Name: _____ Score (Out of 10 points):

1. (3 points) Let X be a set. State the definition of a *metric* on X.

2. A metric space (X, d) is called *bounded* if there exists some real number $R \geq 0$ such that $d(x, y) \leq R$ for all $x, y \in X$. In other words, any pair of points in X are distance at most R apart.

   (a) (2 points) A metric space that is not bounded is called *unbounded*. Negate the definition to give a precise statement of what it means for a metric space to be unbounded.

   (b) (2 points) Show that the real numbers $\mathbb{R}$ with the standard Euclidean metric is unbounded.

   (c) (3 points) Let (X, d) be a metric space. Suppose that there is some element x_0 in X so that $d(x_0, x) \leq 10$ for all $x \in X$. Prove that X is bounded.