Name: _____ Score (Out of 13 points):

1. (5 points) Let X be a set. State the definition of a *metric* on X.

2. (8 points) The following is a list of spaces X and functions $d : X \times X \to \mathbb{R}$ that **fail** to be a metric on X. For each function d, identify an axiom for a metric that fails to hold, and show by counterexample that the axiom fails.

   (a) $X = \mathbb{N} = \{1, 2, 3, \dots\}$, and d is the function $d(x, y) = |x||x - y|$.

   (b) $X = \mathbb{R}$, and d is the constant function $d(x, y) = 10$ for all $x \in X$.

   (c) $X = \mathbb{R}$, and d is the function $d(x, y) = |x^2 - y^2|$.

   (d) $X = \{a, b, c\}$, and d is the function

$$\begin{aligned} d(a, a) &= d(b, b) = d(c, c) = 0 \\ d(a, b) &= d(b, a) = 2 \\ d(b, c) &= d(c, b) = 3 \\ d(a, c) &= d(c, a) = 6. \end{aligned}$$