optimal strategy.*

Notice that the cutoffs depend only on the number of exclusive traders of each type. The precise distribution of tastes of other traders is not relevant once $\lambda(a,a)$ and $\lambda(b,b)$ are given. In particular, the average taste parameter is unimportant. The building decisions of the other agents are also irrelevant. Given $\lambda(a,a)$, $z_1(\gamma)$

is closer to zero the closer r, δ and γ are to 0. Thus, given z, an agent is more likely to be exclusive if the object is very durable (low δ), if the agent is very patient (low r), or if it is unlikely that the agent will trade (low γ). Naturally, if the object is very durable and you are not likely to trade it, *and* you care a lot about the future, then you don't want to get stuck with the wrong house.

It is important to understand the intuition behind this proposition. Because the logic is identical for the b cutoff ($z_3(\gamma)$) we focus on the a cutoff ($z_1(\gamma)$). Consider an agent with a taste parameter $z < 0$ who optimally follows an (a, a) policy. Suppose that upon receiving the trade shock and being matched with someone who has a b, he deviates from his optimal policy. Specifically, suppose he decides to accept the b house but then revert to the trade rule a for subsequent trade shocks. He benefits by avoiding the trade penalty π. There are two costs however. First, he will reside in a house other than his preferred type for some time. This expected loss is $z/(r + \delta + \gamma)$ (recall $z < 0$). Second, he may encounter someone who follows an exclusive a trading rule while he still has the b house. In this case he pays the trade penalty π.[3] This expected cost is $\pi\gamma\lambda(a, a)/(r + \delta + \gamma)$. The discount rate $(r + \delta + \gamma)$ reflects both the agents impatience and the likelihood of moving out of the type b house (which occurs if the house dies or if he gets another trade shock). There are no additional costs because the trade penalties in every other circumstance are the same as if he continued to follow policy (a, a). The net benefit of this deviation is

$$\pi + \frac{z}{r + \delta + \gamma} - \pi\frac{\gamma\lambda(a, a)}{r + \delta + \gamma} \leq 0 \tag{4}$$

The inequality follows because we assumed that it was optimal to follow (a, a). If the agent is indifferent between (a, a) and (a, ab), this expression would hold with equality. Rearranging this expression shows that the expression is zero only if $z = z_1(\gamma)$ as given in Proposition 1.

The $z_2(\gamma)$ cutoffs are of special interest because they determine the equilibrium number of houses of each type built. If $z_2(\gamma) > 0$ for instance, there may be people who prefer b houses but choose to build a's. If an agent builds a house other than the type dictated by his taste parameter z, then we say the agent is *conforming* to the market.

Definition 1. *If $z_2(\gamma) \neq 0$ for some γ then we say that there is conformity in the market. If $z_2(\gamma) > 0$ for all γ then the market conforms on type a houses, and if*

[3] He would not pay this trade penalty if he did not deviate from the (a, a) strategy.

$z_2(\gamma) < 0$ *for all* γ *then the market conforms on type* b *houses. If* $z_2(\gamma) = 0$ *for all* γ *then we say that there is no conformity.*

Like the trading cutoffs $z_1(\gamma)$ and $z_3(\gamma)$ the building cutoffs $z_2(\gamma)$ are determined by the number of extreme traders. In particular, $z_2(\gamma)$ are determined by the difference $\lambda(a,a) - \lambda(b,b)$. If $\lambda(a,a) > \lambda(b,b)$ then $z_2(\gamma) > 0$ for all γ. Only if $\lambda(a,a)$ and $\lambda(b,b)$ are exactly equal will $z_2(\gamma) = 0$.

We now define and characterize the equilibrium.

2.2. Equilibrium

We analyze steady state equilibria. In a steady state, the matching probabilities and the distribution of existing houses remain constant over time. Because the matching probabilities are constant, in a steady state equilibrium, the optimal policies described in the previous section are also constant over time.

To find steady state equilibria we solve a fixed point problem. Given perceived matching probabilities $\lambda(a,a)$, $\lambda(b,b)$, $\lambda(a,ab)$, and $\lambda(b,ab)$, agents follow optimal policies. The policies, in turn, imply matching probabilities. This leads to a mapping from perceived probabilities to implied probabilities. A steady state equilibrium is a fixed point of this mapping. The first step of the mapping is described in Proposition 1. The second step takes the cutoffs $z_1(\gamma)$, $z_2(\gamma)$, and $z_3(\gamma)$ and derives the implied matching probabilities.

Computing $\lambda(a,a)$ and $\lambda(b,b)$ is relatively straightforward as they are simply the numbers of people who follow policies (a,a) and (b,b). Unfortunately, the numbers of people who follow policy (a,ab) and policy (b,ab) are not necessarily equal to $\lambda(a,ab)$ and $\lambda(b,ab)$. While some people in $\lambda(a,ab)$ follow policy (a,ab), others follow policy (b,ab) and have traded for a type a house in the past. To clarify this distinction we introduce the following notation: Let $P_s(x)$ denote the number of people who follow policy s and hold a type x house where $x \in \{a,b\}$ and $s \in \{(a,a),(b,b),(a,ab),(b,ab)\}$. Using this notation, $\lambda(a,ab) = P_{a,ab}(a) + P_{b,ab}(a)$ and $\lambda(b,ab) = P_{b,ab}(b) + P_{a,ab}(b)$. The complication arises because the number of people who follow policy (a,ab), which is $P_{a,ab}(a) + P_{a,ab}(b)$ is different than $\lambda(a,ab) = P_{a,ab}(a) + P_{b,ab}(a)$, the number of people who follow an inclusive trading rule but possess an a.

Given the cutoff functions $z_1(\gamma)$, $z_2(\gamma)$, and $z_3(\gamma)$, one can compute $P_s(x)$ for all s and x and thus solve for $\lambda(a,ab)$ and $\lambda(b,ab)$. As it turns out, only $\lambda(a,a)$ and $\lambda(b,b)$ are necessary for the characterization of the equilibrium. The following lemma completes the description of the fixed point mapping by showing that, given the cutoff functions, there is a unique set of implied matching probabilities.

Lemma 3. *Let $z_1(\gamma), z_2(\gamma)$ and $z_3(\gamma)$ be given. The implied steady state matching probabilities $\lambda(a,a)$ and $\lambda(b,b)$ are given by*

$$\lambda(a,a) = \int_0^\infty F(z_1(\gamma))\, dG(\gamma) \text{ and } \lambda(b,b) = \int_0^\infty [1 - F(z_3)]\, dG(\gamma).$$

Also, there exist unique nonnegative numbers $P_{a,ab}(a)$, $P_{a,ab}(b)$, $P_{b,ab}(a)$ and $P_{b,ab}(b)$ such that $\lambda(a,ab) = P_{a,ab}(a) + P_{b,ab}(a)$ and $\lambda(b,ab) = P_{a,ab}(b) + P_{b,ab}(b)$.

Once we have the matching probabilities, we can find the composition of types in the steady state housing stock. Specifically, the number of type a houses is $\lambda(a,a) + \lambda(a,ab)$ and the number of type b houses is $\lambda(b,b) + \lambda(b,ab)$. The distribution of types in the housing stock is the same as the distribution of types for sale.[4] However, the distribution of houses built differs from the distribution in the housing stock. The reason these numbers differ is that typically one type of house is rejected more often than the other type. When a trade is rejected, agents effectively transform their current house into the type of house they want (at cost π). In reality, houses that are difficult to sell remain vacant for some time, which causes the distribution of houses built to differ from the distribution of houses for sale. Vacancy costs are one interpretation of the trade penalty π.

Below we present a formal definition of a steady state equilibrium.

Definition 2. *A steady state equilibrium consists of four non-negative numbers $\lambda(a,a), \lambda(b,b), \lambda(a,ab)$, and $\lambda(b,ab)$ summing to one, and three cutoff functions $z_1(\gamma), z_2(\gamma)$ and $z_3(\gamma)$ such that*

1. *Given $\lambda(a,a)$, $\lambda(b,b)$, $\lambda(a,ab)$, and $\lambda(b,ab)$, Proposition 1 implies the cutoff functions $z_1(\gamma)$, $z_2(\gamma)$ and $z_3(\gamma)$.*
2. *Given $z_1(\gamma), z_2(\gamma)$ and $z_3(\gamma)$, Lemma 3 implies $\lambda(a,a), \lambda(b,b), \lambda(a,ab)$, and $\lambda(b,ab)$.*

[4] This follows because houses for sale are a random selection from existing houses and because houses do not stay on the market after their initial match.

To prove that an equilibrium exists, recall that the cutoff functions for the exclusive a and b traders ($z_1(\gamma)$ and $z_3(\gamma)$) each depend only on the number of exclusive a and b traders. More precisely, $z_1(\gamma)$ is completely determined once $\lambda(a,a)$ is given and $\lambda(a,a)$ depends only on the cutoff function $z_1(\gamma)$. Similarly $z_3(\gamma)$ depends only on $\lambda(b,b)$ and vice versa. As a result, we can analyze the determination of these cutoffs separately. Define two mappings $L_{a,a} : [0,1] \to [0,1]$ and $L_{b,b} : [0,1] \to [0,1]$ as

$$L_{a,a}(\lambda(a,a)) = \int_0^\infty F(-\pi\{r+\delta+\gamma[1-\lambda(a,a)]\})\, dG(\gamma) \in [0,1] \tag{5}$$

$$L_{b,b}(\lambda(b,b)) = \int_0^\infty [1 - F(\pi\{r+\delta+\gamma[1-\lambda(b,b)]\})]\, dG(\gamma) \in [0,1]. \tag{6}$$

While these mappings may not be continuous (if F has mass points), they are both increasing functions on a compact set which implies (by Tarski's fixed point theorem) that each has at least one fixed point.

