Effects of Voting Technologies and Recount Methods on Votes in Wisconsin and Michigan*

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Were the outcomes in Wisconsin and Michigan in the 2016 presidential election correct? Candidate Trump won both states—by margins over Clinton of 22,748[1] and 10,702[2], respectively—but the results are controversial. One concern is whether the vote tabulation technologies were hacked, as much of the equipment used to tabulate votes in 2016 has been shown to be particularly vulnerable.[3] Russian hacking had already taken place during the campaign, as acknowledged by ODNI (2017), and it seems reasonable that in their efforts to influence the election vote manipulation may have been attempted. Recounts were prompted in both states by the Stein campaign (Gupta 2016; Halderman and Bernhard 2016; Friess 2017).

Using data from the recounts, we present evidence that that the voting technologies used in places that had the votes recounted in these states appear to have treated candidates Trump and Clinton symmetrically. Whether votes cast for Trump or Clinton were counted does not appear to depend on which candidate the vote was for. Presumably a hack intended to benefit or harm one candidate more than the other would cause asymmetric treatment. We find no evidence that that happened. We also find that manual and machine recount methods in Wisconsin seem to have performed similarly.

Our analysis allows us to say whether the recount's adding or subtracting votes in particular Wisconsin wards or Michigan precincts is associated with the type of voting technology used in each place, but it does not allow us to estimate how many votes are affected by the behavior of voting technologies in the two states. So we can't say whether possible misbehavior of technologies affected vote counts by enough to have changed the election outcomes. Nonetheless the analysis adds to confidence that the election outcomes are correct.

[1] Wisconsin margin computed using recounted vote values in Wisconsin Elections Commission (2017*b*).

[2] Michigan margin computed using official values in Johnson (2017*a*).

[3] See California's Top-to-Bottom review (California Secretary of State's Office 2007) and Ohio's Project EVEREST (McDaniel et al. 2007).

1 Recount Data

It is useful to look at raw numbers from the recounts both to show one of the difficulties in the way of estimating the number of affected votes and to explain the basic approach we take to analyzing the data. The following issues with the numerical distributions are by no means the most serious challenge to performing an analysis in terms of exact vote counts, but it's not clear how to resolve them.

The problem with the exact vote counts is that they are mostly small but there are a few relatively large values. We focus on the differences between the recounted vote counts for each candidate and the original vote counts: the original vote count in each ward (Wisconsin) or precinct (Michigan) is subtracted from the recounted vote count. Tables 1 and 2 enumerate the distribution of differences by major party candidate in Wisconsin, and Tables 3 and 4 enumerate the distribution of differences by candidate in Michigan. In all four cases the most frequent difference is zero, meaning the count of votes for the candidate did not change in the recount from the original count. The next most frequent differences are small decreases or increases.

*** Tables 1, 2, 3 and 4 about here ***

The problem is the sporadic double-digit and even a few triple-digit differences: in Wisconsin Trump gains 246 votes in one machine-recounted ward; in Michigan Trump loses 209 votes and Clinton loses 287 votes in absentee (AV) precincts. Probably the large differences are produced by different processes than are the smaller differences, but it is not obvious how to distinguish the processes: simply to declare the larger values are "outliers" (Wand, Shotts, Sekhon, Mebane, Herron and Brady 2001; Mebane and Sekhon 2004) seems incurious about what produced them; to specify a mixture model is challenging given the complexities of technologies and procedures in the states, which we do not elaborate here.[4]

At least in Wisconsin we observe that larger differences tend to be associated with

[4] But see the discussion of DRE usage on page 8.

particular reasons cited to explain recount changes in official "minutes" documents (Wisconsin Elections Commission 2017*d*,*h*). As Table 5 shows, in Wisconsin the largest average differences (in magnitude) occur when the reasons cited are "nonstandard pens or ballots" (mentioned four times) or "voting machine/tabulator error" (mentioned 13 times).[5] Both of these reasons concern features of the voting technologies and so may be worrisome. Many nonzero changes occur ($N = 759$) that lack explanation.

*** Table 5 about here ***

We reduce the differences to signs, focusing merely on whether at each observation—at each ward or precinct—the candidate lost votes in the recount, kept the same number of votes or gained votes. We consider the candidates, Trump and Clinton, together, observing whether at each ward or precinct the two of them lost votes, kept the same number, gained votes, or some combination. We consider two forms of this paired-signs-of-differences measure: one in which the differences are unlabeled, so it is not indicated which candidate has the losses or gains; and one which the differences are labeled, so it is clearly indicated which candidate has the losses or gains. We use multinomial logit regression models to check whether the pattern of differences is associated with voting technology, recount methods and other covariates.[6] The key analytical move is to see whether associations differ when we consider the labeled differences instead of the unlabeled differences. If voting technologies are treating votes for the candidates symmetrically, then labeling the

[5] In Table 1 the biggest increase (from CITY OF MILWAUKEE Ward 34) is not explained but the recounted vote count in Wisconsin Elections Commission (2017*b*) matches the count reported in minutes (Milwaukee County 2016, 17–18), the second biggest (from CITY OF MARINETTE Wards 1,3,5) is explained by "nonstandard pens or ballots" and "voting machine/tabulator error," and the third biggest (from CITY OF MARINETTE Wards 2,4,6) is explained by "nonstandard pens or ballots," "ballots found during recount" and "ballots rejected during recount." In Table 2 the biggest increase (from CITY OF MARINETTE Wards 1,3,5) is explained by "nonstandard pens or ballots" and "voting machine/tabulator error," and the second biggest (from CITY OF MARINETTE Wards 2,4,6) is explained by "nonstandard pens or ballots," "ballots found during recount" and "ballots rejected during recount." The Marinette wards used Eagle opscan machines (vendor Command Central), and minutes mention problems with "improper pens," "Problems with the voting machine rejecting ballots on election night" and "Machine parts were obtained [...] and installed per instructions from Command Central, voting equipment vendor" (Marinette County 2016, 43–44).

