Name: _____ Score (Out of 9 points):

1. (9 points) For each of the following sequences: state the set of all limits, or, if the sequence has no limits, write "`Does not converge`". **No justification necessary.**

   (a) Let $\mathbb{R}$ have the topology $\{U \mid 0 \in U\} \cup \{\varnothing\}$.

   (i) $0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ \cdots$ limits: $\{1\}$

   (ii) $0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ \cdots$ Does not converge.

   (b) Let $\mathbb{R}$ have the topology $\{(a, \infty) \mid a \in \mathbb{R}\} \cup \{\mathbb{R}\} \cup \{\varnothing\}$

   (i) $0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ \cdots$ limits: $(-\infty, 0]$

   (c) Let $\mathbb{R}$ have the cofinite topology $\{U \mid \mathbb{R} \setminus U \text{ is finite}\} \cup \{\varnothing\}$

   (i) $0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ \cdots$ Does not converge

   (ii) $0,\ 1,\ 0,\ 2,\ 0,\ 3,\ 0,\ 4,\ 0,\ 5,\ 0,\ 6,\ \cdots$ limits: $\{0\}$

   (iii) $0,\ 1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \frac{1}{6},\ \frac{1}{7},\ \frac{1}{8},\ \frac{1}{9},\ \frac{1}{10},\ \cdots$ limits: all real numbers

   (d) Let $\{a, b, c, d\}$ have the topology $\Big\{\varnothing, \{a\}, \{a, b, c\}, \{a, d\}, \{a, b, c, d\}\Big\}$.

   (i) $a,\ a,\ a,\ a,\ a,\ a,\ a,\ a,\ a,\ a,\ a,\ a,\ \cdots$ limits: $\{a, b, c, d\}$

   (ii) $b,\ b,\ b,\ b,\ b,\ b,\ b,\ b,\ b,\ b,\ b,\ b,\ b,\ \cdots$ limits: $\{b, c\}$

   (iii) $a,\ d,\ a,\ d,\ a,\ d,\ a,\ d,\ a,\ d,\ a,\ d,\ \cdots$ limits: $\{d\}$