Lemma 4. *The mappings $L_{a,a}$ and $L_{b,b}$ defined by (5) and (6) each have at least one fixed point.*

Any combination of fixed points of these mappings corresponds to equilibrium values of $\lambda(a,a)$ and $\lambda(b,b)$. To complete the construction of an equilibrium, given any fixed points $\lambda(a,a)$ and $\lambda(b,b)$, define $z_1(\gamma)$, $z_2(\gamma)$ and $z_3(\gamma)$ as in Proposition 1. Then $\lambda(a,ab)$ and $\lambda(b,ab)$ are given uniquely by Lemma 3. The resulting probabilities $\lambda(a,a)$, $\lambda(b,b)$, $\lambda(a,ab)$, and $\lambda(b,ab)$, and cutoffs $z_1(\gamma)$, $z_2(\gamma)$ and $z_3(\gamma)$ satisfy the definition of a steady state equilibrium. This establishes the following proposition:

Proposition 2. *Given any distribution of types F and distribution of trade hazards G, there exists at least one steady state equilibrium.*

We now present two examples. Example 1 illustrates a case with a unique equilibrium. Example 2 shows that conformity may arise due to multiplicity of equilibria even when the distribution F is symmetric around zero. Both examples consider the case in which all agents have a common trade hazard γ.

Example 1: Suppose F is uniform on $[-q+\mu, q+\mu]$.[5] Figure 1 shows the fixed

[5] The figure is drawn under the assumption that $-q+\mu < -\pi(r+\delta+\gamma)$.

point mappings $L_{a,a}$ and $L_{b,b}$ for two different values of the mean taste μ. The light dashed line corresponds to $\mu = 0$. In this case, the distribution is symmetric around zero and thus $L_{a,a} = L_{b,b}$. In equilibrium $\lambda(a,a) = \lambda(b,b)$, $z_2 = 0$ and there is no conformity. The dark lines correspond to $\mu > 0$. Because F has shifted to the right, $L_{b,b}$ has shifted up while $L_{a,a}$ has shifted down. In the new equilibrium $\lambda(a,a) < \lambda(b,b)$ and $z_2 < 0$ so the market conforms on b.

Example 2: Suppose F is symmetric about 0 but not uniform. Symmetry implies that the mappings $L_{a,a}$ and $L_{b,b}$ are identical. Figure 2 shows an example with three fixed points: $\lambda_1^* < \lambda_2^* < \lambda_3^*$. By setting $\lambda(a,a) = \lambda_i^*$ and $\lambda(b,b) = \lambda_j^*$ with $i, j = 1$, 2 or 3 we can construct nine possible equilibria. Three of these are non-conforming equilibria (when $\lambda(a,a) = \lambda(b,b)$) while the other six are conforming equilibria (when $\lambda(a,a) \neq \lambda(b,b)$).

Notice that conformity is a generic property of equilibrium. Non-conforming equilibria occur only in knife-edge cases in which $\lambda(a,a) = \lambda(b,b)$.

3. Comparative Statics and Welfare

In this section we analyze the relationship between conformity and the underlying parameters of the model. We pay particular attention to durability and the frequency of trade. We then discuss the welfare properties of the equilibrium.

3.1. Comparative Statics

We use comparative statics to highlight several features of the equilibrium. Specifically, we consider how variations in the durability of the good and the subjective time discount factor affect the equilibrium. The frequency of trade also matters both for individual behavior and for the equilibrium. We consider variations in the individual trade hazards holding the aggregate distribution of trade hazards fixed. We also consider shifts in the distribution of trade hazards itself.

To facilitate the analysis, we place restrictions on the distribution F to rule out multiple equilibria. The following assumption provides sufficient conditions for a unique equilibrium.

Assumption 1. *F has a density function f which is symmetric about the mean μ, and quasi-concave with $f(\mu) < \frac{1}{\pi\bar{\gamma}}$ where $\bar{\gamma}$ is the mean trade hazard.*

Under assumption 1, we can now present the following proposition:

Proposition 3. *If F satisfies Assumption 1 then*

1. *The equilibrium is unique.*

2. *The market conforms to the mean taste whenever $\mu \neq 0$ (i.e., the market conforms on a if $\mu < 0$ and conforms on b if $\mu > 0$).*

3. *If $\mu \neq 0$, an increase in durability (lower δ) or patience (lower r) increases conformity (i.e., $z_2(\gamma)$ increases for all γ if $\mu < 0$ and decreases for all γ if $\mu > 0$).*

4. *Agents with a greater likelihood of trade conform more.*

Part 1 of the proposition (uniqueness) follows from the bound on f in Assumption 1 which ensures that the fixed point mappings $L_{a,a}$ and $L_{b,b}$ never have slopes greater than 1. Part 2 of the proposition demonstrates that conformity is a generic property within this class of distributions.

The third result in the proposition demonstrates that there is more conformity for goods that are more durable and when consumers care more about the future. Trading for a good that you don't prefer is costly if the good is expected to survive for a long time and if you care about the future. Consequently, if the good is more durable or if people are more patient, more agents follow exclusive trading strategies. Our distributional assumptions guarantee that the increase in exclusive trading is greatest for the good that the market conforms to. Because conformity depends on the relative number of exclusive traders, conformity increases as the good becomes more durable or as people become more patient.

The final part of the proposition says that agents who trade frequently conform more. If an agent finds himself in the trade stage often, he will reduce the incidence of the trade penalty by conforming. Agents who rarely trade can indulge in goods that satisfy their idiosyncratic tastes.

The last result suggests that regions with high turnover (i.e., where everyone has a high trade hazard) may also be regions of greater conformity. Below, we show that this is not necessarily true. The heterogeneity in trade hazards γ allows us to distinguish between individual trade hazards and aggregate trade hazards. Holding aggregate trade hazards fixed, an increase in an individual's trade hazard necessarily increases conformity for that agent. While increases in aggregate trade hazards must increase conformity for some agents, it need not increase conformity for all.

To formalize this, we consider a rightward shift of the distribution G. Specifically, write each agent's trade hazard as $(\gamma + \theta)$ where $\theta \geq 0$ is common to all agents and γ is the agent's idiosyncratic trade hazard. We can then associate a marginal increase in the aggregate trade frequency with a marginal increase in θ at $\theta = 0$. Differentiating the $z_2(\gamma)$ cutoff with respect to θ and evaluating at $\theta = 0$ gives the following expression:

$$\left.\frac{\partial z_2(\gamma)}{\partial \theta}\right|_{\theta=0} = \underbrace{\frac{z_2(\gamma)}{\gamma}}_{\text{"Exposure Effect"}} + \underbrace{\gamma\pi\left[\left.\frac{\partial \lambda(a,a)}{\partial \theta}\right|_{\theta=0} - \left.\frac{\partial \lambda(b,b)}{\partial \theta}\right|_{\theta=0}\right]}_{\text{"Composition Effect"}}.$$

The change in $z_2(\gamma)$ depends on two terms. The first term always has the same sign as $z_2(\gamma)$ and thus always serves to increase conformity. Using our expression for $z_2(\gamma)$ we can rewrite this term as $\pi[\lambda(a,a) - \lambda(b,b)]$ which is independent of γ. This term captures the change in $z_2(\gamma)$ from an increase in the trade hazard holding the number of exclusive traders ($\lambda(a,a)$ and $\lambda(b,b)$) fixed. Because the agent trades more frequently, he encounters exclusive traders more often and thus chooses to conform more. We refer to this term as the "exposure effect."

The second term, the "composition effect," captures the effects on $z_2(\gamma)$ holding a given agent's trade hazard fixed but considering variations in the number of exclusive traders caused by the shift in the distribution G. It turns out that the composition effect can be either positive or negative depending on the distributions F and G. Notice however, the importance of composition effect depends positively on γ. Thus, for agents with sufficiently small trade hazards, the exposure effect dominates and conformity increases. For agents with sufficiently high trade hazards, the composition effect dominates and thus they may or may not conform more. If the composition effect is the same sign as $z_2(\gamma)$ then conformity increases with the general frequency of trade. Rather than encouraging market diversity, increased trade in durable goods encourages conformity.

We summarize this result in the following proposition.

