Problem Set 1
International Equilibrium

1. a. Explain how it is possible for a production function to have the property of constant or increasing returns to scale and still satisfy the Law of Diminishing Returns.

   b. If a production function has only one input, say labor, and displays constant returns to scale, does it then violate the Law of Diminishing Returns?

   c. Suppose that a production function has two inputs, K and L, and that, contrary to the Law of Diminishing Returns, an increase in L alone always causes output to rise by the same proportion that L has increased. Show that the production function must therefore display increasing returns to scale.

   d. Differentiate the definition of "Homogeneous of Degree k" with respect to λ, and evaluate the result at $\lambda=1$. Then use this result to show that, if factors of production are paid the value of their marginal products as assumed under perfect competition, then factor payments will exactly equal the value of output if returns to scale are constant, but will exceed it if returns to scale are increasing. What does the latter result tell you about the compatibility of perfect competition with increasing returns to scale?

2. a. In the 2×2 production model with constant returns to scale, show that if the two industries have the same factor intensity for all factor prices, then the production possibility frontier (PPF) is a straight line.

   b. Use the Edgeworth-Bowley Box diagram to illustrate the effects on production possibilities of an increase in the endowment of capital holding the endowment of labor constant. Show how the changes in the box diagram translate to changes in the PPF.

3. In the 2×2 model of production and trade, suppose that a country is initially exporting good X and that its production possibilities then expand (we don't know why), making it possible for it to produce more of both goods. Assuming that the country is small enough that the world market price does not change as a result, which of the following *must* be true, which of the following *might or might not* be true, and which of the following *cannot* be true?

   a. It will produce more of good X.

   b. It will produce more of good Y.

   c. Its income will rise.

   d. It will import more of good Y.