Problem Set #2 - Answers
Due September 28, 1998

1. In the village of Cheng Kin, province of Yu Wing-lun, a thriving market has long existed for Ho Ho-shuens, known colloquially as hohos because they make people feel so happy. Hohos are manufactured from the finest raw materials in a large number of local factories, and it is well-established that the supply curve for hohos, accurately reflecting their marginal cost, is given by the following equation:

$$P = 120 + 4Q$$

where Q is the quantity of hohos in thousands, and P is their price in local currency, tingting per hoho (☺/hh.). Demand for hohos has been stable for many years, given by the equation

$$P = 930 - 0.5Q$$

Price, as you should verify, has been ☺840 per hh.

Verifying price:

$$P = 120 + 4Q \quad \& \quad P = 930 - 0.5Q$$
$$\Rightarrow \quad 120 + 4Q = 930 - 0.5Q \quad \Rightarrow \quad 4.5Q = 810 \quad \Rightarrow \quad Q = 180$$
$$\Rightarrow \quad P = 120 + 4(180) = 840$$

The mayor of Cheng Kin, Kin-chung Clinton, has just learned that hohos can be used effectively to enhance productivity of government employees, and he proposes to have the government buy 20,000 of them.

a) What percentage of current hoho production would this government order be?

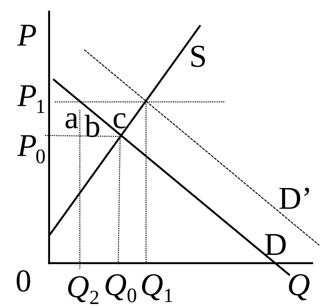
As derived above, Q=180 thousand hh. The order for 20 thousand hh is therefore 20/180=1/9=11% of current production.

b) What would be the new equilibrium price of hohos if this order were added to the current market? How much, therefore, will the 20,000 hohos cost the government?

To find the new equilibrium price and quantity, it is simplest to first find the new demand curve including the government purchase. The private demand curve is $P = 930 - 0.5Q$, which can be expressed also as $0.5Q = 930 - P$ or $Q = 1860 - 2P$. Adding the government demand of 20 thousand hh to this, the new demand curve is $Q = 1880 - 2P$. Substituting this into the supply curve, we get

$P = 120 + 4Q = 120 + 4(1880 - 2P) = 7640 - 8P$ or $9P = 7640 \Rightarrow P = 848.9$.
Private-sector demand is therefore $Q = 1860 - 2(848.9) = 162.2$ and total production is 162.2 + 20 = 182.2. Thus the new equilibrium price is ☺848.9 per hh and government expenditure on 20,000 Hohos is $848.9 \times 20{,}000 = 16{,}978{,}000$ or ☺16.978 mil.

c) Calculate the effects of this purchase on private consumer and producer surplus in the hoho market.



As shown in the figure at the right (which is not drawn to scale, in order to make it easier to see), the government demand shifts the demand curve to the right, raising both equilibrium price and equilibrium quantity. Private-sector demanders, given by the old demand curve, lose from this. Their loss in consumer surplus is the area a+b, or the rise in price times the average of their old and new quantities purchased. That is, they lose

$$(P_1 - P_0)\frac{(Q_0 + Q_2)}{2} = (848.9 - 840)(180 + 162.2)/2 = 8.9 \times 171.1 = ☺1.539 \text{ mil.}$$

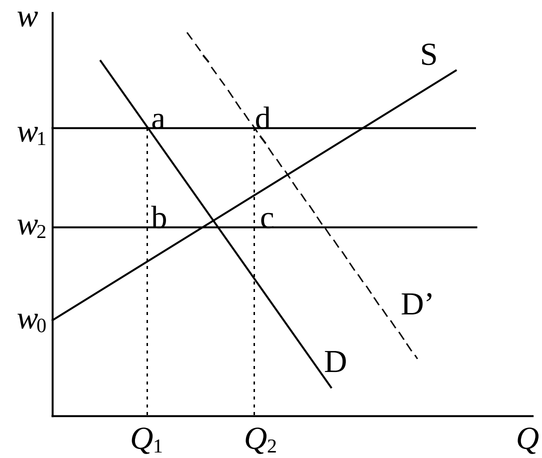
Producers gain the increase in producer surplus equal to area a+b+c, or the change in price times the average of their old and new quantity sold:

$$(P_1 - P_0)\frac{(Q_1 + Q_0)}{2} = (848.9 - 840)(182.2 + 180)/2 = 8.9 \times 181.1 = ☺1.629 \text{ mil.}$$

d) What is the net social cost of this policy?