Proposition 4. *Assume that agents trade hazards are written as $(\gamma + \theta)$ as described above and consider an F which satisfies Assumption 1. Then, for a marginal increase in θ at $\theta = 0$, there exists $\hat{\gamma} \in (0, \infty]$ such that conformity increases for all agents with $\gamma < \hat{\gamma}$ and decreases for all agents with $\gamma > \hat{\gamma}$.*

Similar arguments hold for changes in the trade penalty π. There are two effects, an exposure effect and a composition effect. The exposure effect always causes conformity to increase while the composition effect may cause conformity to increase or decrease.[6]

3.2. Welfare

In markets for durable goods, conformity is the rule rather than the exception. Here, we consider the welfare implications of conformity. While conformity has an obvious cost – people live in houses that they don't prefer – it also has benefits. First, conformity lowers search costs. Naturally, social welfare is lower when houses are vacant or when people are forced to conduct protracted searches for a good match. Second, when houses are exchanged, the new owner may not be an ideal match for the house. The potential mismatch of preferences and allocations is another cost to society that conformity mitigates. Thus, a social planner may desire some conformity in durable goods markets.

Below, we explore the welfare implications of conformity in our model. To facilitate the exposition, we simplify the model first by restricting attention to situations in which all agents have the same trade hazard and all are perfectly patient (i.e., $r = 0$). This second assumption allows us to restrict our attention to the steady state flow of total welfare.[7] The following Lemma provides an expression for the flow of social welfare in the steady state when all agents have a common trade hazard. Because there is a single trade hazard, we can consider a single set of cutoffs z_1, z_2, and z_3.

Lemma 5. *For any given cutoffs z_1, z_2 and z_3, let $P_{a,ab}(a)$, $P_{b,ab}(a)$, $P_{a,ab}(b)$ and $P_{b,ab}(b)$ be given as in Lemma 3 and define $\psi_{a,ab} = P_{a,ab}(a) / (F(z_2) - F(z_1))$ and $\psi_{b,ab} = P_{b,ab}(b) / (F(z_3) - F(z_2))$. Then, the flow of social welfare is*

$$\begin{aligned} W(z_1, z_2, z_3) = {} & \{1 - \pi\gamma[\lambda(b,b) + \lambda(b,ab)]\} F(z_1) \\ & + \int_{z_1}^{z_2} \left\{\psi_{a,ab}[1 - \pi\gamma\lambda(b,b)] + \left(1 - \psi_{a,ab}\right)[1 + z - \pi\gamma\lambda(a,a)]\right\} dF(z) \\ & + \int_{z_2}^{z_3} \left\{\left(1 - \psi_{b,ab}\right)[1 - \pi\gamma\lambda(b,b)] + \psi_{b,ab}[1 + z - \pi\gamma\lambda(a,a)]\right\} dF(z) \\ & + \int_{z_3}^{\infty} \{1 + z - \pi\gamma[\lambda(a,a) + \lambda(a,ab)]\} dF(z) - \delta c. \end{aligned} \tag{7}$$

[6] Our model assumes that π is common to all agents. If we allowed for heterogeneity in trade penalties we would obtain a result similar to Proposition 4 for trade penalties.

[7] All of the conclusions in this section hold for $r > 0$ provided that r is sufficiently small.

Each term in the welfare function has a natural economic interpretation. The first term is the flow of welfare to the (a, a) traders. Since they are exclusive traders, they always hold type a houses and thus they all receive a flow utility of 1. In addition, they sometimes incur the trade penalty by matching with (b, b) and (b, ab) traders. The second term is the flow of welfare for agents who follow the (a, ab) policy. These individuals sometimes hold a houses but sometimes hold b's. The fraction of time these agents hold the a house is $\psi_{a,ab}$. In this case, their flow utility is 1 less the trade hazard of meeting an exclusive b trader. The remaining fraction of time $\left(1 - \psi_{a,ab}\right)$ they hold b houses. In this case, their flow utility is $1 + z$ less the trade hazard of meeting an exclusive a trader. The remaining terms have analogous interpretations. All agents are equally likely to experience the depreciation shock so all welfare flows are reduced by δc.

We now consider the welfare implications of increasing or decreasing conformity. Conformity can increase welfare through two separate channels. First, increased conformity can reduce the incidence of the trade penalty. Second, increased conformity means that the average type resides in his preferred house more often. Differentiating expression (7) with respect to z_2, one can show that

$$\frac{\partial W}{\partial z_2} = z_2 \frac{\partial \lambda\left(a, ab\right)}{\partial z_2} + \left\{ \frac{\partial \psi_{a,ab}}{\partial z_2} \int_{z_1}^{z_2} \left[z_2 - z\right] dF\left(z\right) + \frac{\partial \psi_{b,ab}}{\partial z_2} \int_{z_2}^{z_3} \left[z - z_2\right] dF\left(z\right) \right\}. \tag{8}$$

Let z_1^*, z_2^* and z_3^* be equilibrium cutoffs. If $\partial W\left(z_1^*, z_2^*, z_3^*\right)/\partial z_2$ is the same sign as z_2^*, then there is too little conformity in equilibrium. To see this, suppose $z_2^* > 0$ so the market conforms on a. If $\partial W/\partial z_2 > 0$ then the flow of social welfare would increase if z_2 were greater, that is, if there were even more conformity.

The first term in (8) gives the change in welfare that arises due to changes in the incidence of trade penalties caused by an increase in z_2. It can be shown that $\partial \lambda\left(a, ab\right)/\partial z_2 > 0$ so the sign of the first term is the same as the sign of z_2. The second term captures the change in welfare due to changes in residence patterns among the the inclusive traders caused by an increase in z_2. Unlike the first term, the second does not necessarily have the same sign as z_2. The following proposition provides conditions under which the second term has the same sign as z_2 and thus there is too little conformity in equilibrium.

Proposition 5. *Let the average preference of the inclusive traders be* $\mu_{ab} = \int_{z_1}^{z_3} z dF\left(z\right)/\left[F\left(z_3\right) - F\left(z_1\right)\right]$, *let* F *satisfy Assumption 1, and let* z_1^*, z_2^* *and* z_3^* *be equilibrium cutoffs. If either (i)* $\operatorname{sign}\left(\mu_{ab}\right) = \operatorname{sign}\left(\mu\right)$, *or (ii)* $\left|\mu\right| \geq \frac{\pi(\gamma+\delta)}{2}$, *then* $\text{sign}\left(\partial W\left(z_1^*, z_2^*, z_3^*\right)/\partial z_2\right) = \text{sign}\left(z_2^*\right)$ *and there is too little conformity in equilibrium.*

In words, if the inclusive traders on average prefer the same type of house as the society as a whole, or if the average preference is sufficiently strong, there is too little conformity in equilibrium. Note, while condition (i) is defined in terms of endogenous variables, (ii) relies only on exogenous parameters μ, π, γ and δ.[8]

A Numerical Illustration Here we consider the equilibrium and optimal level of conformity for a continuum of agents when the taste parameter is distributed normally. We maintain the assumption of a common trade hazard. We numerically solve for the equilibrium and calculate the optimal level of conformity.

In Figure 3, we plot the optimal the cutoffs and the equilibrium cutoffs for several different means of the distribution $F(z)$. As mentioned above, we assume that the distribution of tastes F is normal with unit variance and with mean μ. The other parameters are $r = 0.02$, $\delta = 0.05$, $\gamma = 0.10$, and $\pi = 5$ (the building cost c matters neither for equilibrium nor for welfare comparisons). The discount rate and depreciation rate are roughly in line with their real-world counterparts. The trade hazard rate implies that people move roughly once every ten years. The figure shows that for $\mu > 0$ there is conformity in equilibrium ($z_2 < 0$). Not surprisingly, the equilibrium level of conformity rises with the mean. Notice that the optimal level of conformity rises even faster than the equilibrium level of conformity. Also, as μ rises, the cutoffs for the exclusive a and b traders both fall.

4. Discussion and Extensions

The model could be extended in a variety of potentially interesting dimensions. Here we briefly discuss several possible extensions. One extension would be to allow for more realistic trading mechanisms. Allowing agents to negotiate over prices or to match repeatedly would change the equilibrium but most features of our analysis would likely survive. As long as durable goods are held by agents with different preferences, there is pressure to conform under any trading mechanism. To the extent that bargaining improves efficiency, there would be even more conformity with such negotiations. The inefficiencies in our model, will also likely

[8] While environments that do not satisfy the conditions in Proposition 5 often still have too little conformity, it is possible to construct examples with too much conformity. See House and Ozdenoren [2006].

be present with other trading mechanisms. If preferences are private information, other mechanisms cannot in general produce efficient allocations (Myerson and Satterthwaite (1983)). Even if trading mechanisms were efficient (which requires observable preferences), there would still be inefficiency at the build stage to the extent that agents receive only some of the surplus in the trade stage and thus do not fully internalize the value of conformity at the build stage.[9]

Introducing a rental market would allow agents to enjoy a durable without worrying about resale. Intuitively, the option to rent is more valuable if one has unusual tastes. For instance, luxury or exotic cars are often leased while more mainstream cars are usually bought outright. This is exactly the pattern one would expect given our model. People who lease exotic cars do not want to conform and thus do not want to participate in the secondary market. In contrast, people who own mainstream cars can easily find buyers if they need to sell.

