Discounting

Discounting is a procedure for evaluating benefits and costs that occur at different times. Let $X = (X_0, X_1, \ldots, X_t, \ldots)$ be a sequence of benefits or costs (if negative), X_t, that occur at times t periods (usually years) from the present, where the present is t=0. The object is to find a single amount, V(X), at time 0 that a decision-maker would view as equivalent to X, in the sense that they would be indifferent between having X and having V(X) instead. In general, if there were no markets for borrowing and lending, V(X) would be subjective, depending on the decision-maker's various options at various times. However, if the decision maker has the option of borrowing or lending arbitrary amounts at a given interest rate, r per period, then they can borrow against any future receipts and save for future payments at this rate r, and therefore use the markets to convert any stream of benefits and costs into an actual present sum, that we call the present discounted value.

Present Discounted Value: For any sequence of positive and/or negative payments X_t, $t=0,\ldots,\infty$, (including implicit ones in the form of consumer surplus, for example), the present discounted value is

$$V(X) = \sum_{t=0}^{\infty} \frac{X_t}{(1+r)^t}$$

This formula is reasonably general, and it can be applied to both finite and infinite payments streams, so long as the interest rate r is constant over time. If t is measured in years, then r should be the interest rate per year, but the period of analysis can be something else, so long as the interest rate is defined for it as well.

Special Cases:

In all of the cases considered below, the payment in year zero is taken to be zero. Since a payment in the present requires no discounting (and is divided in the formula above by $(1+r)^0=1$), these can be simply added to the formulas provided.

1. $X_t = X$, for $t=1,\ldots,\infty$:

$$V_1 = \frac{X}{r}$$

because $V_1 = \sum_{t=1}^{\infty} \frac{X}{(1+r)^t} = \frac{X}{1+r} + \sum_{t=2}^{\infty} \frac{X}{(1+r)^t} = \frac{X}{1+r} + \frac{1}{1+r} \sum_{t=1}^{\infty} \frac{X}{(1+r)^t} = \frac{X}{1+r} + \frac{1}{1+r} V_1$
implies $(1+r)V_1 = X + V_1$ implies $V_1 = X / r$.

2. $X_t = X$, for t=1,…,T:

$$V_1 = \frac{X}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

because

$$V_1 = \sum_{t=1}^{T}\frac{X}{(1+r)^t} = \sum_{t=1}^{\infty}\frac{X}{(1+r)^t} - \sum_{t=T+1}^{\infty}\frac{X}{(1+r)^t} = \sum_{t=1}^{\infty}\frac{X}{(1+r)^t} - \frac{1}{(1+r)^T}\sum_{t=1}^{\infty}\frac{X}{(1+r)^t}$$

$$= \frac{X}{r} - \frac{1}{(1+r)^T}\frac{X}{r} = \frac{X}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

3. $X_t = X(1+a)^{t-1}$ for t=1,…, T

$$V_3 = \frac{X}{r-a}\left(1 - \left(\frac{1+a}{1+r}\right)^T\right)$$