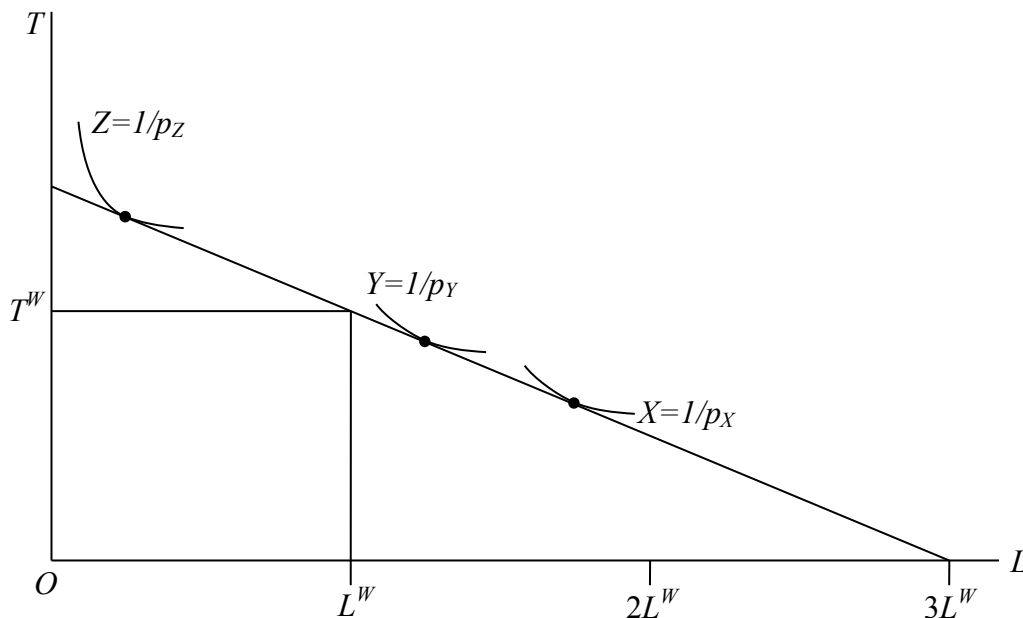


Problem Set #2 - *Answers*

1. Consider an integrated world economy (IWE) with the endowments of land, T^w , and labor, L^w , shown below. The IWE produces three goods, X, Y, and Z, with the equilibrium prices that are associated with the unit value isoquants indicated.



- a) What is world income in units of the same numeraire used for p_x , p_y , and p_z ?

Ans: World income is 1, since the world endowment is on the unit isocost line.

- b) What are the shares of labor and land in world income?

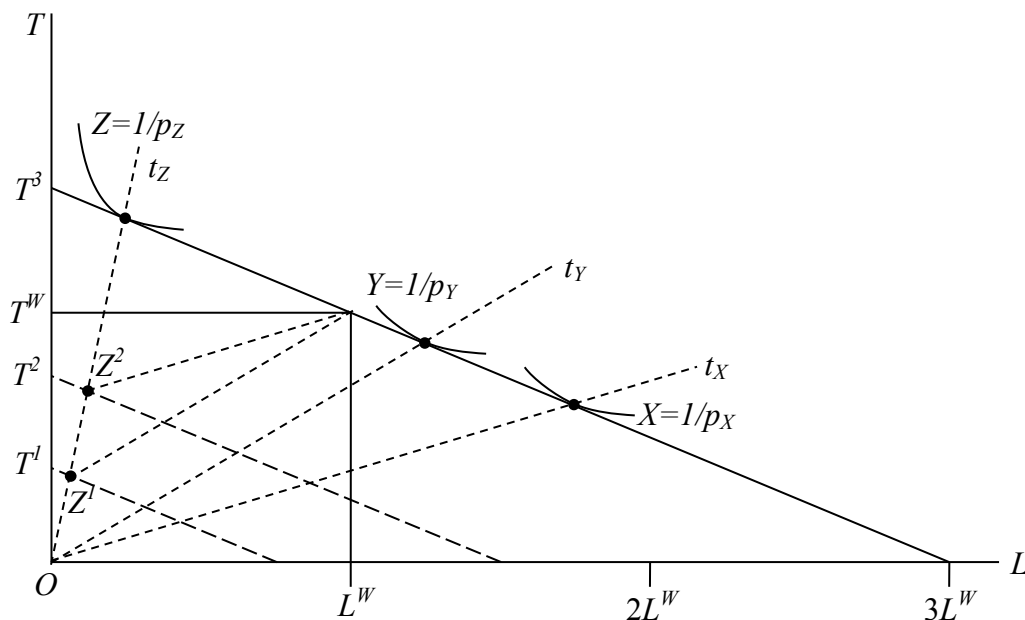
Ans: Labor earns 1/3 of world income, land 2/3. (In units of, say, labor, labor's income is L^w while land's income is $(3L^w - L^w)$).