Another extension for subsequent study would be to consider imperfect competition in our framework. To get intuition for how imperfect competition might affect the results, suppose each type of durable is supplied by a different firm (i.e., a duopoly). In this case, the building prices would exceed marginal cost c by a type-specific markup. Since the duopolists compete over the same group of potential buyers (the buyers at the z_2 margin), the change in demand caused by price changes (i.e., dQ/dP) is the same for both firms. Suppose $\mu < 0$ and the two firms were to charge the same price. In this case, our analysis applies and there is conformity on type a and higher demand for a houses. Consequently, the elasticity of demand $((dQ/dP) \times P/Q)$ will be lower for the firm that produces a houses and therefore this firm has a greater incentive to increase its price. Thus, in the duopoly there is a conformity premium. By reducing demand for the a house, the conformity premium reduces conformity relative to the competitive case. Interestingly, in durable goods markets, imperfect competition inefficiently increases product diversity.

As another extension, one could consider the market for new homes in a model with a small city and a large city. In our model, agents have to switch houses because they must move from one city to another. If one city were larger than the other, conformity would differ across the locations. In particular, if the trade shock reflected job-to-job transitions, there would be greater conformity in the

[9] Under Nash bargaining, every efficient exchange occurs but the traders only capture half of the surplus and thus would not conform enough. Moreover, with Nash bargaining and repeated matching, owning a house that conforms to the majority taste would increase the value of an agent's outside option and provide greater incentive to conform.

smaller city. In the large city, changing jobs does not necessarily imply that you must move. In contrast, in the small city, changing jobs often requires moving to the large city and thus you must trade your house. Thus, the trade hazard is greater for people living in the small city and they conform more.

Finally, one could test empirical predictions of the model. The model predicts that, all else equal, there should be more conformity for goods with lower depreciation rates. Furthermore, individuals that are likely to move should conform and purchase typical houses. Tenured professors for example should live in houses with more "character" compared to untenured professors who should own typical houses (casual observation suggests that this is indeed the case).

5. Related Literature

Our paper is related with three separate lines of research. The first is the literature on durable goods in matching models. Two papers are particularly noteworthy. Wheaton (1990) considers a search model of housing with two types of occupants (families and singles) and two types of houses (large and small). The focus of his paper is on the optimal level of search intensity. Smith (1997) considers a matching model with many types of agents with idiosyncratic tastes and many types of perfectly durable goods. Smith uses his model to demonstrate the "risk-increasing" nature of trades. In both Smith (1997) and Wheaton (1990), the supply of durable goods is exogenous. As a result, neither Smith nor Wheaton address conformity as we do here.[10]

Second, our paper is related to the literature on liquidity and matching models. (See among others Kiyotaki and Wright (1989) and (1993)). In these models, fiat money has value because it provides liquidity. In our model, the value of a good reflects both its intrinsic utility and the liquidity it provides. Unlike our model however, in the money-search literature, there is typically an exogenous double-coincidence of wants problem. Agents cannot produce the good they consume. In our model, agents can produce the good they consume so there is no exogenous double coincidence problem. Indeed, if depreciation rates are very high, the goods are essentially non-durable and there is no conformity – agents simply produce the good they prefer. The source of the double-coincidence problem in our model is

[10]There is also a literature on the provision and resale of durable goods in market settings (see Waldman (2003) for a summary). Following Akerlof (1970), much of this literature focuses on how adverse selection problems affect the provision and resale of durables. See Hendel and Lizzeri (1999), (2002), and House and Leahy (2004)) for recent contributions.

durability. Durable goods survive over long periods of time and may be consumed by many different people. Unless markets are very efficient at matching buyers and sellers, durability endogenously creates a double-coincidence problem.

Finally, there is the literature on conformity itself as in Bernheim (1994). The main difference between our environment and Bernheim's is that the desire to conform in our model arises endogenously through resale concerns while in Bernheim's model, agents conform because their preferences place weight on public perceptions of their type.[11]

6. Conclusion

A consumer's demand for a durable good is governed not only by his individual preferences but also by the preferences of other market participants. This interdependence of preferences arises because of the inevitable resale of durable goods. If a majority of the people who buy durables want goods with certain features, the original owners choose to buy goods with these features even if they do not like them. The incentive to conform to the average taste is strongest for long-lived durable goods and for people who trade frequently. For non-durable goods or for durables that are rarely traded, there is little incentive to conform.

There are two features which lead to conformity in our model. First, because there is a chance that agents will have to sell their house, they care about its resale value. The lower the depreciation rate is, and the more likely it is that they will have to enter the resale market, the more they care about the resale value. In the model, the resale value of the home is determined simply by the likelihood that it will be accepted in trade. Second, frictions in the resale market (due to matching) generate the possibility that the house will be purchased by someone with different preferences from the current owner. As a result, the resale value depends on the average preferences of the buyers in the resale market.

In equilibrium there is typically too little conformity relative to the social optimum. By not conforming, agents impose two negative externalities on other agents. First, people with typical preferences incur greater search costs because of excessive product diversity in the secondary market. Second, some people with moderate tastes settle goods they do not prefer. Because the original buyers do not fully internalize these costs, conformity is inefficiently low.

[11] Other papers dealing with conformity include Akerlof (1980), Kandori (1992), and Okuno-Fujiwara and Postlewaite (1995). Also related is the literature on product diversity. See Spence (1976), Dixit and Stiglitz (1977), Mankiw and Whinston (1987), and Tirole (1993) chapter 7.

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Appendix: Proofs of the Propositions

This appendix provides proofs of the propositions. Proofs of the Lemmas are available in the online appendix.

Proposition 1 *Given non-negative* $\lambda(a,a)$, $\lambda(b,b)$, $\lambda(a,ab)$, *and* $\lambda(b,ab)$ *summing to 1, define* $z_1(\gamma)$, $z_2(\gamma)$, *and* $z_3(\gamma)$ *as follows:*

$$z_1(\gamma) = -\pi\{r+\delta+\gamma[1-\lambda(a,a)]\}$$

$$z_2(\gamma) = \gamma\pi[\lambda(a,a)-\lambda(b,b)]$$

$$z_3(\gamma) = \pi\{r+\delta+\gamma[1-\lambda(b,b)]\}$$

For an agent with parameters z, γ, *if* $z \leq z_1(\gamma)$ *then* (a,a) *is optimal; if* $z_1(\gamma) \leq z \leq z_2(\gamma)$, (a,ab) *is optimal; if* $z_2(\gamma) \leq z \leq z_3(\gamma)$, (b,ab) *is optimal and if* $z_3(\gamma) \leq z$, (b,b) *is optimal.*

Proof. We focus on the continuation value when an agent receives the build shock. Agents follow one of the four strategies $\{a,a\}$, $\{a,ab\}$, $\{b,ab\}$, and $\{b,b\}$. We calculate the building continuation value for each strategy. The optimal strategy is the one with the highest value in the building stage. Suppose that an agent with parameters z, γ follows $\{a,a\}$. We write $v_{a,a}(.)$ and $\tau_{a,a}(.)$ to denote the value of following this strategy and the expected value of receiving the trade shock. Note these strategies may not be optimal (i.e. $v_{a,a}(x) \leq V(x)$ and $\tau_{a,a}(x) \leq T(x)$). We consider each policy in turn.

1. Strategy $\{a,a\}$. If he follows $\{a,a\}$ then he always rejects b in the trade stage and $\tau_{a,a}(b)$ is irrelevant. The trade value $\tau_{a,a}(a) = v_{a,a}(a) - \pi[\lambda(b,ab)+\lambda(b,b)]$ and thus the consumption stage value is,

$$rv_{a,a}(a) = 1-\delta c-\gamma\pi[\lambda(b,ab)+\lambda(b,b)]. \tag{8}$$

2. Strategy $\{a,ab\}$. Because the agent accepts either a or b in trade, we must calculate the trade value of a and b. Similarly, we must consider the consumption value of a and b. The trade value of possessing a is $\tau_{a,ab}(a) = v_{a,ab}(a) + \lambda(b,ab)[v_{a,ab}(b)-v_{a,ab}(a)] - \pi\lambda(b,b)$. The trade value of possessing b is $\tau_{a,ab}(b) = v_{a,ab}(a) + [\lambda(b,b)+\lambda(b,ab)][v_{a,ab}(b)-v_{a,ab}(a)] - \pi\lambda(a,a)$. Thus $v_{a,ab}(a)$ and $v_{a,ab}(b)$ satisfy

$$rv_{a,ab}(a) = 1-\delta c+\gamma\{\lambda(b,ab)[v_{a,ab}(b)-v_{a,ab}(a)]-\pi\lambda(b,b)\}$$

$$\begin{aligned} rv_{a,ab}(b) &= 1+z-\delta c-\delta[v_{a,ab}(b)-v_{a,ab}(a)] \\ &\quad +\gamma\{[\lambda(b,b)+\lambda(b,ab)][v_{a,ab}(b)-v_{a,ab}(a)]-\pi\lambda(a,a)\} \\ &\quad -\gamma[v_{a,ab}(b)-v_{a,ab}(a)] \end{aligned}$$

Solving for $v_{a,ab}(a)$ gives

$$rv_{a,ab}(a) = 1+\gamma\left\{\lambda(b,ab)\left[\frac{z+\gamma\pi[\lambda(b,b)-\lambda(a,a)]}{r+\delta+\gamma[1-\lambda(b,b)]}\right]-\pi\lambda(b,b)\right\}-\delta c.$$

Consider an agent who receives the build shock. If he follows $\{a,ab\}$, his payoff is $v_{a,ab}(a)-c$ while if he follows $\{a,a\}$ his payoff is $v_{a,a}(a)-c$. The agent prefers $\{a,ab\}$ to $\{a,a\}$ if $v_{a,ab}(a) > v_{a,a}(a) \Leftrightarrow$

$$z > -\pi\{r+\delta+\gamma[1-\lambda(a,a)]\} \equiv z_1(\gamma)$$

Any agent with parameters z, γ with $z > z_1(\gamma)$ will prefer $\{a,ab\}$ to $\{a,a\}$.