[6] Multinomial logit regression models are estimated using the `multinom()` function in the `nnet` package (Venables and Ripley 2002) for **R** (R Development Core Team 2011).

differences should not produce a different impression of associations with the technologies than analyzing the unlabeled differences does.

A key assumption that motivates the analysis is that vote counts that were both manually cast and manually recounted—votes originally cast on paper and recounted by hand—are correct, so that any difference between a manually recounted count and the corresponding original count implies that the original count is in error. We say that only recounts produced by manual tabulation are known to be correct partly because important suspicions involve hacking of or errors in any machine technology. An assumption to trust manually tabulated counts involves further assumptions regarding soundness of the chain of custody of ballots, trustworthiness of manual tabulators and other procedural details that we do not spell out (see e.g. Stark and Wagner 2012).

2 Analysis Motivation

To describe the analysis plan more precisely, let loss, same and gain values be denoted, respectively, -1, 0 and 1. Then

$$S_0 = \{00, \text{-1-1}, 11, \text{-10}, 01, \text{-11}\}\,,$$

contains all possible combinations of recount-minus-original count changes (loss, same or gain) for the two candidates Trump and Clinton *without* noting which candidate has which kind of change. For instance, the value -11 denotes a case where one candidate lost votes and the other candidate gained votes. Contrasted to the unlabeled outcomes in S_0 are the labeled outcomes in

$$S_1 = \{\text{T0C0}, \text{T-1C-1}, \text{T1C1}, \text{T0C-1}, \text{T-1C0}, \text{T0C1}, \text{T1C0}, \text{T-1C1}, \text{T1C-1}\}\,.$$

where for instance `T0C0` indicates the number of votes for both candidates' counts are the same in the recounted and originally counted data, `T0C-1` means Trump counts are the same while Clinton has fewer in the recounted data than in the originally counted data, and `T0C1` means Trump counts are the same while Clinton has more in the recounted data than in the originally counted data.

We estimate a multinomial regression model in which the possible outcome categories are the elements of S_0 and another multinomial regression model using the same regressors in which the possible outcome categories are the elements of S_1. In the model for S_0 00 is the reference category and in the model for S_1 T0C0 is the reference category: the same observations belong to these two categories. In a multinomial regression model, coefficients for all categories other than the reference category measure differences between each category and the reference category. So coefficients in the models measure differences between outcomes where at least one candidate's vote count changes between the original and recounted counts and the outcome where neither candidate's outcome changes.

The idea that the voting technologies or recount methods do not treat the two candidates differently implies that for a particular predictor the coeffients for each of the following outcomes are the same: (a) -10, `T0C-1` and `T-1C0`; (b) 01, `T0C1` and `T1C0`; (c) -11, `T-1C1` and `T1C-1`. The exception to the coefficient equality expectation may be intercept terms because for instance the sets of observations with S_1 values `T0C-1` or `T-1C0` are subsets of the set of observations with S_0 values -10: `T0C-1` and `T-1C0` may have more negative intercept values because there are fewer such observations than there are -10 observations.

If the identity of the candidate is irrelevant to the vote tabulation and recount processes, then apart from intercept terms that capture the reduced frequency of each of the pairs of candidate-labeling categories $\{\text{T0C-1}, \text{T-1C0}\}$, $\{\text{T0C1}, \text{T1C0}\}$ and $\{\text{T-1C1}, \text{T1C-1}\}$ compared to the unlabeled categories -10, 01 and -11, the coefficients of regressors for the unlabeled variable categories S_0 should not differ significantly from the corresponding coefficients for the candidate-labeling categories S_1. So non-intercept coefficients for each set of categories in the same row in Table 6 should not differ from one another. Coefficients for outcomes in the first three rows of Table 6 are not informative about Trump and Clinton being treated differently, but coefficients for outcomes in the last three rows can be informative.

*** Table 6 about here ***

We test whether coefficients for the same regressor differ between `T0C-1` and `T-1C0`, between `T0C1` and `T1C0`, or between `T-1C1` and `T1C-1`. Specifically for the coefficients of voting technology and recount method variables we test whether each of the following differences between coefficients is zero: a coefficient in `T0C-1` minus the corresponding coefficient in `T-1C0`; a coefficient in `T0C1` minus the corresponding coefficient in `T1C0`; or a coefficient in `T-1C1` minus the corresponding coefficient in `T1C-1`. In particular, finding that the coefficients for the type of voting technology used originally to tabulate the votes differ raises suspicions about that technology, and finding that the coefficients for the method used for the recount differ raises suspicions about the recount method.

Whether an effect of the voting technology or of recount method variables is connected to the operations of the machines or merely to other features that happen to be collocated with the machines is a question we cannot resolve with kinds of data that we have. We include additional covariates as regressors in the models because the voting technology and recount method variables are associated with them—depend on them in the regression sense of dependence. By including the additional regressors, we hope that the partial effects we estimate for the voting technology and recount method variables are more validly interpretable as reflecting operations of the technologies and recount procedures.

We use additional covariates that we happen to have for each ward or precinct observation. Given the hypothesis that the recounted counts do not differ systematically from the original counts, studying outcomes that originate in differences between recounted and original vote counts should remove dependence on features that affect voting in the election. Nonetheless we include as a regressor the proportion of votes for Clinton, computed using the recounted vote counts. Anything related to vote choices is necessarily captured by these actual vote proportions, if we assume the recounted votes are accurate. We also include other variables, described below for each state.

