Problem Set #2 - Answers
Due September 23, 1997

1. Restaurants on Mackinac Island have unusually high costs because the island is not accessible by car or truck. Even though there is a very famous Mackinac Bridge that runs between the two peninsulas of Michigan, it goes right past Mackinac Island, which can be reached only by boat or airplane. This is part of the appeal of the island, which bans automobiles, to the many tourists who visit there every summer.

   Let us suppose that the government of the island is considering building a tunnel from the mainland to the island that would be kept secret from the public and used only for trucking in supplies to the restaurants. This would lower the cost of providing meals by, let's say, \$2 per meal. Before doing a full benefit-cost analysis of this proposal, however, the question has arisen as to who exactly will benefit from it. Will it be primarily the owners of the restaurants on the island, or will it be primarily the tourists who eat at them?

   The graphs below show three different ideas about what the restaurant market on Mackinac Island may look like, differing in terms of how costs currently depend upon output. Figure A shows constant cost suppliers, Figure B shows increasing cost suppliers, and Figure C shows costs and therefore supply turning vertical at some capacity output, $\overline{Q}$, a limit that is assumed to have been reached. Initial price and quantity are the same in all three cases.

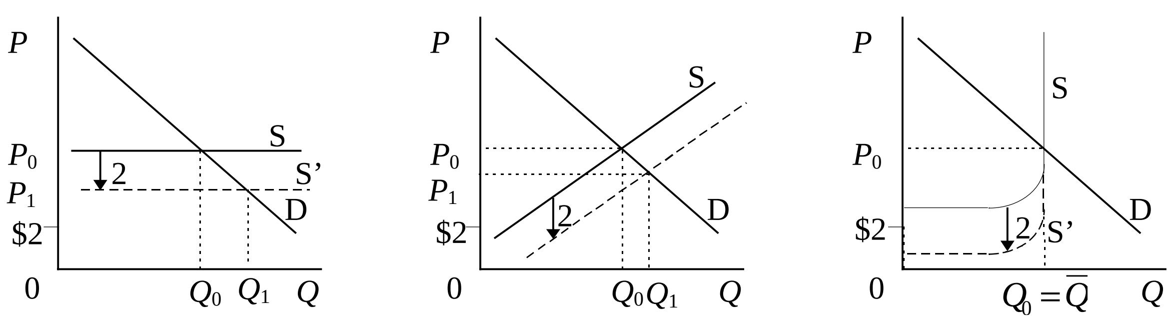


Figure A   Figure B   Figure C

   a) For each supply curve, show the new equilibrium price and quantity if costs per unit of production are reduced by \$2 for every feasible level of output. (Amounts are not numbered in the graphs, but the size of the cost reduction is indicated by showing where \$2 is on the vertical axis, measuring from the origin at 0.) How does the price change compare (larger, smaller, equal, opposite, etc.) to the change in costs?

As shown in the diagrams above, the supply curve shifts down by 2 in each case. In cases A and B, this leads to new lower price P_1 and higher quantity Q_1 as shown. In case C, however, because the downward shift of the supply curve causes the vertical portion merely to slide along itself, there is no change in the intersection with demand and no change in price or quantity. Price falls by the same amount at cost in case A, by less than the fall in cost in case B, and by nothing at all in case C.

b) Indicate in your diagrams the welfare effects on suppliers and demanders in each case.

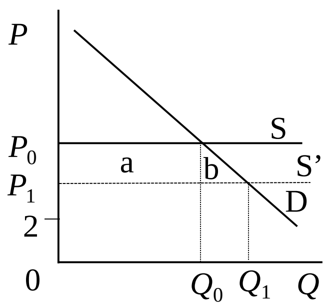


Figure A

In Case A, because price falls by the same amount as cost, there is no benefit to suppliers. They do produce more, but they break even on that production. Demanders gain consumer surplus equal to area (a+b).