3. Strategy $\{b,ab\}$. Following the argument above we find $\tau_{b,ab}(a) = v_{b,ab}(b)+[\lambda(a,a)+\lambda(a,ab)][v_{b,ab}(a)-v_{b,ab}(b)] - \pi\lambda(b,b)$ and $\tau_{b,ab}(b) = v_{b,ab}(b)+\lambda(a,ab)[v_{b,ab}(a)-v_{b,ab}(b)]-\pi\lambda(a,a)$. Thus $v_{a,ab}(a)$ and $v_{a,ab}(b)$ satisfy

$$\begin{aligned} rv_{b,ab}(a) &= 1+\delta[v_{b,ab}(b)-v_{b,ab}(a)-c] \\ &\quad +\gamma(v_{b,ab}(b)-v_{b,ab}(a)-[\lambda(a,a)+\lambda(a,ab)][v_{b,ab}(b)-v_{b,ab}(a)]-\pi\lambda(b,b)) \end{aligned}$$

$$rv_{b,ab}(b) = 1+z-\delta c+\gamma[-\lambda(a,ab)[v_{b,ab}(b)-v_{b,ab}(a)]-\pi\lambda(a,a)]$$

Solving for $v_{b,ab}(b)$ gives

$$rv_{b,ab}(b) = 1 + z + \gamma \left[-\lambda(a,ab) \left[\frac{z + \gamma\pi[\lambda(b,b) - \lambda(a,a)]}{r + \delta + \gamma[1 - \lambda(a,a)]} \right] - \pi\lambda(a,a) \right] - \delta c.$$

An agent who receives the build shock prefers $\{b,ab\}$ to $\{a,ab\}$ if $v_{b,ab}(b) \geq v_{a,ab}(a) \Longleftrightarrow$

$$\begin{aligned} & 1 + z + \gamma \left[-\lambda(a,ab) \left[\frac{z + \gamma\pi[\lambda(b,b) - \lambda(a,a)]}{r + \delta + \gamma[1 - \lambda(a,a)]} \right] - \pi\lambda(a,a) \right] - \delta c \\ \geq \quad & 1 + \gamma \left\{ \lambda(b,ab) \left[\frac{z + \gamma\pi[\lambda(b,b) - \lambda(a,a)]}{r + \delta + \gamma[1 - \lambda(b,b)]} \right] - \pi\lambda(b,b) \right\} - \delta c. \end{aligned}$$

This expression can be rewritten as

$$(z + \gamma\pi[\lambda(b,b) - \lambda(a,a)])\,\Omega \geq 0$$

where

$$\Omega \equiv 1 - \frac{\gamma\lambda(a,ab)}{r + \delta + \gamma[1 - \lambda(a,a)]} - \frac{\gamma\lambda(b,ab)}{r + \delta + \gamma[1 - \lambda(b,b)]}$$

We now show that $\Omega > 0$. Without loss of generality assume that $\lambda(b,b) \geq \lambda(a,a)$. Then,

$$\Omega \geq 1 - \gamma \left[\frac{\lambda(b,ab) + \lambda(a,ab)}{r + \delta + \gamma[1 - \lambda(b,b)]} \right].$$

$\Omega > 0$ if $r + \delta + \gamma[1 - \lambda(b,b) - \lambda(b,ab) - \lambda(a,ab)] > 0$ which is satisfied since $\lambda(b,ab) + \lambda(a,ab) + \lambda(a,a) + \lambda(b,b) = 1$. Because $\Omega > 0$, the agents prefers $\{b,ab\}$ to $\{a,ab\}$ whenever

$$z \geq \gamma\pi[\lambda(a,a) - \lambda(b,b)] \equiv z_2(\gamma).$$

Agents (z, γ) with $z > z_2(\gamma)$ prefer $\{b,ab\}$ to $\{a,ab\}$.

4. Strategy $\{b,b\}$. An agent who follows $\{b,b\}$ always rejects a so $\tau_{b,b}(a)$ is irrelevant. $\tau_{b,b}(b) = v_{b,b}(b) - \pi[\lambda(a,ab) + \lambda(a,a)]$. Thus $v_{b,b}(b)$ is

$$rv_{b,b}(b) = 1 + z - \delta c - \gamma\pi[\lambda(a,ab) + \lambda(a,a)]$$

An agent who receives the build shock prefers $\{b,b\}$ to $\{b,ab\}$ if $v_{b,b}(b) > v_{b,ab}(b) \Leftrightarrow$

$$z > \pi[r + \delta + \gamma[1 - \lambda(b,b)]] \equiv z_3(\gamma)$$

Since $z_1(\gamma) < z_2(\gamma) < z_3(\gamma)$ any agent with $z < z_1(\gamma)$ prefers $\{a,a\}$ to all of the other strategies. To see this note that such an agent prefers $\{a,a\}$ to $\{a,ab\}$ by case one above. However, by case two, he also prefers $\{a,ab\}$ to $\{b,ab\}$ and by case three prefers $\{b,ab\}$ to $\{b,b\}$. Thus $\{a,a\}$ is optimal for this agent. Similar arguments imply that for $z_1(\gamma) < z < z_2(\gamma)$ the optimal strategy is $\{a,ab\}$; for $z_2(\gamma) < z < z_3(\gamma)$ the optimal strategy is $\{b,ab\}$ and for $z > z_3(\gamma)$ the optimal strategy is $\{b,b\}$. ■

Proposition 2 *Given any F and G, there exists at least one steady state equilibrium.*

Proof. By Lemma 4 $L_{a,a}$ and $L_{b,b}$ have at least one fixed point. Let $\lambda(a,a)$ and $\lambda(b,b)$ be fixed points of $L_{a,a}$ and $L_{b,b}$. We construct an equilibrium as follows: Use $\lambda(a,a)$ and $\lambda(b,b)$ to construct $z_1(\gamma)$, $z_2(\gamma)$, and $z_3(\gamma)$ from Proposition 1. With $z_1(\gamma)$, $z_2(\gamma)$, and $z_3(\gamma)$ compute $\lambda(a,a)$, $\lambda(b,b)$, $\lambda(a,ab)$, and $\lambda(b,ab)$ with Lemma 3. By construction $\lambda(a,a)$, $\lambda(b,b)$, $\lambda(a,ab)$, and $\lambda(b,ab)$ are equilibrium matching probabilities. ■

Proposition 3 *If F satisfies Assumption 1 then (1) The equilibrium is unique; (2) There is conformity whenever $\mu \neq 0$ and the market conforms to the mean taste (the market conforms on a if $\mu < 0$ and conforms on b if $\mu > 0$); (3) If $\mu \neq 0$, an increase in durability (lower δ) or patience (lower r) causes conformity to increase (i.e., $z_2(\gamma)$ increases if $\mu < 0$ and decreases if $\mu > 0$ for all γ); (4) All else equal, agents with a greater likelihood of trade conform more.*

Proof.

1. By proposition 2, $\lambda(a,a)$ is a fixed point of the mapping $L_{a,a}$,

$$\lambda(a,a) = L_{a,a}(\lambda(a,a)) = \int_0^\infty F(-\pi\{r+\delta+\gamma[1-\lambda(a,a)]\})\,dG(\gamma)$$

By assumption F has a density and thus we can calculate the derivative of $L_{a,a}$. This derivative is

$$0 \leq \frac{\partial L_{a,a}(\lambda(a,a))}{\partial\lambda(a,a)} = \int_0^\infty f(z_1(\gamma))\,\pi\gamma dG(\gamma) < f(\mu)\,\pi\int_0^\infty \gamma dG(\gamma) = f(\mu)\,\pi\bar{\gamma} < 1.$$

Where the second inequality follows since $f(z) \leq \mu$ and the last inequality follows from Assumption 1. Thus there can be at most one fixed point. Similar arguments hold for $L_{b,b}$. Since existence of at least one equilibrium is guaranteed by proposition 2, the equilibrium is unique.

2. Because f is symmetric about μ, if $\mu = 0$ then $F(-x) = 1 - F(x)$ and thus the unique equilibrium must have $\lambda(a,a) = \lambda(b,b)$ and $z_2(\gamma) = 0$ for all γ (no conformity). Let λ^* be the equilibrium $\lambda^* = \lambda(a,a) = \lambda(b,b)$ for $\mu = 0$. For any $l \geq \lambda^*$ we must have $L_{a,a}(l) \leq l$ (since the derivative of $L_{a,a}$ is less than 1 by part 1) and for any $l \leq \lambda^*$ we must have $L_{b,b}(l) \geq l$ by the same reasoning.