- c) Identify geometrically the maximum and minimum shares of world income that can be spent on each good in the IWE equilibrium and still have this be an equilibrium. (Note for example that income cannot all be spent on good Z, since the Z industry would not employ all of the labor at these prices.)

Ans: Since Z is the only good employing a land-labor ratio higher than the world endowment, a positive amount of it must be produced in order to fully employ the factors. Its minimum output will be obtained when the rest of the economy is as land intensive as possible, hence producing only good Y, as at Z_1 below. Its maximum output will be obtained when the rest of the economy is as labor intensive as possible, hence producing only good X, as at Z_2 below. The corresponding shares of expenditure can be found by valuing

these outputs in terms of land on the vertical axis and comparing to total income, also measured there as T_3 . Corresponding shares of X and Y can be read analogously:

	Shares of		
	X	Y	Z
min.	0	0	OT^1/OT^3
max.	T^2T^3/OT^3	T^1T^3/OT^3	OT^2/OT^3

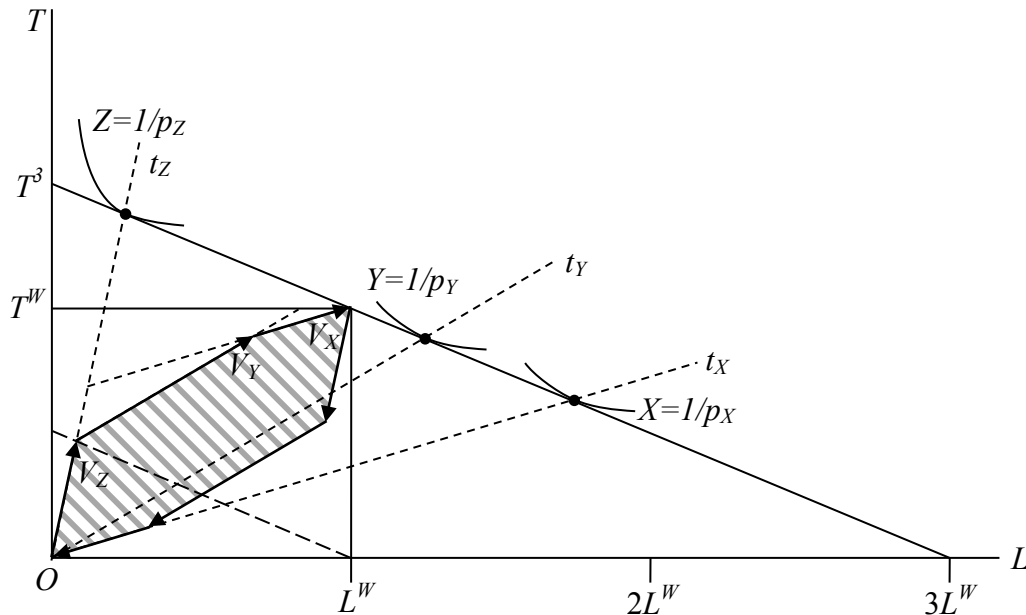


- d) Suppose that $1/3$ of world income is spent on good Z in this IWE. Find geometrically the factors employed in producing each good in the IWE equilibrium.