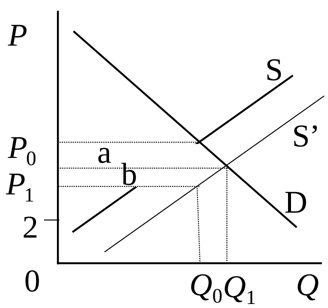


Figure B

In Case B, the price falls by less than 2, so that the gain in consumer surplus is smaller, area a shown to the left of the demand curve. At the same time, since cost has fallen by more than price, suppliers also gain, area b to the left of the supply curve.

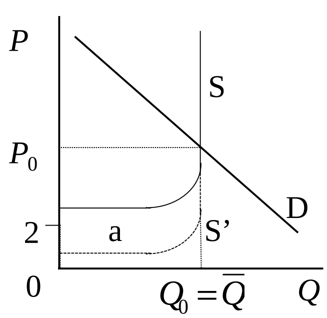


Figure C

In Case C there has been no change in price paid by demanders, and therefore they gain nothing. Suppliers benefit, however, from the lower cost without any change in the price they receive. Their gain is shown as area a, which is equal to $2\overline{Q}$.

c) Would it matter to the results of a benefit-cost evaluation of the tunnel which of these cases is correct? Why, or why not?

There are two reasons why it might matter. First, the size of the total gain to suppliers and demanders together is different in each case. It is smallest in Case C, where the gain, as noted, is $2\overline{Q} = 2Q_0$. In Case A, the gain (all to consumers) is larger by area b, the triangle $2(Q_1 - Q_0)/2 = (Q_1 - Q_0)$. Case B is in between, also larger than Case C by this expression, but the quantity change is smaller than in Case A because the price change is smaller. Thus the social gain from the tunnel is greater the flatter is the supply curve.

On the other hand, from the standpoint of Mackinac Island there is another difference. Most of the demanders in island restaurants are tourists from outside the island, while the suppliers are residents. Therefore the benefit to Mackinac Island alone is smallest in Case A when the supply curve is horizontal, and greatest in Case C with vertical supply.

d) In addition to the cost of building the tunnel and the benefits and costs you identified in part (b), what other considerations do you think should be brought into a benefit-cost analysis of this project? (There is no single correct answer here. I'm looking for a paragraph or so speculating on what other sources of gain and loss might exist for project like this.)

The most important other consideration is all of the other businesses, besides restaurants, whose costs will also be reduced by the tunnel. The benefits to them, as well as to their workers and to the workers in the restaurants, could easily be greater than the benefits to the restaurants alone. And just as tourists benefit from the reduced prices of meals, they will also benefits from reduced prices in other shops on the island. There may be benefits as well to businesses whose costs are not reduced significantly, such as hotels. To the extent that the lower prices of other tourist services attract additional tourists, these businesses will find the prices that they can charge going up. All of these benefits to businesses will likely extend to the local government of Mackinac Island, which will collect greater taxes from all of this economic activity. In addition, assuming that the government bears the cost of disposing of garbage from the island, this cost will presumably go down if garbage can be trucked to the mainland through the tunnel, instead of whatever is done with it now. These benefits to the local government, of course, really reflect benefits to the islands residents, whose taxes finance the government. On the negative side, we should perhaps also consider the noise and pollution that may be caused by the trucks arriving through the tunnel, even if it is possible to keep them out of sight. And there is also the danger that, once the complete ban on motor vehicles has been breached, the tradition of keeping out cars and other vehicles may be more difficult to maintain.

2. In the village of Shuk Ping, province of Suk Chun, a thriving market has long existed for Wing Kis, also known colloquially as Ki Wings, because of their ability to fly back and forth at a moment's notice. Ki Wings are manufactured from the finest raw materials in a large number of local factories, and it is well-established that the supply curve for Ki Wings, accurately reflecting their marginal cost, is given by the following equation:

$$P = 120 + 4Q$$

where Q is the quantity of Ki Wings in thousands, and P is their price in Ching Ching per Ki Wing (☺/kw.). Demand for Ki Wings has been stable for many years, given by the equation

$$P = 930 - 0.5Q$$

Price, as you should verify, has been ☺840 per kw.