   Consider $\mu > 0$ (the argument for $\mu < 0$ is identical). For any given $l \in [0,1]$, and for all γ,

$$F(-\pi\{r+\delta+\gamma[1-l]\};\mu>0) < F(-\pi\{r+\delta+\gamma[1-l]\};\mu=0)$$

$$1 - F(\pi\{r+\delta+\gamma[1-l]\};\mu>0) > 1 - F(\pi\{r+\delta+\gamma[1-l]\};\mu=0)$$

   Therefore, integrating over all γ we have $L_{a,a}(l)\,|_{\mu>0} < L_{a,a}(l)\,|_{\mu=0}$ and $L_{b,b}(l)\,|_{\mu>0} > L_{b,b}(l)\,|_{\mu=0}$. When $\mu > 0$, $l \in [\lambda^*, 1]$ cannot be a fixed point of $L_{a,a}$ since $L_{a,a}(l)\,|_{\mu>0} < L_{a,a}(l)\,|_{\mu=0} \leq l$ for $l \in [\lambda^*, 1]$. Similarly, $l \in [0, \lambda^*]$ cannot be a fixed point of $L_{b,b}$. Because the equilibrium is unique, we conclude that for $\mu > 0$, the equilibrium satisfies $\lambda(b,b) > \lambda^* > \lambda(a,a)$. This implies that $z_2(\gamma) < 0$ for all γ so the market conforms on b.

3. If $\mu > 0$ then $\int_0^\infty [1 - F(z_3(\gamma))]\,dG(\gamma) = \lambda(b,b) > \lambda(a,a) = \int_0^\infty F(z_1(\gamma))\,dG(\gamma)$ by part (2). This implies that $z_2(\gamma) < 0$. Differentiating gives

$$\frac{\partial z_2(\gamma)}{\partial\delta} = \pi\gamma\left[\frac{\partial\lambda(a,a)}{\partial\delta} - \frac{\partial\lambda(b,b)}{\partial\delta}\right]$$

$$\frac{\partial\lambda(a,a)}{\partial\delta} = -\frac{\pi\int f(z_1(\gamma))\,dG(\gamma)}{1-\pi\int f(z_1(\gamma))\,\gamma dG(\gamma)} \text{ and } \frac{\partial\lambda(b,b)}{\partial\delta} = -\frac{\pi\int f(z_3(\gamma))\,dG(\gamma)}{1-\pi\int f(z_3(\gamma))\,\gamma dG(\gamma)}$$

   Lemma 6 implies $f(z_3(\gamma)) \geq f(z_1(\gamma))$ for all γ. Moreover, Assumption 1 guarantees that $1 - \pi\int f(z_1(\gamma))\,\gamma dG(\gamma) > 0$ and $1 - \pi\int f(z_3(\gamma))\,\gamma dG(\gamma) > 0$. Thus

$$\frac{\partial z_2(\gamma)}{\partial\delta} = \pi\gamma\left[-\frac{\pi\int f(z_1(\gamma))\,dG(\gamma)}{1-\pi\int f(z_1(\gamma))\,\gamma dG(\gamma)} + \frac{\pi\int f(z_3(\gamma))\,dG(\gamma)}{1-\pi\int f(z_3(\gamma))\,\gamma dG(\gamma)}\right] > 0$$

   An increase in durability implies a reduction in $z_2(\gamma)$ for all γ. The proof for r is identical.

4. The proof follows immediately by observing that $z_2(\gamma) = \pi\gamma[\lambda(a,a) - \lambda(b,b)]$.

■

Proposition 4 *Assume that trade hazards are $(\gamma + \theta)$ as described in the text ($\theta = 0$ is the original equilibrium) and consider an F satisfying Assumption 1. Then, for a marginal increase in θ at $\theta = 0$, there exists $\hat{\gamma} \in (0, \infty]$ such that conformity increases for all agents with $\gamma < \hat{\gamma}$ and decreases for all agents with $\gamma > \hat{\gamma}$.*

Proof. Differentiating $\lambda(a,a)$ and $\lambda(b,b)$ with respect to θ and evaluating at $\theta=0$ gives

$$\begin{aligned}\left.\frac{\partial\lambda(a,a)}{\partial\theta}\right|_{\theta=0} &= \frac{-\pi(1-\lambda(a,a))\int f(z_1(\gamma,0))\,dG(\gamma)}{1-\int f(z_1(\gamma,0))\,\gamma dG(\gamma)},\\ \left.\frac{\partial\lambda(b,b)}{\partial\theta}\right|_{\theta=0} &= \frac{-\pi(1-\lambda(b,b))\int f(z_3(\gamma,0))\,dG(\gamma)}{1-\int f(z_3(\gamma,0))\,\gamma dG(\gamma)}.\end{aligned}$$

The conformity cutoff is $z_2(\gamma)=(\gamma+\theta)\pi[\lambda(a,a)-\lambda(b,b)]$. Differentiating z_2 with respect to θ and evaluating at $\theta=0$ we obtain:

$$\left.\frac{\partial z_2(\gamma)}{\partial\theta}\right|_{\theta=0}=\frac{z_2(\gamma)}{\gamma}+\gamma\pi\left[\left.\frac{\partial\lambda(a,a)}{\partial\theta}\right|_{\theta=0}-\left.\frac{\partial\lambda(b,b)}{\partial\theta}\right|_{\theta=0}\right].$$

Suppose $\mu>0$ ($\mu<0$ is symmetric.) In this case $z_2(\gamma)<0$. If the term in square brackets is negative then $\left.\frac{\partial z_2(\gamma)}{\partial\theta}\right|_{\theta=0}<0$ for all γ so we set $\hat{\gamma}=\infty$. If the term is positive then let $\hat{\gamma}$ be

$$\hat{\gamma}=\frac{-[\lambda(a,a)-\lambda(b,b)]}{\left[\left.\frac{\partial\lambda(a,a)}{\partial\theta}\right|_{\theta=0}-\left.\frac{\partial\lambda(b,b)}{\partial\theta}\right|_{\theta=0}\right]}>0.$$

By construction $\left.\frac{\partial z_2(\gamma)}{\partial\theta}\right|_{\theta=0}<0$ for $\gamma<\hat{\gamma}$ and $\left.\frac{\partial z_2(\gamma)}{\partial\theta}\right|_{\theta=0}>0$ for $\gamma>\hat{\gamma}$ which proves the result. ■

Proposition 5 *Assume that all agents have a common γ, let F satisfy Assumption 1, and let z_1^*, z_2^* and z_3^* be the unique equilibrium cutoffs. If either (i) μ_{ab} is the same sign as μ, or (ii) $|\mu|\geq\frac{\pi(\gamma+\delta)}{2}$, then* sign $\left(\frac{\partial W(z_1^*z_2^*z_3^*)}{\partial z_2}\right)=$ sign (z_2^*) *and there is too little conformity in equilibrium.*

Proof. Differentiating W from Lemma 5 with respect to z_2 gives

$$\begin{aligned}\frac{\partial W}{\partial z_2} &= -\pi\gamma\frac{\partial\lambda(b,ab)}{\partial z_2}F(z_1)\\ &+\int_{z_1}^{z_2}\left\{\left[\frac{\partial P_{a,ab}(a)}{\partial z_2}\frac{1}{F(z_2)-F(z_1)}-\frac{P_{a,ab}(a)}{[F(z_2)-F(z_1)]^2}f(z_2)\right][z_2-z]\right\}dF(z)\\ &+f(z_2)\left[\left(\frac{P_{a,ab}(a)}{F(z_2)-F(z_1)}\right)[1-\pi\gamma\lambda(b,b)]+\left(1-\frac{P_{a,ab}(a)}{F(z_2)-F(z_1)}\right)[1+z_2-\pi\gamma\lambda(a,a)]\right]\\ &+\int_{z_2}^{z_3}\left\{\left[\frac{\partial P_{b,ab}(b)}{\partial z_2}\left(\frac{1}{F(z_3)-F(z_2)}\right)+\frac{P_{b,ab}(b)}{(F(z_3)-F(z_2))^2}f(z_2)\right][z-z_2]\right\}dF(z)\\ &-f(z_2)\left[\left(1-\frac{P_{b,ab}(b)}{F(z_3)-F(z_2)}\right)[1-\pi\gamma\lambda(b,b)]+\left(\frac{P_{b,ab}(b)}{F(z_3)-F(z_2)}\right)[1+z_2-\pi\gamma\lambda(a,a)]\right]\\ &-\pi\gamma\frac{\partial\lambda(a,ab)}{\partial z_2}[1-F(z_3)]\end{aligned}$$