Ans: Since world income in labor units is $3L^W$, an isocost line through L^W indicates factors worth $1/3$ of world income. Using the factor ray for good Z , this is enough to identify the factors employed in sector Z as vector V_Z . There is only one way that the remaining factors can be fully employed in producing X and Y , as shown below (after part (e)) by the vectors V_X and V_Y .

- e) Under the demand assumption of part (d), show the allocations of labor and land endowments to two countries that would be consistent with their having equal factor prices if they had free trade in goods and no trade in factors.

Ans: The vectors found in part (d) and their mirror image form the boundary of the FPE lens, shown as the shaded region below.

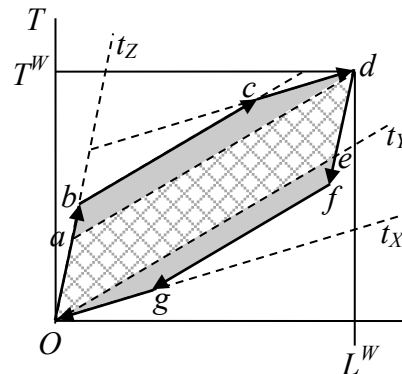
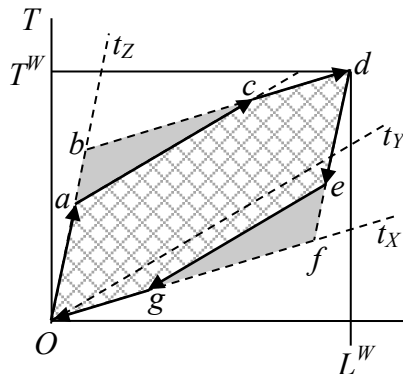


- f) If factor endowments were allocated randomly to the two countries, with every allocation within the IWE box having positive probability, how would the probability of factor price equalization change if the share of good Z in income were to increase from the 1/3 of part (d) to the maximum you found in part (c)?

Ans: The FPE region would increase from the cross-hatched area Oacdeg on the left below, to the larger shaded area Obdf. Since the latter includes the former, the probability of FPE must rise as long as there is any positive probability of selecting the allocations in either of the two triangles abc or gfe.

- g) Repeat (f) for good Y.

Ans: Here the FPE region shrinks from the shaded area Obcdfg on the right below to the cross-hatched area Oade, and the probability of FPE falls.



2. Tests of the standard HOV Theorem, $F^i = V^i - s^i V^W$, require data on factor endowments of the entire world. This problem deals with a related result, due to Bob Staiger, that compares factor contents and endowments for only a pair of countries, and therefore requires data only on two countries.

In a world of many countries, identical constant-returns-to-scale technologies, identical homothetic preferences, and factor price equalization, consider any two countries, A and B, and let F^A and F^B be the vectors of the factor contents of the net exports of each, both measured using the actual factor requirements in use in country A, A^A . Thus $F^i = A^A T^i$, $i = A, B$, where T^i is country i 's vector of net exports.

- a) Show that for any factor, k , if trade of both countries is balanced, then

$$\frac{F_k^A}{Y^A} - \frac{F_k^B}{Y^B} = \frac{V_k^A}{Y^A} - \frac{V_k^B}{Y^B}$$

where V^A and V^B are the countries' vectors of factor endowments and Y^A and Y^B are their (scalar) incomes.

Ans: The usual HOV derivation yields the following, with E^i =expenditure:

$$\begin{aligned} F^i &= AT^i = AX^i - AC^i = V^i - A s^i C^W \\ &= V^i - s^i V^W = V^i - (E^i / Y^W) V^W \end{aligned}$$

Balanced trade implies $E^i = Y^i \Rightarrow F^i = V^i - (Y^i / Y^W) V^W$. Thus

$$\left(\frac{F_k^i}{Y^i} \right) = \left(\frac{V_k^i}{Y^i} \right) - \left(\frac{V_k^W}{Y^W} \right)$$

which implies the result, since the two world endowment terms cancel out.