Verifying price:

$$P = 120 + 4Q \quad \& \quad P = 930 - 0.5Q$$
$$\Rightarrow \quad 120 + 4Q = 930 - 0.5Q \quad \Rightarrow \quad 4.5Q = 810 \quad \Rightarrow \quad Q = 180$$
$$\Rightarrow \quad P = 120 + 4(180) = 840$$

The mayor of Shuk Ping, Yuen-ling St. Vincent Millay, has just learned that Ki Wings can be used effectively to enhance productivity of government bureaucrats, and proposes to have the government buy 20,000 of them.

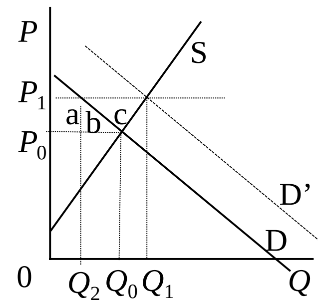
a) What percentage of current Ki Wing production would this government order be?

As derived above, Q=180 thousand kw. The order for 20 thousand kw. is therefore 20/180=1/9=11% of current production.

b) What would be the new equilibrium price of Ki Wings if this order were added to the current market? How much, therefore, will the 20,000 Ki Wings cost the government?

To find the new equilibrium price and quantity, it is simplest to first find the new demand curve including the government purchase. The private demand curve is $P = 930 - 0.5Q$, which can be expressed also as $0.5Q = 930 - P$ or $Q = 1860 - 2P$. Adding the government demand of 20 thousand kw. to this, the new demand curve is $Q = 1880 - 2P$. Substituting this into the supply curve, we get $P = 120 + 4Q = 120 + 4(1880 - 2P) = 7640 - 8P$ or $9P = 7640 \Rightarrow P = 848.9$. Private-sector demand is therefore $Q = 1860 - 2(848.9) = 162.2$ and total production is 162.2 + 20 = 182.2. Thus the new equilibrium price is ☺848.9 per kw. and government expenditure on 20,000 Ki Wings is $848.9 \times 20{,}000 = 16{,}978{,}000$ or ☺16.978 mil.

c) Calculate the effects of this purchase on consumer and producer surplus in the Ki Wing market.



As shown in the figure at the right (which is not drawn to scale, in order to make it easier to see), the government demand shifts the demand curve to the right, raising both equilibrium price and equilibrium quantity. Private-sector demanders, given by the old demand curve, lose from this. Their loss in consumer surplus is the area a+b, or the rise in price times the average of their old and new quantities purchased. That is, they lose

$$(P_1 - P_0)\frac{(Q_0 + Q_2)}{2} = (848.9 - 840)(180 + 162.2)/2 = 8.9 \times 171.1 = ☺1.539 \text{ mil.}$$

Producers gain the increase in producer surplus equal to area a+b+c, or the change in price times the average of their old and new quantity sold:

$$(P_1 - P_0)\frac{(Q_1 + Q_0)}{2} = (848.9 - 840)(182.2 + 180)/2 = 8.9 \times 181.1 = ☺1.629 \text{ mil.}$$

d) What is the net social cost of this policy?

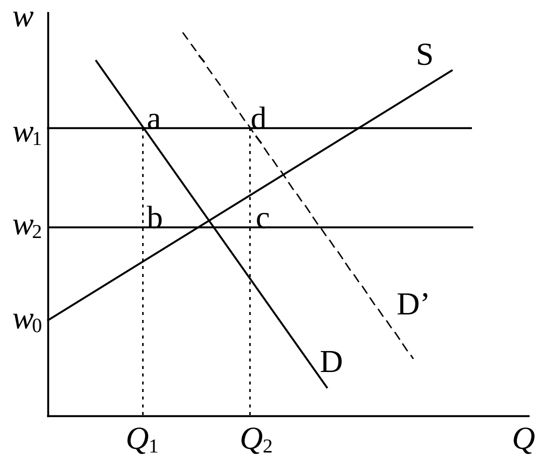
Gain to producers ☺1.629 mil.