The 3rd and 5th lines add to zero. We can now write the derivative as

$$\begin{aligned}\frac{\partial W}{\partial z_2} &= \pi\gamma\frac{\partial\lambda(a,ab)}{\partial z_2}(F(z_1)-[1-F(z_3)])\\ &+\int_{z_1}^{z_2}\left\{\left[\frac{\partial P_{a,ab}(a)}{\partial z_2}\left(\frac{1}{F(z_2)-F(z_1)}\right)-\frac{P_{a,ab}(a)}{[F(z_2)-F(z_1)]^2}f(z_2)\right][z_2-z]\right\}dF(z)\\ &+\int_{z_2}^{z_3}\left\{\left[\frac{\partial P_{b,ab}(b)}{\partial z_2}\left(\frac{1}{F(z_3)-F(z_2)}\right)+\frac{P_{b,ab}(b)}{(F(z_3)-F(z_2))^2}f(z_2)\right][z-z_2]\right\}dF(z)\end{aligned}$$

where we have used $\frac{\partial\lambda(b,ab)}{\partial z_2}=-\frac{\partial\lambda(a,ab)}{\partial z_2}$. Lemma 9 shows that $\frac{\partial\lambda(a,ab)}{\partial z_2}>0$, so the first term has the same sign as z_2. Lemma 10 shows that the sum of the remaining two terms has the same sign as

$$[\gamma[1-F(z_1)]+\delta]\left[\int_{z_1}^{z_2}[z_2-z]\,dF(z)\right]-[\gamma F(z_3)+\delta]\left[\int_{z_2}^{z_3}[z-z_2]\,dF(z)\right] \tag{9}$$

Using the expressions for z_1 and z_3 when $r = 0$ we can rewrite this expression as

$$(-z_1)(z_2)(F(z_2) - F(z_1)) + (z_3)(z_2)(F(z_3) - F(z_2)) - \left\{(-z_1)\int_{z_1}^{z_2} zdF(z) + (z_3)\int_{z_2}^{z_3} zdF(z)\right\}$$

The mean taste of the inclusive traders μ_{ab} is

$$\mu_{ab} = \frac{\int_{z_1}^{z_3} zdF(z)}{F(z_3) - F(z_1)} = \frac{\int_{z_1}^{z_2} zdF(z) + \int_{z_2}^{z_3} zdF(z)}{F(z_3) - F(z_2) + F(z_2) - F(z_1)}$$

Using this in the expression above gives

$$\begin{array}{l}(z_2)\left\{(-z_1)(F(z_2) - F(z_1)) + (z_3)(F(z_3) - F(z_2)) - \int_{z_2}^{z_3} zdF(z)\right\} \\ -\mu_{ab}(-z_1)[F(z_3) - F(z_2) + F(z_2) - F(z_1)]\end{array} \tag{10}$$

Note that $\int_{z_2}^{z_3} zdF(z) < z_3 \int_{z_2}^{z_3} dF(z) = z_3(F(z_3) - F(z_2))$ so that

$$\begin{aligned} & (-z_1)(F(z_2) - F(z_1)) + (z_3)(F(z_3) - F(z_2)) - \int_{z_2}^{z_3} zdF(z) \\ > \; & (-z_1)(F(z_2) - F(z_1)) + (z_3)(F(z_3) - F(z_2)) - z_3(F(z_3) - F(z_2)) \\ = \; & (-z_1)(F(z_2) - F(z_1)) > 0 \end{aligned}$$

As a result, the first term in (10) has the same sign as z_2. Thus, if $-\mu_{ab}$ has the same sign as z_2, the sign of $\frac{\partial W}{\partial z_2}$ is the same as the sign of z_2. By Assumption 1, $-\mu$ has the same sign as z_2. Thus if μ_{ab} has the same sign as μ, there is too little conformity. This establishes (i.).

To establish (ii.) recall that the sign of (**??**) is the same as the sign of (9) Using integration by parts, we can write the first integral as

$$\int_{z_1}^{z_2} [z_2 - z]\, dF(z) = \int_{z_1}^{z_2} F(z)\, dz - z_3 F(z_1).$$

Similarly, the second integral is

$$\int_{z_2}^{z_3} [z - z_2]\, dF(z) = -F(z_3)\, z_1 - \int_{z_2}^{z_3} F(z)\, dz.$$

Using these expressions and since $z_2 = z_1 + z_3$ we have $z_3 = z_2 - z_1 > 0$ and $-z_1 = z_3 - z_2 > 0$, we can rewrite the expression as

$$\frac{(-z_1)}{\pi}\left[\int_{z_1}^{z_2} [F(z) - F(z_1)]\, dz\right] - \frac{(z_3)}{\pi}\left[\int_{z_2}^{z_3} [F(z_3) - F(z)]\, dz\right].$$

Assume that $\mu < 0$ (the proof for $\mu > 0$ is analogous). Since $z_2 > 0 > \mu$, F is concave for all $z \in [z_2, z_3]$. Thus, by Jensen's inequality,

$$\int_{z_2}^{z_3} [F(z_3) - F(z)]\, dz < \frac{1}{2}(-z_1)(F(z_3) - F(z_2))$$

F may or may not be concave for all $z \in [z_1, z_2]$. However, because of the symmetry of F, if $\mu \leq \frac{z_1 + z_2}{2}$ then,

$$\int_{z_1}^{z_2} [F(z) - F(z_1)]\, dz \geq \frac{1}{2}(z_3)(F(z_2) - F(z_1)).$$

Thus if $(-z_1)\frac{1}{2}(z_3)(F(z_2) - F(z_1)) - z_3\frac{1}{2}(-z_1)(F(z_3) - F(z_2)) > 0$ expression (9) must also be greater than zero and there is too little conformity. Dividing by $(-z_1)\frac{1}{2}(z_3)$ (a positive number) gives $(F(z_2) - F(z_1)) - (F(z_3) - F(z_2))$ which by Lemma 11 is always positive for $\mu < 0$. Thus, if $\mu \leq \frac{z_1 + z_2}{2}$ there is too little conformity. Since $\mu < 0$, $z_2 > 0$. Moreover, $z_1 = -\pi[\delta + \gamma[1 - F(z_1)]] > -\pi[\delta + \gamma]$. Thus, if $\mu < \frac{-\pi(\gamma + \delta)}{2} \leq \frac{z_1}{2} \leq \frac{z_1 + z_2}{2}$ there is too little conformity. This establishes (ii.). ■

Lemma 1 *If an agent builds type x, then he accepts type x in trade.*

Lemma 2 *Given matching probabilities $\lambda(x,\tau)$ for $x \in \{a,b\}$ and $\tau \in \{a,b,ab\}$ with $\sum_{\tau\in\{a,b,ab\}}\sum_{y\in\{a,b\}}\lambda(y,\tau) = 1$ there exist unique values $V(a)$, $V(b)$, $T(a)$, $T(b)$, and B satisfying (1), (2), and (3).*

Lemma 3 *Let $z_1(\gamma)$, $z_2(\gamma)$ and $z_3(\gamma)$ be given. The implied matching probabilities $\lambda(a,a)$ and $\lambda(b,b)$ are $\lambda(a,a) = \int_0^\infty F(z_1(\gamma))\,dG(\gamma)$ and $\lambda(b,b) = \int_0^\infty [1-F(z_3)]\,dG(\gamma)$. Furthermore, there exist unique nonnegative numbers $P_{a,ab}(a)$, $P_{a,ab}(b)$, $P_{b,ab}(a)$ and $P_{b,ab}(b)$ such that the implied matching probabilities $\lambda(a,ab)$ and $\lambda(b,ab)$ are $\lambda(a,ab) = P_{a,ab}(a) + P_{b,ab}(a)$ and $\lambda(b,ab) = P_{a,ab}(b) + P_{b,ab}(b)$.*

Lemma 4 *The mappings $L_{a,a}$ and $L_{b,b}$ defined by (5) and (6) each have at least one fixed point.*

Lemma 5 *Assume that all agents have a common γ. Given cutoffs z_1, z_2, z_3, let $P_{a,ab}(a)$, $P_{b,ab}(a)$, $P_{a,ab}(b)$ and $P_{b,ab}(b)$ be given by Lemma 3 and define $\psi_{a,ab} = P_{a,ab}(a)/[F(z_2)-F(z_1)]$ and $\psi_{b,ab} = P_{b,ab}(b)/[F(z_3)-F(z_2)]$. Then, the flow of social welfare is*

$$
\begin{aligned}
W(z_1,z_2,z_3) \;=\; & \{1-\pi\gamma[\lambda(b,b)+\lambda(b,ab)]\}F(z_1) \\
& + \int_{z_1}^{z_2}\left\{\psi_{a,ab}[1-\pi\gamma\lambda(b,b)] + \left(1-\psi_{a,ab}\right)[1+z-\pi\gamma\lambda(a,a)]\right\}dF(z) \\
& + \int_{z_2}^{z_3}\left\{\left(1-\psi_{b,ab}\right)[1-\pi\gamma\lambda(b,b)] + \psi_{b,ab}[1+z-\pi\gamma\lambda(a,a)]\right\}dF(z) \\
& + \int_{z_3}^{\infty}\{1+z-\pi\gamma[\lambda(a,a)+\lambda(a,ab)]\}dF(z) - \delta c.
\end{aligned}
$$

Lemma 6 *If a density function $f(z)$ with mean μ satisfies the following conditions: (1) Symmetry (S): for any x, $f(\mu+x) = f(\mu-x)$, and $F(\mu+x) = 1-F(\mu-x)$; and (2) Quasi-Concave (QC): for any fixed x, the set $\{y : f(y) \geq f(x)\}$ is convex; then for any $z < z'$, $F(z) \geq 1-F(z') \Leftrightarrow f(z) \geq f(z')$.*