- b) Of the assumptions listed at the start of this question, which if any can be relaxed and still have the relationship in part (a) be valid?

Ans: Countries other than A and B don't really matter. For this result, we just need identical technologies, identical preferences, and FPE in countries A and B. Then their two consumption vectors will both be proportional to a common vector, $\bar{C}$, of consumption per dollar of income:

$C^A = Y^A \bar{C}$, $C^B = Y^B \bar{C}$. Thus

$$F^i = AX^i - Y^i A \bar{C} \Rightarrow \frac{F_k^i}{Y^i} = \frac{V_k^i}{Y^i} - A \bar{C}_k$$

which also gives the result.

- c) Suppose now that trade is not necessarily balanced. Can you derive a relationship analogous to that in part (a) that relates the factor contents of the two countries trade to their factor endowments using only data from the two countries themselves?

Ans: Just replace Y^i with the countries' expenditures, E^i . The result is then

$$\frac{F_k^A}{E^A} - \frac{F_k^B}{E^B} = \frac{V_k^A}{E^A} - \frac{V_k^B}{E^B}$$

- d) Returning to balanced trade, suppose contrary to the above that factor k in country A is more productive in all uses than its counterpart in all other countries by a multiple λ . How must the assumptions and relationship in part (a) be modified in order to remain valid and still include, except for λ , only the same observable variables?

Ans: Assume identical technologies in terms of effective factor units, and that factor prices are equalized for these units. Effective factor endowments are then $\tilde{V}^i = V^i$ except for $\tilde{V}_k^A = \lambda V_k^A$. Letting $\tilde{A}$ be the effective factor requirements matrix, common to both countries by these assumptions, then measured factor requirements in country A are $(\lambda-1)\%$ smaller than $\tilde{A}$: $A^A = \tilde{A}$ except $A_k^A = \tilde{A}_k / \lambda$ where the subscript indicates the k^{th} row.

In effective units, the model is identical to HOV. Hence

$$\tilde{F}^i = \tilde{A}T^i = \tilde{A}X^i - \tilde{A}C^i = \tilde{V}^i - \left(\frac{Y^i}{Y^W} \right) \tilde{A}C^W, \quad i = A, B$$

Dividing by Y^i and subtracting country B from country A,

$$\frac{\tilde{F}_k^A}{Y^A} - \frac{\tilde{F}_k^B}{Y^B} = \frac{\tilde{V}_k^A}{Y^A} - \frac{\tilde{V}_k^B}{Y^B}$$

To express this in terms of observed factors and factor requirements, use

$$F_k^i = A_k^i T^i = \frac{1}{\lambda} \tilde{A}_k T^i = \frac{1}{\lambda} \tilde{F}_k^i \quad \text{and} \quad \tilde{V}_k^A = \lambda V_k^A$$

Then

$$\frac{\lambda F_k^A}{Y^A} - \frac{\lambda F_k^B}{Y^B} = \frac{\lambda V_k^A}{Y^A} - \frac{\lambda V_k^B}{Y^B}$$

3. a) Does the “Friends and Enemies” version of the Stolper-Samuelson Theorem work for groups of goods? That is, with arbitrary numbers of n goods and m factors, suppose that $\hat{p}_i = \rho > 0$ for all i in a group G of $n_G < n$ goods and $\hat{p}_i = 0 \forall i \notin G$. Is it then true that there must exist at least one factor, j_F , such that $\hat{w}_{j_F} > \rho$ and at least one other factor, j_E , such that $\hat{w}_{j_E} < 0$? To keep things simple, assume that every factor is employed in positive amounts in every industry.