Cost to consumers –1.539

Cost to government –16.978

Net cost to society–☺16.887 mil. plus any gain in government productivity

3. The figure at the right shows, as solid lines, the initial supply and demand for labor and a minimum wage, w_1. The demand curve then shifts to the right, to D', as a result of increased employment by government. Assuming that available jobs are allocated randomly among those who want to work at wage w_1, determine the following:



a) The quantity of labor employed before and after the increase in government demand for labor.

Before: Q_1 ; After: Q_2 .

b) The effect of the increased demand on the welfare of suppliers and demanders of labor.

Demanders are unaffected, since they continue to get all the labor they want at wage w_1 . Suppliers gain, since $Q_2 - Q_1$ of them are now employed who were not before. Since they are selected at random from among those willing to work at wage their marginal cost of working ranges from w_0, the intercept of the supply function, to w_1 itself. With the linear supply curve, their average marginal cost of working is therefore $w_2 = (w_1 - w_0)/2$, which is half way between w_0 and w_1. The surplus of the new workers is the excess of their wage, w_1, over this average, or the rectangle bounded by the points labeled a, b, c, and d.

c) Is it possible that the increase in government employment is socially beneficial even if there is no social value to what they do in their new jobs? If so, identify the gain to society. If not, determine how productive they must be in their new jobs in order for this policy to be beneficial for society as a whole.

No. The workers cost the government w_1, while their gain is on average only $w_1 - w_2$ (and at most only $w_1 - w_0$), so employing them for no purpose is creating a net social cost of $w_1 - (w_1 - w_2) = w_2$ and cannot be beneficial. It would be better just to give them money without requiring them to work. However, if they are productive, then their productivity only needs to be greater than w_2 for their employment to be socially beneficial, since this would be enough to turn the net social cost into a net social benefit.

4. Calculate the present discounted value of the projects listed in the table below, which reports for each of four projects, a, b, c, and d, the relevant interest rate, r, and the benefits (positive) and costs (negative) in the present (t=0), and t years from the present.

Project	Interest rate	Benefits (+) and Costs (–) in present (0) and future years, t= 0	1	2	3	4…9	10	11	12…∞
a)	5%	–700	300	400					
b)	3%	5	–5	–5	–5	–5	–5		
c)	7%	–200	14	14	14	14	14	14	14
d)	10%							100	100

a) $PV(a) = -700 + \frac{300}{1.05} + \frac{400}{(1.05)^2} = -700 + 285.7 + 362.8 = -51.5$

Uses general formula, $PV = \sum_{t=0}^{T} \frac{X_t}{(1+r)^t}$

b) $PV(b) = 5 + \frac{-5}{0.03}\left[1 - \frac{1}{(1.03)^{10}}\right] = 5 - 166.7[1 - 0.744] = -37.651$

Uses formula for constant X_t, t=1,…,T, $PV = \frac{X}{r}\left[1 - \frac{1}{(1+r)^T}\right]$

c) $PV(c) = -200 + \frac{14}{0.07} = -200 + 200 = 0$

Uses formula for constant X_t, t=1,…,∞, $PV = \frac{X}{r}$

d) $PV(d) = \frac{100}{0.1} - \frac{100}{.01}\left[1 - \frac{1}{(1.1)^{10}}\right] = 1000 - 1000[1 - .3855] = 385.5$

Uses both of the formulas in (b) and (c) by evaluating the infinite sum, then subtracting the missing finite sum. This could also be done more directly by realizing that in year 10, this will be a constant amount each year from the next year on, and therefore will have the value in year 10 of X/r=100/.1=1000. Then just use the general formula to discount this back to the present, $1000/(1.1)^{10}$.