3 Wisconsin

Table 7 shows the frequency distribution of voting technology and recount method types across all Wisconsin wards for which the total number of recounted votes across all presidential candidates is positive ($n = 3,500$). Several of the machines that are part of the technologies (and some of the machines that are part of accessibility technologies) are depicted in Figure 1.[7] Each municipality has its own technology: Figure 2 shows how the technologies are distributed across municipalities.[8] Except for the four wards that report using a combination of Eagle and Insight technologies, most types of voting technology occur with sufficient frequency to support informative statistical analysis. In the multinomial regression models we use "None" as the reference category for the set of dummy variables that represent the Voting Technology variable and "Hand" as the reference category for the Recount Method variable.

*** Figures 1 and 2 and Table 7 about here ***

In addition to the types of systems listed as Voting Technology all wards also have "accessibility technology" (Wisconsin Elections Commission 2017*f*). Table 8 shows the pattern in which Voting Technology overlaps in wards with Accessibility Technology. Voters can choose which mode to use to vote. While all the voting technologies except "None" are opscan systems, several of the accessibility systems are Direct Record Electronic (DRE) systems (Accuvote TSX, Edge and iVotronic; Automark and

[7]For descriptions of these technologies see Verified Voting Foundation (2017).

[8]Category "Other" in Figure 2 contains the technologies Populex 2.3, Vote-Pad and "Edge; Automark."

ExpressVote are ballot marking devices, ImageCast Evolution and Populex 2.3 are accessible ballot marking and scanning devices).[9] As Table 9 shows many wards have some votes cast using DRE systems.

*** Tables 8 and 9 about here ***

The greatest challenge to estimating the association between Voting Technology and votes is that we rarely know precisely which mode was used to record each vote. Votes cast using DRE systems were not changed in the recount, but only rarely are all ballots reported as having been cast using DREs.[10] This is especially important to note because if DRE machines were corrupted, the paper audit trail generated by the machines would likely reflect the manipulated votes. If voters fail to verify that their vote has been correctly recorded by the machine (which may occur, see Campbell and Byrne 2009), then neither the paper trail nor this analysis of recount data would detect manipulation: our key assumption would provide more complete confidence if the "None" category for Voting Technology included only votes cast manually on paper, but such is not the case. If a sufficient fraction of voters successfully verify their vote as recorded on the paper, this is in principle enough to detect manipulation—but we have no data regarding such verifications.

[9] Problems that required "programmer" or vendor Command Central help to resolve or that may suggest there was some kind of software error are reported for the Edge machine in several county minutes files. In at least seven wards a programmer or Command Central had to help to retrieve ballots (TOWN OF ARLAND Ward 1 and TOWN OF CUMBERLAND Ward 1 (Barron County Board of Canvass 2016, 11–12); TOWN OF GILMANTON Ward 1 (County of Buffalo 2016, 14); TOWN OF RUSK Ward 1 and VILLAGE OF WEBSTER Wards 1-2 (Burnett County Board of Canvassers 2016, 15, 27); TOWN OF HARRISON Ward 1 (Grant County 2016, 22); TOWN OF OCONTO FALLS Ward 1-2 (Oconto County Board of Canvass 2016, 46)). In at least nine wards the machine count was wrong (TOWN OF RED CEDAR Ward 1-3, TOWN OF WILSON Ward 1 and CITY OF MENOMONIE Wards 5,7 (Dunn County 2016, 13, 23, 34); TOWN OF BEETOWN Ward 1, TOWN OF BLOOMINGTON Ward 1, TOWN OF BOSCOBEL Wards 1-2 (Grant County 2016, 10, 12–13); TOWN OF CHASE Wards 1-5 (Oconto County Board of Canvass 2016, 22); TOWN OF HELVETIA Wards 1-2 (Waupaca County 2016, 8); TOWN OF WAUTOMA Ward 1-3 (Waushara County Board of Canvassers 2016, 20)). In at least four wards ballots did not print out or needed to be reprinted (TOWN OF STANFOLD Ward 1 (Barron County Board of Canvass 2016, 22); TOWN OF COLBURN Ward 1 and TOWN OF GOETZ Wards 1-2 (Chippewa County Board of Canvass 2016, 13, 20); CITY OF BERLIN Ward 1-6 (Green Lake County Board of Canvassers 2016, 2)). Overall the minutes report 41 wards with Edge machines and explicitly described problems and 1270 with Edge machines but nothing reported regarding them. Problem reports are not always associated with nonzero changes in vote counts.

[10] In Wisconsin Elections Commission (2017*a*) only 21 wards report a positive number of DRE votes and zero votes cast using other modes, which are `Paper Ballots`, `Optical Scan Ballots`, and `Auto-Mark`.

Only a few incidences of incorrect votes recorded on the paper audit trail were reported in Wisconsin, and the reported discrepancies are small; while this does not rule out large-scale DRE tampering, it does narrow the likelihood that it occurred. We focus on the Voting Technology systems, but some ballots in each case may be produced using accessibility technology.[11]

Table 10 shows the frequencies of the paired-signs-of-differences variables. Unlabeled change frequencies appear at the top of the table, labeled changes at the bottom. All categories occur sufficiently frequently to support the multinomial regression model analysis.

*** Table 10 about here ***

Regressions not reported here show that several variables relate to Voting Technology and Recount Method when either is used as the outcome variable in a ward-level multinomial regression analysis. These variables are Clinton (HRC) vote proportion, a ratio of two different estimates of the number of registered voters,[12] the proportion of DRE votes, the absentee proportion,[13] turnout[14] and county total votes.[15]

As reported in Table 11, likelihood-ratio tests reject the hypothesis that using the labeled categories (S_0) is not significantly better than using the unlabeled categories (S_1). The hypothesis is rejected whether or not covariates in addition to the Voting Technology and Recount Method variables are included as regressors.

[11] Wisconsin Elections Commission (2016) shows that DS200 goes with accessibility technology `ES&S ExpressVote` (ballot marking) technology in 333 cases, with `ES&S Automark` (ballot marking) 1141 times with `ES&S iVotronic` (touchscreen) technology in one case. ImageCast Evolution technology always goes with `ImageCast Evolution` accessibility technology. M100 technology goes with accessibility technology `ES&S Automark` 183 times and `ES&S iVotronic` 21 times.