Ans: Yes. We know that, for all i , $\hat{p}_i$ is a weighted average of $\hat{w}_j$:

$$\hat{p}_i = \sum_j \theta_{ji} \hat{w}_j, \quad \text{where} \quad \sum_j \theta_{ji} = 1; \theta_{ji} > 0$$

For any $i \in G$,

$$\hat{p}_i = \sum_j \theta_{ji} \hat{w}_j = \rho > 0 \Rightarrow \exists j_{F'} \text{ s.t. } \hat{w}_{j_{F'}} > 0$$

For any $i \notin G$,

$$\hat{p}_i = \sum_j \theta_{ji} \hat{w}_j = 0 \quad \& \quad \hat{w}_{j_{F'}} > 0 \Rightarrow \exists j_E \text{ s.t. } \hat{w}_{j_E} < 0$$

Then again, for any $i \in G$,

$$\hat{p}_i = \sum_j \theta_{ji} \hat{w}_j = \rho > 0 \quad \& \quad \hat{w}_{j_E} < 0 \Rightarrow \exists j_F \text{ s.t. } \hat{w}_{j_F} > \rho$$

Thus $\hat{w}_{j_F} > \hat{p}_i \quad \forall i \quad \& \quad \hat{w}_{j_E} < \hat{p}_i \quad \forall i$.

- b) Now suppose, again with arbitrary numbers of goods and factors, and all factors employed in strictly positive amounts in all goods, that prices change by amounts that are not all equal, so that at least some relative prices change. Can we be sure that at least one factor is made unambiguously better off and at least one factor is made unambiguously worse off?

Ans: Yes. Let $\hat{i}$ be the good (or one of the goods) whose price rises the most,

$\hat{p}_{\hat{i}} \geq \hat{p}_i \quad \forall i$, and $\hat{\bar{i}}$ be the good (or one of the goods) whose price rises the least, $\hat{p}_{\bar{i}} \leq \hat{p}_i \quad \forall i$. Since the price changes are not all equal, $\hat{p}_{\hat{i}} > \hat{p}_{\bar{i}}$.

Using the same reasoning as in part (a), since each of these price changes is a weighted average of all the factor price changes with positive weights on each factor, it follows first that $\exists j_{F'} \text{ s.t. } \hat{w}_{j_{F'}} \geq \hat{p}_{\hat{i}}$, then that

$\exists j_E \text{ s.t. } \hat{w}_{j_E} < \hat{p}_{\bar{i}}$, and finally that $\exists j_F \text{ s.t. } \hat{w}_{j_F} > \hat{p}_{\bar{i}}$. From this it follows that $\hat{w}_{j_F} > \hat{p}_i \quad \forall i$ and $\hat{w}_{j_E} < \hat{p}_i \quad \forall i$.

- c) In parts (a) and (b) we found that $\hat{w}_{j_F} > \hat{p}_i \quad \forall i \quad \& \quad \hat{w}_{j_E} < \hat{p}_i \quad \forall i$. Suppose that we are in the “Extreme Specific Factors Model” in which all factors are unable to move across industries. Why, in that model, does this conclusion not hold? What in the proofs of (a) and (b) prevents the argument from going through? Is it still true that owners of some factors unambiguously gain and owners of some factors unambiguously lose?

Ans: The conclusion cannot hold because each factor’s price change will exactly equal that of the price of the good in which it is employed. Therefore it cannot be strictly greater or strictly less than all price changes. The proofs fail because the assumption that all factors are employed in all sectors does not hold in the Extreme Specific Factors Model, where, for example, labor employed in sector X is not, by definition, employed at all in any other sector. However, the same arguments used in (a) and (b) but without all θ ’s being positive can still be used to obtain: $\hat{w}_{j_F} \geq \hat{p}_i \quad \forall i \quad \& \quad \hat{w}_{j_E} \leq \hat{p}_i \quad \forall i$.