Lemma 7 *For the model with a single γ (i.e., degenerate G), define*

$$r_a = \frac{\delta+\gamma F(z_1)}{F(z_3)-F(z_2)}, \text{ and } r_b = \frac{\delta+\gamma[1-F(z_3)]}{F(z_2)-F(z_1)},$$

then, the following statements are true

$$
\begin{aligned}
P_{a,ab}(b) &= \frac{\gamma r_b}{\gamma r_a+\gamma r_b+r_a r_b}(F(z_2)-F(z_1)) \text{ and } P_{a,ab}(a) = \frac{\gamma r_a + r_a r_b}{\gamma r_a+\gamma r_b+r_a r_b}(F(z_2)-F(z_1)) \\
P_{b,ab}(a) &= \frac{\gamma r_a}{\gamma r_a+\gamma r_b+r_a r_b}(F(z_3)-F(z_2)) \text{ and } P_{b,ab}(b) = \frac{\gamma r_b + r_a r_b}{\gamma r_a+\gamma r_b+r_a r_b}(F(z_3)-F(z_2))
\end{aligned}
$$

Lemma 8

$$\frac{\partial P_{b,ab}(b)}{\partial z_2} + \frac{P_{b,ab}(b)}{F(z_3)-F(z_2)}f(z_2) = -f(z_2)\frac{P_{b,ab}(b)}{(F(z_3)-F(z_2))}\frac{P_{b,ab}(a)}{(F(z_3)-F(z_2))}\left[\frac{F(z_3)-F(z_2)}{F(z_2)-F(z_1)} + \frac{\gamma}{\gamma+r_a}\right] < 0$$

$$\frac{\partial P_{a,ab}(a)}{\partial z_2} - \frac{P_{a,ab}(a)}{F(z_2)-F(z_1)}f(z_2) = f(z_2)\frac{P_{a,ab}(a)}{F(z_2)-F(z_1)}\frac{P_{a,ab}(b)}{F(z_2)-F(z_1)}\left[\frac{F(z_2)-F(z_1)}{F(z_3)-F(z_2)} + \frac{\gamma}{\gamma+r_b}\right] > 0$$

Lemma 9 $\frac{\partial\lambda(a,ab)}{\partial z_2} > 0$.

Lemma 10 *The sign of*

$$
\begin{aligned}
&\left(\frac{1}{F(z_2)-F(z_1)}\right)\left[\frac{\partial P_{a,ab}(a)}{\partial z_2} - P_{a,ab}(a)\frac{f(z_2)}{F(z_2)-F(z_1)}\right]\int_{z_1}^{z_2}[z_2-z]\,dF(z) \\
&+\left(\frac{1}{F(z_3)-F(z_2)}\right)\left[\frac{\partial P_{b,ab}(b)}{\partial z_2} + P_{b,ab}(b)\frac{f(z_2)}{F(z_3)-F(z_2)}\right]\int_{z_2}^{z_3}[z-z_2]\,dF(z)
\end{aligned}
$$

is the same as the sign of

$$[\gamma[1-F(z_1)]+\delta]\left[\int_{z_1}^{z_2}[z_2-z]\,dF(z)\right] - [\gamma F(z_3)+\delta]\left[\int_{z_2}^{z_3}[z-z_2]\,dF(z)\right]$$

Lemma 11 $F(z_2)-F(z_1) \gtreqless F(z_3)-F(z_2)$ *if* $\mu \lesseqgtr 0$.

FIGURE 1: UNIQUE EQUILIBRIA WITH A UNIFORM DISTRIBUTION

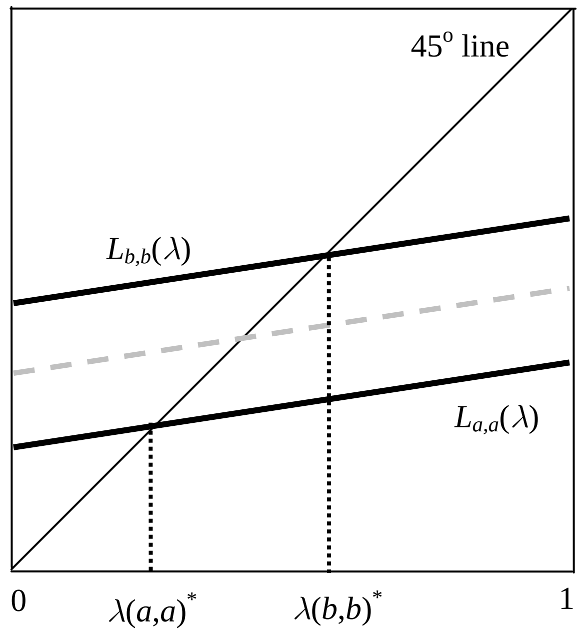


FIGURE 2: MULTIPLE EQUILIBRIA WITH A SYMMETRIC DISTRIBUTION

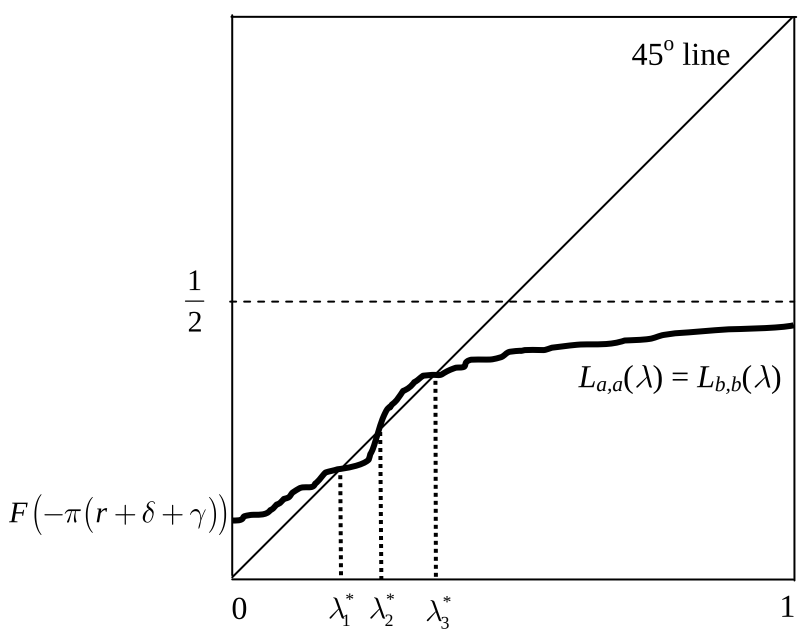
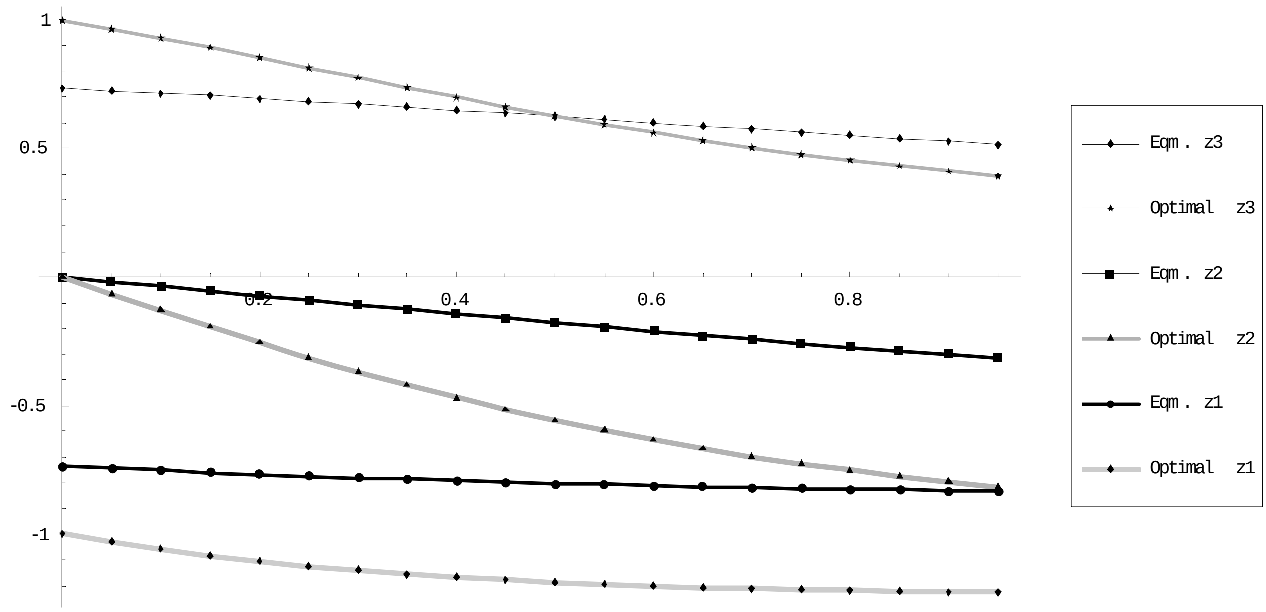


FIGURE 3: EQUILIBRIUM AND OPTIMAL z_1, z_2, z_3 FOR DIFFERENT MEANS.



The figure shows the equilibrium cutoffs (the solid dark lines) and the optimal cutoffs (the light shaded lines) for different means of $F(z)$. In each case, F is normal with a unit variance. The cutoffs are plotted on the vertical axis while the mean is on the horizontal axis.