[12] The ratio is the number of registered voters from Wisconsin Elections Commission (2017*g*), over the number of registered voters from Wisconsin Elections Commission (2017*a*).

[13] The "proportion" is the ratio of `Absentee Issued` to `Total Voters`, both from Wisconsin Elections Commission (2017*a*). In one ward the ratio is greater than 1: in "VILLAGE OF FOOTVILLE Ward 1" the ratio is 556/410.

[14] Turnout is computed using the ratio of the recounted `Total Votes` from Wisconsin Elections Commission (2017*b*) over the number of registered voters from Wisconsin Elections Commission (2017*g*).

[15] For numerical stability in the multinomial regression estimation software, the total of the recounted votes in each county is divided by the state total of the recounted votes.

*** Table 11 about here ***

Tests for the hypothesis that there is no difference between coefficients for the outcomes listed in the last three rows of Table 6 show that a few of the voting technologies are associated with significant differences when additional covariates are excluded, but no test statistics are significantly large when the additional covariates are included.[16] Table 12 reports test statistics using two multinomial regression models.[17] The first is a model that includes Voting Technology and Recount Method variables and no other covariates (in the top part of the table), and the second is a model that includes those variables along with the additional covariates (bottom part). The differences between the statistics when additional covariates are omitted or included is testimony to the need to include these covariates: omitting them produces spurious coefficient differences. We interpret only the tests when all the covariates are included in the model.

*** Table 12 about here ***

When the additional covariates are included as regressors in the model, no statistics significantly reject the hypothesis of no difference between coefficients. To evaluate the tests we use false discovery rate adjustment for multiple testing (Benjamini and Hochberg 1995), considering all 27 tests in the bottom of Table 12 as simultaneous independent tests.

The focus on Voting Technology may be inappropriately narrow: perhaps distortions in votes originate in Accessibility Technology. We expand the analysis by adding the Accessibility Technology variable as a regressor. In light of ImageCast Evolution being both Voting and Accessibility Technology and because of the low frequencies for "Edge; Automark," "Populex 2.3" and "Vote Pad" technologies, these types are combined as "other" Accessibiity Technology. When the additional covariates are also included as regressors in the model, and using false discovery rate adjustment for multiple testing, no statistics (Table 13) significantly reject the hypothesis of no difference between coefficients.

[16] We use two-tailed tests at test level $\alpha = .05$.

[17] Test statistics are t-statistics for the difference between coefficients for the same regressor for the two categories that are being compared.

*** Table 13 about here ***

A specific suspicion in the election is that some vendors may have corrupted votes using the software they installed in voting technology. Figure 3 shows how the vendors are distributed across municipalities. As the top part of Table 14 shows, several opscan system vendors provided several different types of voting technology. As the bottom part of the table shows, various kinds of accessibility technology are collocated in wards with the vendors' opscan systems.

*** Figure 3 and Table 14 about here ***

We repeat the analysis based on multinomial regression analysis of the paired-signs-of-differences variable, except replacing the Voting Technology variable with the Vendor variable.

Coefficient difference test results appear in Table 15. When the additional covariates are included as regressors in the model, only one statistic significantly rejects the hypothesis of no difference between coefficients (using false discovery rate adjustment for multiple testing). A significant difference appears only for None-Hand ($t = -3.1$) for the T0C1 − T1C0 difference. As previously in Table 12, the None-Hand coefficients are intercept terms, so it is reasonable not to give much attention to this difference. But if we wished to interpret the significant difference associated with None Vendor and Hand Recount Method in Table 15, we would say it suggests that with that combination of procedures, other things equal, Trump gains and Clinton stays the same (T1C0) more frequently than Clinton gains and Trump stays the same (T0C1). Coefficient estimates (not shown) suggest that, other things equal, with voting on paper, ballot marking or DRE technology (all possible with Vendor "None") and with a manual recount ("Hand"), T0C1 and T1C0 both occur less frequently than does T0C0. If we assume that the manually recounted results are correct, then these results suggest that votes on paper or via ballot marking or DRE technology tended to undercount Trump's votes more often than Clinton's votes.[18]

*** Table 15 about here ***

When the Accessible Technology variable is added as a regressor in Vendor models, however, with false discovery rate correction for multiple testing none of the coefficient difference test results are significant (Table 16). This means that the significant test result observed in Table 15 is not meaningfully relevant for understanding the effects of voting technology on votes.

*** Table 16 about here ***

4 Michigan

Table 17 shows the frequency distribution of types of voting technology across all Michigan precincts for which the total number of recounted votes across all presidential candidates is positive ($n = 3,051$). Each city or township has its own technology: Figure 4 shows how the technologies are distributed across cities and townships. All types of voting technology occur with sufficient frequency to support informative statistical analysis.

*** Table 17 about here ***

Table 18 shows the frequencies of the paired-signs-of-differences variables. Unlabeled change frequencies appear at the top of the table, labeled changes at the bottom. All categories occur sufficiently frequently to support the multinomial regression model analysis.

*** Table 18 about here ***

[18]Even if we treated these estimates as credible, which we don't in light of what happens when Accessible Technology is introduced as a regressor, for reasons stated above we don't think one can say much specifically about DRE technologies. We'd need either voter testimony or observations of recounts of DRE VVPATs changing counts.

Regression analysis reported in Table 19 shows that several variables relate to Voting Technology when it is used as the outcome variable in a precinct-level multinomial regression analysis. These variables are Clinton (HRC) vote proportion,[19] turnout,[20] active voter proportion[21] and county vote population.[22]

*** Table 19 about here ***

As reported in Table 20, likelihood-ratio tests reject the hypothesis that using the labeled categories (S_0) is not significantly better than using the unlabeled categories (S_1). The hypothesis is rejected whether or not covariates in addition to the Voting Technology variables are included as regressors.