Gain to producers	☺1.629 mil.
Cost to consumers	–1.539
Cost to government	<u>–16.978</u>
Net cost to society	–☺16.887 mil plus any gain in government productivity

2. The figure at the right shows, as solid lines, the initial supply and demand for labor and a minimum wage, w_1. The demand curve then shifts to the right, to D', as a result of increased employment by government. Assuming that available jobs are allocated randomly among those who want to work at wage w_1, determine the following:



   a) The quantity of labor employed before and after the increase in government demand for labor.

      Before: Q_1 ; After: Q_2 .

   b) The effect of the increased demand on the welfare of suppliers and demanders of labor.

      Demanders are unaffected, since they continue to get all the labor they want at wage w_1 . Suppliers gain, since $Q_2 - Q_1$ of them are now employed who were not before. Since they are selected at random from among those willing to work at wage their marginal cost of working ranges from w_0, the intercept of the supply function, to w_1 itself. With the linear supply curve, their average marginal cost of working is therefore $w_2 = (w_1 - w_0)/2$, which is half way between w_0 and w_1. The surplus of the new workers is the excess of their wage, w_1, over this average, or the rectangle bounded by the points labeled a, b, c, and d.

   c) Is it possible that the increase in government employment is socially beneficial even if there is no social value to what they do in their new jobs? If so, identify the gain to society. If not, determine how productive they must be in their new jobs in order for this policy to be beneficial for society as a whole.

      No. The workers cost the government w_1, while their gain is on average only $w_1 - w_2$ (and at most only $w_1 - w_0$), so employing them for no purpose is creating a net social cost of $w_1 - (w_1 - w_2) = w_2$ and cannot be beneficial. It would be better just to give them money without requiring them to work. However, if they are productive, then their productivity only needs to be greater than w_2 for their employment to be socially beneficial, since this would be enough to turn the net social cost into a net social benefit.

3. Calculate the present discounted value of the projects listed in the table below, which reports for each of four projects, a, b, c, and d, the relevant interest rate, r, and the benefits (positive) and costs (negative) in the present (t=0), and t years from the present.

Project	Interest rate	Benefits (+) and Costs (–) in present (0) and future years, t= 0	1	2	3	4…9	10	11	12…∞
a)	5%	–700	300	400					
b)	3%	5	–5	–5	–5	–5	–5		
c)	7%	–200	14	14	14	14	14	14	14
d)	10%							100	100

a) $PV(a) = -700 + \frac{300}{1.05} + \frac{400}{(1.05)^2} = -700 + 285.7 + 362.8 = -51.5$

Uses general formula, $PV = \sum_{t=0}^{T} \frac{X_t}{(1+r)^t}$

b) $PV(b) = 5 + \frac{-5}{0.03}\left[1 - \frac{1}{(1.03)^{10}}\right] = 5 - 166.7[1 - 0.744] = -37.651$

Uses formula for constant X_t, t=1,…,T, $PV = \frac{X}{r}\left[1 - \frac{1}{(1+r)^T}\right]$

c) $PV(c) = -200 + \frac{14}{0.07} = -200 + 200 = 0$

Uses formula for constant X_t, t=1,…,∞, $PV = \frac{X}{r}$

d) $PV(d) = \frac{100}{0.1} - \frac{100}{.01}\left[1 - \frac{1}{(1.1)^{10}}\right] = 1000 - 1000[1 - .3855] = 385.5$

Uses both of the formulas in (b) and (c) by evaluating the infinite sum, then subtracting the missing finite sum. This could also be done more directly by realizing that in year 10, this will be a constant amount each year from the next year on, and therefore will have the value in year 10 of X/r=100/.1=1000. Then just use the general formula to discount this back to the present, $1000/(1.1)^{10}$.