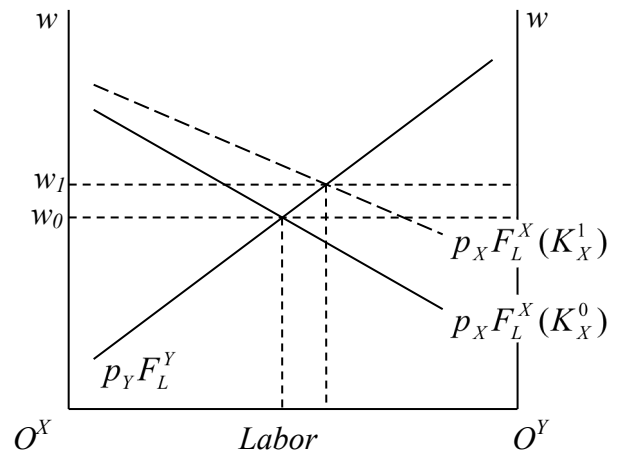
With price changes not all the same, that is enough to assure factors that unambiguously win and lose, but only with the additional assumption that owners of all factors consume all goods. That is needed, as one can easily

see in the Extreme Specific Factors Model. If instead, factors there all consumed only the goods they help to produce, then price changes would be irrelevant to their welfare and none would ever gain or lose.

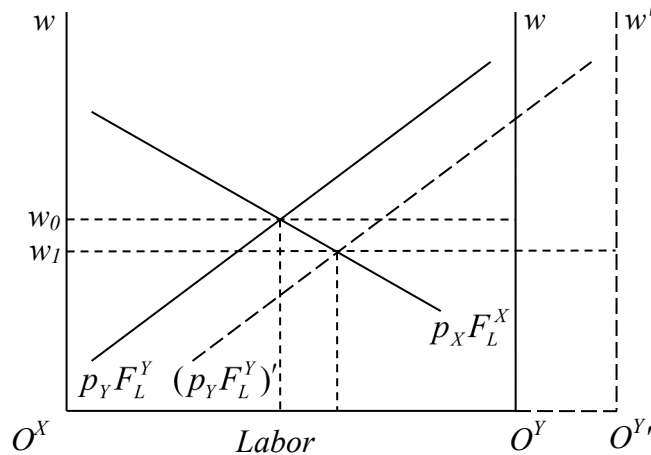
4. In the two-good Heckscher-Ohlin model, the Rybczynski Theorem says that, at fixed prices and with factor-price equalization, an increase in the endowment of (only) one factor will cause the output of one good to fall and output of the other good to increase by a larger percentage than the increase in the factor endowment itself. How many, if any, of these results are also valid in the two-good specific factors (Ricardo-Viner) model? That is,

- a) Is it true for any factor that an increase in its endowment reduces the output of one good, and, if so, is this true for all factors?

Ans: Yes, it is true for either specific factor. Adding more of a specific factor increases the marginal product of labor in that sector, increasing demand for labor there. This causes the equilibrium wage to rise reducing employment and output in the other sector. (With constant returns to scale, and increase in the specific factor shifts the marginal product curve horizontally by the same percentage as the increase in the specific factor, as shown.)



However, it is not true of all factors, since it is not true for the mobile factor, labor. An increase in the endowment of labor increases employment in both sectors, raising output in both. See the diagram below, where the horizontal dimension expands with an increase in the labor endowment, shifting the right-hand vertical axis to the right and the Y-sector labor demand curve to the right along with it.