*** Table 20 about here ***

Whether or not the additional covariates are included as regressors in the model, none of the statistics significantly rejects the hypothesis of no difference between coefficients. We use false discovery rate adjustment for multiple testing, considering all 18 tests in the bottom of Table 21 as simultaneous independent tests.

*** Table 21 about here ***

None of the voting technologies show signs of treating the candidates asymmetrically. The subset of absentee precincts does not differ significantly from the collection of all precincts. In Michigan the pattern of losses or gains due to the recount appears to be unrelated to the voting technology used to cast and record the votes.

[19]HRC vote proportion is computed using recounted vote counts in Bureau of Elections (2017*b*).

[20]Turnout is the ratio of the precinct total of votes cast for president in the recount data (from Bureau of Elections (2017*b*).) over the total number of registered voters in the town the precinct is in (from Bureau of Elections (2017*a*)).

[21]The active voter proportion is the ratio of `ActiveVoters` over `RegisteredVoters`, both town-level variables from Bureau of Elections (2017*a*)

[22]For numerical stability in the multinomial regression estimation software, the total of the recounted votes in each county is divided by the state total of the recounted votes.

5 Conclusion

In neither Wisconsin nor Michigan do we find evidence that vote tabulation systems are associated with distortion in votes for either Trump or Clinton. Our analysis addresses only Wisconsin wards and Michigan precincts for which recounts occurred and for which we have data from the officially produced data files. While the recount in Wisconsin covered the whole state, the recount in Michigan did not. We have nothing to say about Michigan precincts that were not recounted, apart from noting that severe problems have been noted to have occurred in Detroit (Johnson 2017*b*).

Likewise one of our key assumptions is that hand recounted ballots that were originally cast manually on paper provide "true" tabulations, and in Wisconsin about half of the votes were recounted by machine. Even though we find no significant differences between Hand and Machine recounted ballots in Wisconsin in our coefficient-difference tests, if the same machines were used to recount as to originally tabulate votes, and these machines were corrupted, then the recount data provides no veneration of those results.

Our analysis is limited in that we have avoided studying the magnitudes of losses or gains in the candidates' votes counts. It may be that an analysis that used the recount structure to analyze the exact vote counts would reach different conclusions.

For both states, however, and especially for Wisconsin, we think the prospects are not good for using the kinds of data we have assembled to produce more exact statistical estimates—using the exact vote counts—of the effects voting technologies (and recount methodologies) may have had. In Wisconsin the profound problem is that we cannot be sure which technology was used to produce the record of each vote, and cases of machine recounting do not meet sufficiently rigorous standards to establish the correct outcome. In Michigan a problem is that someone decided whether a recount was done, those decisions were based on vastly more information than we have as analysts, and there is no reason to believe these decisions are unrelated to features associated with both voting technologies and potential distortions in votes. In fact such a self-selection concern affects all the data we have, given that someone chose which voting technologies to implement in each jurisdiction and then someone chose which modality to use to cast, count and record each vote: self-selections qualify as well the analysis we have reported here.

The best way to get evidence about whether the vote counts are correct is to perform either a risk-limiting audit (Lindeman and Stark 2012) or a full manual retabulation. Neither is likely to occur in either state. Such evidence about the accuracy of the vote counts would still leave the problem of determining whether voting technologies—or something else—distorted votes.

A significantly stronger way to rule out hacking would be forensic analysis of the machines that tabulated the votes themselves. Closely examining the code that is running during the election can greatly bolster confidence in the output of voting machines. This is also unlikely to happen in either state: even if one could get access to the code that should have been running during the election, there is no way to know whether that was the code the machines were then executing. While sophisticated malicious code on the machines might disguise itself well enough to evade detection, the combination of audits and electronic forensics would provide a significant challenge to any would-be attacker. Until more light is shed on what actually goes on during American elections, we cannot have the utmost confidence that democracy is being carried out to its fullest extent.

Nonetheless it is no longer appropriate to say that there is no evidence regarding possible hacking of the voting technology in the 2016 election. Our analysis provides evidence that the voting technology did not distort the votes in Wisconsin or Michigan: the voting technologies all treated votes for Trump or Clinton the same way; how a vote was treated appears not to have depended on which candidate the vote was for. Presumably a hack intended to benefit or harm one candidate more than the other would cause asymmetric treatment. We have evidence—albeit weak evidence—that that didn't happen. The idea has not been entirely ruled out, but it is now less credible, much moreso than had recounts never occurred.

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Table 1: Trump: recounted votes minus original votes, Wisconsion

	-25	-18	-16	-11	-10	-9	-7	-6	-5	-4	-3	-2	-1	0
Hand	1	1	0	0	1	1	2	2	5	9	15	43	167	1457
Machine	0	0	1	1	0	0	2	1	2	4	9	18	58	810
Mixed	0	0	0	0	0	0	0	0	0	2	3	3	21	199

	1	2	3	4	5	6	7	8	9	10	11	14	23	29
Hand	199	57	39	11	7	4	3	2	1	1	1	2	1	2
Machine	100	27	7	7	3	2	2	0	1	2	0	1	0	0
Mixed	31	8	3	1	2	0	0	1	0	0	0	0	0	0

	31	32	39	50	65	246
Hand	0	1	1	1	1	0
Machine	1	0	0	0	0	1
Mixed	0	0	0	0	0	0

Note: number of precincts that have each displayed value for the difference between the recounted vote total and the original vote total for Trump in a precinct. Recounted and original vote counts from Wisconsin Elections Commission (2017*b*). Recount methods gleaned from Wisconsin Elections Commission (2017*c*) and from county minutes at Wisconsin Elections Commission (2017*e*).

