Name: _____ Score (Out of 9 points):

1. (9 points) Consider the following topological spaces X and their subsets S. In each case, compute the interior $\text{Int}(S)$, the closure $\overline{S}$, and the boundary ∂S. **No justification necessary.**

Let $X = \{a, b, c, d\}$ with the topology $\Big\{\varnothing, \{a\}, \{b, c\}, \{a, b, c\}, \{a, d\}, \{a, b, c, d\}\Big\}$.

(a) Let $S = \{a,\ d\}$.

$\text{Int}(S)$: $\{a,d\}$ $\overline{S}$: $\{a, d\}$ ∂S: $\varnothing$

(b) Let $S = \{a,\ c\}$.

$\text{Int}(S)$: $\{a\}$ $\overline{S}$: $\{a, b, c, d\}$ ∂S: $\{b, c, d\}$

Let $X = \mathbb{R}$ with the topology $\mathcal{T} = \{U \mid 0 \in U\} \cup \{\varnothing\}$.

(c) Let $S = \{0, 1\}$.

$\text{Int}(S)$: $\{0,1\}$ $\overline{S}$: $\mathbb{R}$ ∂S: $\mathbb{R}\backslash\{0,1\}$

(d) Let $S = \{1\}$.

$\text{Int}(S)$: $\varnothing$ $\overline{S}$: $\{1\}$ ∂S: $\{1\}$

Let $X = \mathbb{R}$ with the topology $\mathcal{T} = \{(a, \infty) \mid a \in \mathbb{R}\} \cup \{\varnothing\} \cup \{\mathbb{R}\}$.

(e) Let $S = (-\infty, 0)$.

$\text{Int}(S)$: $\varnothing$ $\overline{S}$: $(-\infty, 0]$ ∂S: $(-\infty, 0]$

(f) Let $S = (0, \infty)$.

$\text{Int}(S)$: $(0,\infty)$ $\overline{S}$: $\mathbb{R}$ ∂S: $(-\infty, 0]$