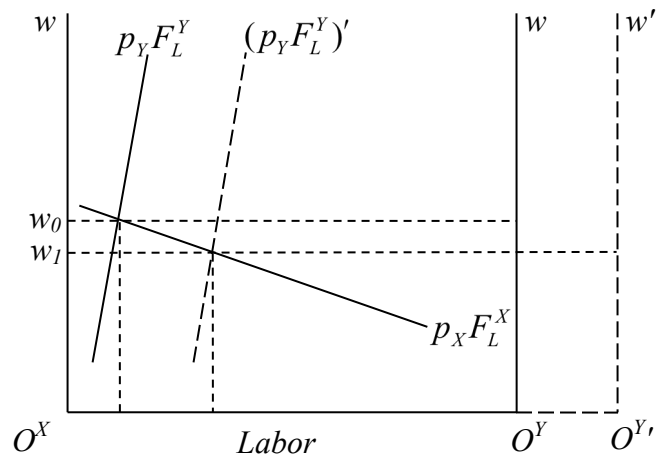


- b) Is it true for any factor that an increase in its endowment increases the output of some good more than in proportion to the endowment increase, and, if so, is this true for all factors?

Ans: When endowment of a specific factor increases, output in its sector will expand by the same proportion only if labor employment in that sector likewise expands in that proportion as well. (I'm assuming, clearly, constant returns to scale.) But that will not happen, since the wage rises, as shown in the first figure of part (a).

When endowment of labor increases, diminishing returns will prevent all sectors from expanding by as much, and it is therefore certainly possible that all will expand by less than the proportional increase in labor. However, it is also possible for one sector to expand by more, if the sector was initially very small and if its marginal product of labor is relatively flat, as shown below. In that figure, output of industry X, given by the area under the F_L^X curve, has more than doubled while the increase in labor endowment is only a fraction.

Thus the statement is certainly not true for all factors, and it may not be true for any. But it may, under certain circumstances, be true of labor.



5. In the following modification of the Spence-Dixit-Stiglitz utility function, determine the value of the parameter ν that will neutralize the effect of increased variety on utility. That is, assuming that all varieties are priced the same, find the value of ν such that an increase in n , holding prices and total expenditure constant, will leave utility unaffected.

$$U = \left(\sum_{i=1}^n n^{\nu} c_i^{\beta} \right)^{\frac{1}{\beta}}$$

Ans: With identical prices, p , symmetry requires that

$$c_i = \frac{E}{np} \quad \forall i$$

where E is expenditure. Then

$$\begin{aligned} U &= \left(\sum_{i=1}^n n^{\nu} \left(\frac{E}{np} \right)^{\beta} \right)^{\frac{1}{\beta}} = \frac{E}{p} \left(\sum_{i=1}^n n^{\nu} n^{-\beta} \right)^{\frac{1}{\beta}} = \frac{E}{p} (n n^{\nu} n^{-\beta})^{\frac{1}{\beta}} \\ &= \frac{E}{p} n^{\frac{1+\nu-\beta}{\beta}} \end{aligned}$$

To neutralize the effect of n on U , set its exponent in the last line to zero. Thus

$$\nu = \beta - 1$$

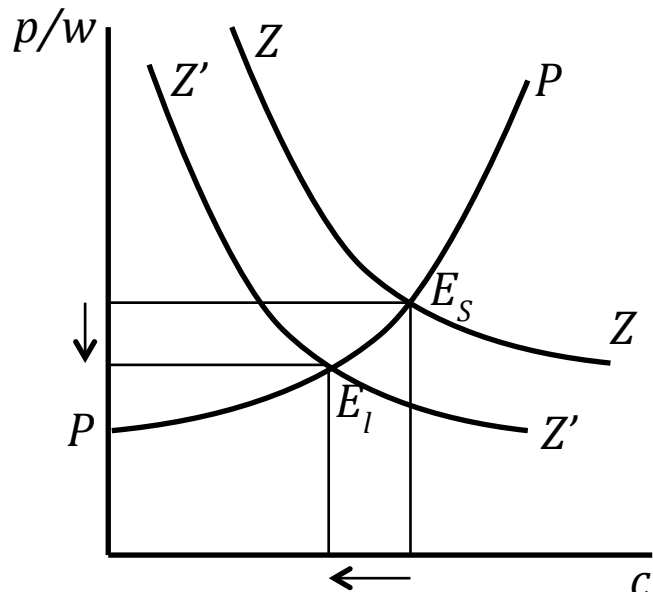
That is, the SDS utility function but without any variety effect on utility is