Table 2: Clinton: recounted votes minus original votes, Wisconsion

	-30	-18	-17	-14	-12	-10	-8	-7	-6	-5	-4	-3	-2	-1
Hand	1	0	1	0	1	0	0	1	0	5	6	17	52	161
Machine	0	1	0	1	0	1	2	2	1	4	6	8	15	82
Mixed	0	0	0	0	0	0	0	0	1	0	1	4	6	25

	0	1	2	3	4	5	6	7	8	9	10	11	13	14
Hand	1457	187	79	22	10	9	5	8	4	0	2	1	1	1
Machine	734	126	31	18	6	4	6	5	2	1	0	0	0	1
Mixed	199	23	6	1	3	3	1	0	0	1	0	0	0	0

	15	17	19	22	24	33	68	79
Hand	1	1	1	1	1	0	1	1
Machine	0	1	1	0	0	1	0	0
Mixed	0	0	0	0	0	0	0	0

Note: number of precincts that have each displayed value for the difference between the recounted vote total and the original vote total for Clinton in a precinct. Recounted and original vote counts from Wisconsin Elections Commission (2017*b*). Recount methods gleaned from Wisconsin Elections Commission (2017*c*) and from county minutes at Wisconsin Elections Commission (2017*e*).

Table 3: Trump: recounted votes minus original votes, Michigan

	-209	-25	-19	-10	-8	-7	-6	-5	-4	-3	-2	-1	0	1
PCT	0	1	2	1	1	1	2	1	4	12	25	119	1306	370
AV	1	0	0	1	0	0	0	0	0	2	10	45	810	123

	2	3	4	5	6	7	8	10	11	15	16	24	26
PCT	111	34	11	4	2	2	0	1	1	1	1	1	1
AV	29	8	2	0	2	0	1	2	0	0	0	0	0

Note: number of precincts that have each displayed value for the difference between the recounted vote total and the original vote total for Trump in a precinct. Precinct types and recounted and original vote counts from Bureau of Elections (2017*b*).

Table 4: Clinton: recounted votes minus original votes, Michigan

	-287	-41	-29	-24	-20	-8	-7	-6	-5	-4	-3	-2	-1	0
PCT	0	1	1	1	1	0	1	1	4	2	8	35	139	1182
AV	1	0	0	0	0	1	0	1	1	4	6	13	78	757

	1	2	3	4	5	6	7	10	16	20	23	25	26
PCT	418	121	58	23	6	5	1	2	1	1	1	1	1
AV	119	41	9	0	3	0	2	0	0	0	0	0	0

Note: number of precincts that have each displayed value for the difference between the recounted vote total and the original vote total for Clinton in a precinct. Precinct types and recounted and original vote counts from Bureau of Elections (2017*b*).

Table 5: Recounted Votes Minus Original Votes, Mean by Reason, Wisconsin

Reason	N^a	Trump	Clinton
Ballots rejected during recount	316	$-.199$	.0158
Ballots found during recount	72	1.38	3.38
Nonstandard pens or ballots	4	13.8	16.9
Ballots marked incorrectly	296	.993	1.17
Lost ballots	23	-1.43	-1.17
Human counting error	37	.0213	-1.23
Paper jam	21	$-.870$	$-.696$
Ballots wrongfully rejected	73	1.09	1.82
Voting machine error	13	7.56	7.83
No explanation	759	.680	.389

Note: mean of nonzero differences between the recounted and original vote count in Wisconsin wards. [a] Number of occurrences of each reason. Multiple reasons are cited for some wards.

Table 6: Categories That Should Have the Same Coefficients

S_0 category	S_1 category
00	T0C0
-1-1	T-1C-1
11	T1C1
-10	T0C-1, T-1C0
01	T0C1, T1C0
-11	T-1C1, T1C-1

Table 7: Wisconsin Ward Voting Technologies and Recount Methods

Voting Technology		Recount Method	
None	850	Hand	2126
Accuvote-OS	154	Machine	1066
DS200	1475	Mixed	286
Eagle	294	other	22
Eagle; Insight	4		
ImageCast Evolution	287		
Insight	229		
M100	205		

Note: number of wards using each type of Voting Technology or recount method. Voting technology taken from Wisconsin Elections Commission (2016). Recount methods gleaned from Wisconsin Elections Commission (2017*c*) and from county minutes at Wisconsin Elections Commission (2017*e*).

Table 8: Wisconsin Ward Voting and Accessibility Technologies

	Accessibility Technology			
Voting Technology	Accuvote TSX	Automark	Edge	Edge; Automark
None	1	64	727	0
Accuvote-OS	120	0	34	0
DS200	0	1141	0	0
Eagle	0	8	286	0
Eagle; Insight	0	0	4	0
ImageCast Evolution	0	0	0	0
Insight	0	0	229	0
M100	0	183	1	1

	Accessibility Technology				
Voting Technology	ExpressVote	ImageCast Evolution	Populex 2.3	Vote Pad	iVotronic
None	0	0	2	9	47
Accuvote-OS	0	0	0	0	0
DS200	333	0	0	0	1
Eagle	0	0	0	0	0
Eagle; Insight	0	0	0	0	0
ImageCast Evolution	0	287	0	0	0
Insight	0	0	0	0	0
M100	0	0	0	0	20

Note: number of wards using each type of Voting Technology and Accessibility Technology by Vendor. Technologies taken from Wisconsin Elections Commission (2016).

Table 9: Wisconsin Ward Voting Technologies by DRE Votes

Voting Technology	Some DRE Votes? No	Yes
None	83	765
Accuvote-OS	119	35
DS200	1458	16
Eagle	87	205
Eagle; Insight	4	0
ImageCast Evolution	282	5
Insight	21	208
M100	186	19

Note: number of wards using each type of Voting Technology with or without any DRE votes. DRE vote counts come from Wisconsin Elections Commission (2017*a*).

Table 10: Recounted-minus-original Changes Frequencies, Wisconsin

Unlabeled Changes

same-same	loss-loss	gain-gain	loss-same	gain-same	loss-gain
2073	113	200	394	553	167

Labeled Changes: Trump first, Clinton second

same-same	loss-loss	gain-gain	same-loss	loss-same	same-gain	gain-same	loss-gain	gain-loss
2073	113	200	213	181	300	253	83	84

Table 11: Tests for Unlabeled Versus Labeled Categories, Wisconsin

covariates	LR statistic	df	p-value
Voting Technology and Recount Method	1460.2	33	0
Voting Technology and Recount Method and covariates	1340.1	51	0

Note: the LR statistic is the difference in model deviances. The p-value is the upper-tail probability for a χ^2_{df} distribution.

Table 12: Coefficient Difference Test Statistics, Wisconsin

Model Excluding Additional Regressors

Variable	Category	T0C-1 − T-1C0	T0C1 − T1C0	T-1C1 − T1C-1
	None-Hand[a]	−1.5	−2.3	−.3
Voting Technology	Accuvote-OS	−.6	.2	−1.3
	DS200	1.9	3.9	1.3
	Eagle	.9	1.1	.7
	Eagle; Insight	−1e-05	−.003	7e-06
	ImageCast Evolution	1.7	.7	.04
	Insight	−1.4	.6	−.9
	M100	2.2	1.7	−.1
Recount Method	Machine	1.6	1.1	−.3
	Mixed	1.5	−.7	−.2

Model Including Additional Regressors[b]

Variable	Category	T0C-1 − T-1C0	T0C1 − T1C0	T-1C1 − T1C-1
	None-Hand[a]	−1.7	−3.0	−.8
Voting Technology	Accuvote-OS	−.5	.2	−1.7
	DS200	−.2	.9	−1.2
	Eagle	−.004	−.2	−.4
	Eagle; Insight	−2e-05	−.02	−.0005
	ImageCast Evolution	.8	.2	−1.3
	Insight	−1.6	.3	−1.4
	M100	1.1	.8	−1.1
Recount Method	Machine	1.6	.7	−.9
	Mixed	1.7	−.05	−.1

Note: t-statistics for differences between pairs of coefficients. Reference categories in the multinomial regression models are `T0C0`, “None” Voting Technology and “Hand” Recount Method. [a] “None-Hand” is the Intercept term. [b] Additional regressors are HRC proportion, registered voter ratio, DRE proportion, Absentee proportion, turnout, reasons and county total votes.

Table 13: Coefficient Difference Test Statistics, Wisconsin

Model Excluding Additional Regressors

Variable	Category	T0C-1 − T-1C0	T0C1 − T1C0	T-1C1 − T1C-1
	None-Automark-Hand[a]	1.1	−.9	.8
Voting Technology	Accuvote-OS	−1.5	−.7	−.2
	DS200	−.3	1.8	.03
	Eagle	.9	.9	.6
	Eagle; Insight	−2e-05	−.005	9e-06
	ImageCast Evolution	.1	−.007	.003
	Insight	−1.3	.4	−.9
	M100	.3	1.1	−.7
Accessibility Technology	Accuvote TSX	.8	.8	−.4
	Edge	−1.6	.07	−.9
	ExpressVote	−2.3	−2.4	−1.4
	iVotronic	−2.1	−1.0	−.1
	other	−.1	.01	−.01
Recount Method	Machine	1.6	1.2	−.3
	Mixed	1.6	−.5	−.5

Model Including Additional Regressors[b]

Variable	Category	T0C-1 − T-1C0	T0C1 − T1C0	T-1C1 − T1C-1
	None-Automark-Hand[a]	−.6	−2.8	−.3
Voting Technology	Accuvote-OS	−1.3	−1.0	−.5
	DS200	−1.0	.7	−1.0
	Eagle	.5	−.4	−.5
	Eagle; Insight	4e-05	−.004	−.0002
	ImageCast Evolution	.02	−.0005	−.0009
	Insight	−1.1	−.008	−1.4
	M100	−.009	.9	−1.0
Accessibility Technology	Accuvote TSX	.9	1.3	−.4
	Edge	−1.5	.3	.07
	ExpressVote	−1.3	.3	−1.1
	iVotronic	−1.8	−.4	−.05
	other	−.02	.0008	−.0003
Recount Method	Machine	1.9	.8	−.7
	Mixed	1.9	.5	−.5

Note: t-statistics for differences between pairs of coefficients. Reference categories in the multinomial regression models are `T0C0`, "None" Voting Technology, "Automark" Accessibility Technology and "Hand" Recount Method. [a] "None-Automark-Hand" is the Intercept term. [b] Additional regressors are HRC proportion, registered voter ratio, DRE proportion, Absentee proportion, turnout, reasons and county total votes.

Table 14: Wisconsin Ward Voting Technologies by Vendor

	Vendor				
Voting Technology	None	Command Central	Dominion	ES&S	Optech
None	850	0	0	0	0
Accuvote-OS	0	33	121	0	0
DS200	0	0	0	1475	0
Eagle	0	281	0	0	13
Eagle; Insight	0	4	0	0	0
ImageCast Evolution	0	0	287	0	0
Insight	0	218	11	0	0
M100	0	0	0	205	0

	Vendor				
Accessibility Technology	None	Command Central	Dominion	ES&S	Optech
Accuvote TSX	1	0	120	0	0
Automark	64	2	0	1324	6
Edge	727	534	12	1	7
Edge; Automark	0	0	0	1	0
ExpressVote	0	0	0	333	0
ImageCast Evolution	0	0	287	0	0
Populex 2.3	2	0	0	0	0
Vote Pad	9	0	0	0	0
iVotronic	47	0	0	21	0

Note: number of wards using each type of Voting Technology or Accessibility Technology by Vendor. Technologies and Vendors taken from Wisconsin Elections Commission (2016).