$$U = \left(\sum_{i=1}^n n^{\beta-1} c_i^{\beta} \right)^{\frac{1}{\beta}}$$

6. Suppose a world of many countries, each like the countries modeled in Krugman (1979) but with labor endowments, L_j , that may be different. That is, consumers in country j maximize a utility function $U^j = \sum_{i=1}^{n_j} v(c_i^j)$ where $v(c_i^j)$ is per capita consumption of the i^{th} variety in country j , while n_j firms, one for each variety i , produce a quantity $x_i^j = L_j c_i^j$ incurring a labor cost of $l_i^j = \alpha + \beta x_i^j$ with $\alpha, \beta > 0$. In equilibrium, firms charge prices p_j that maximize their profits given outputs of other firms; the wage, w^j , is determined to clear the labor market ($L_j = \sum_{i=1}^n l_i^j$); and free entry of firms drives profits to zero: $\pi_i^j = p_i^j x_i^j - w^j l_i^j = 0$.

- a) Following Krugman, suppose first that the v functions are such that $v' > 0$, $v'' < 0$, and $d\varepsilon/dc_i^j < 0$ where ε is the (positive) elasticity of demand facing any firm as a function of its consumers' per capita demand. Holding constant the world's population, $L^W = \sum_j L_j$, determine how the following variables depend on an individual country's population, L_j .

Ans: For an individual country, and also for the world, this is exactly the model of Krugman (1979), and we can analyze it using his diagram, reproduced at the right. The figure shows the "effect" of an increase in the population, as valid for a comparison of two countries in autarky as for a single country (or world) that experiences an increase in population. As in Krugman, such an increase causes a fall in each consumer's consumption of each variety, c , as they also consume a larger number of varieties.



The ratio of price to wage falls, implying an increase in the real wage.

- i) The ratio of the wage to any good's price, w_j/p_j , in autarky.

Ans: As long as the countries are not trading, then each will have its variables determined in a diagram like this, and countries with larger populations will correspond to the equilibrium E_l while smaller countries will be at E_s . Thus the larger is a country's population, the larger will be the ratio of its wage to the price of a typical variety.

- ii) The ratio of the wage to any good's price, w_j/p_j , in free trade.

Ans: If all countries are trading freely, then wages and prices are determined in the world market, a single wage and price prevailing in all countries. In that case, the ratio of wage to price is the same in all.

- iii) The gain in consumer utility of going from autarky to free trade.

Ans: All consumers gain going from autarky to free trade, since their only income is wages and the real wage rises in every country. However, since consumers in small countries start out with lower wages, and consumers everywhere end up the same, the size of the gain from trade is larger for consumers in small countries. Consumers also gain in all countries from access to more varieties of the differentiated product, and since small countries have fewer varieties in autarky than large countries, consumers in small countries gain more from variety as well as from the increase in their wage relative to price.

- b) Now suppose instead that $d\epsilon/dc_j = 0$. Which of your answers in part (a) are altered, and why?

Ans: The sign of $d\epsilon/dc_j$ does not matter for the ZZ curve. It does matter, however, for the PP curve, which reflects monopoly markup prices for the producers of the differentiated good. If the demand elasticity that they face does not fall as consumption levels rise, then the markup will remain constant and the price charged by firms will also be constant. That is, the PP curve will be horizontal. Thus the ratio of wage to price does not after all depend on population. The answers to part (a) are changed as follows:

- i) Country size will no longer matter for the ratio of wage to price in autarky. All countries will have the same ratio.*
- ii) Country size still does not matter for the ratio of wage to price in free trade (thus no change).*
- iii) Country size will no longer matter for the change in the wage-price ratio going from autarky to free trade. However, it does still matter for the size of gains from trade, since this includes the variety effect that is still present.*