Table 15: Coefficient Difference Test Statistics, Wisconsin

Model Excluding Additional Regressors

Variable	Category	T0C-1 − T-1C0	T0C1 − T1C0	T-1C1 − T1C-1
	None-Hand[a]	−1.5	−2.3	−.3
Vendor	Command Central	−.4	.8	−.2
	Dominion	1.2	.7	−.5
	ES&S	2.4	3.8	1.0
	Optech	.0004	−.05	.001
Recount Method	Machine	1.3	1.2	−.2
	Mixed	1.2	−.6	−.5

Model Including Additional Regressors[b]

Variable	Category	T0C-1 − T-1C0	T0C1 − T1C0	T-1C1 − T1C-1
	None-Hand[a]	−1.8	−3.1	−.7
Vendor	Command Central	−1.1	−.03	−1.1
	Dominion	.6	.5	−1.8
	ES&S	.2	1.1	−1.5
	Optech	−.001	−.5	−.002
Recount Method	Machine	1.7	.9	−1.0
	Mixed	1.5	.3	−.5

Note: t-statistics for differences between pairs of coefficients. Reference categories in the multinomial regression models are `T0C0`, "None" Vendor and "Hand" Recount Method. [a] "None-Hand" is the Intercept term. [b] Additional regressors are HRC proportion, registered voter ratio, DRE proportion, Absentee proportion, turnout, reasons and county total votes.

Table 16: Coefficient Difference Test Statistics, Wisconsin

Model Excluding Additional Regressors

Variable	Category	T0C-1 − T-1C0	T0C1 − T1C0	T-1C1 − T1C-1
	None-Automark-Hand[a]	1.2	−.6	1.2
Vendor	Command Central	−.3	.7	−.1
	Dominion	.8	−.4	−.03
	ES&S	−.2	1.5	−.6
	Optech	−.002	−.1	−.006
Accessibility Technology	Accuvote TSX	−1.4	.4	−1.1
	Edge	−1.7	−.2	−1.2
	ExpressVote	−2.6	−2.2	−1.2
	iVotronic	−2.1	−1.3	−.02
	other	−1.1	.4	−.5
Recount Method	Machine	1.4	1.3	−.3
	Mixed	1.1	−.7	−.6

Model Including Additional Regressors[b]

Variable	Category	T0C-1 − T-1C0	T0C1 − T1C0	T-1C1 − T1C-1
	None-Automark-Hand[a]	−.7	−2.7	−.2
Vendor	Command Central	−.3	−.03	−1.1
	Dominion	.6	−.4	−.2
	ES&S	−.7	.8	−1.3
	Optech	5e0-6	−.4	−.0008
Accessibility Technology	Accuvote TSX	−1.0	.6	−.9
	Edge	−1.6	.05	−.1
	ExpressVote	−1.7	.3	−1.1
	iVotronic	−1.7	−.4	−.04
	other	−.9	.5	−.4
Recount Method	Machine	2.0	1.0	−.9
	Mixed	1.4	.5	−.7

Note: t-statistics for differences between pairs of coefficients. Reference categories in the multinomial regression models are `T0C0`, "None" Vendor and "Hand" Recount Method. [a] "None-Automark-Hand" is the Intercept term. [b] Additional regressors are HRC proportion, registered voter ratio, DRE proportion, Absentee proportion, turnout, reasons and county total votes.

Table 17: Michigan Precinct Voting Technologies

	Precincts			
	All		Recounted	
Technology	PCT	AV	PCT	AV
ES&S M100	2490	2021	1362	768
Premier Accuvote	579	492	348	132
Sequoia Optech Insight	323	151	298	126

Note: number of precincts using each type of Voting Technology or recount method. "PCT" denotes in-person precincts and "AV" denotes absentee precincts. Voting technology taken from Bureau of Elections (2017*a*). Precinct type and recounted status from Bureau of Elections (2017*b*).

Table 18: Recounted-minus-original Changes Frequencies, Michigan

Unlabeled Changes

same-same	loss-loss	gain-gain	loss-same	gain-same	loss-gain
1552	22	203	160	791	323

Labeled Changes: Trump first, Clinton second

same-same	loss-loss	gain-gain	same-loss	loss-same	same-gain	gain-same	loss-gain	gain-loss
1552	22	203	100	60	464	327	146	177

Table 19: Equipment Multinomial Regression, Michigan

Variable	ES&S M100	Sequoia Optech Insight
Intercept	-12.0	6.98
	(1.81)	(1.85)
AV	.292	.451
	(.136)	(.164)
HRC proporition	$-.247$	$-.245$
	(.408)	(.460)
active proportion	11.5	-9.12
	(1.876)	(1.93)
turnout	1.80	3.16
	(.426)	(.435)
county vote	47.4	11.5
	(2.73)	(3.23)

Note: multinomial regression model coefficient estimates, standard errors in parentheses. Reference category is Premier Accuvote. $n = 3,051$. Residual deviance: 3525.212.

Table 20: Tests for Unlabeled Versus Labeled Categories, Michigan

covariates	LR statistic	df	p-value
Voting Technology	1714.7	18	0
Voting Technology and covariates	1553.4	30	0

Note: the LR statistic is the difference in model deviances. The p-value is the upper-tail probability for a χ^2_{df} distribution.

Table 21: Coefficient Difference Test Statistics, Michigan

Model Excluding Additional Regressors

Variable	Category	T0C-1 − T-1C0	T0C1 − T1C0	T-1C1 − T1C-1
Voting Technology	Premier Accuvote[a]	.6	1.7	−2.1
	Sequoia Optech Insight	−.2	.4	1.8
	ES&S M100	.4	−.4	1.9
	Accuvote (AV)	.4	.2	−1.0
	Insight (AV)	1.0	.3	.5
	M100 (AV)	−.2	.8	−.2

Model Including Additional Regressors[b]

Variable	Category	T0C-1 − T-1C0	T0C1 − T1C0	T-1C1 − T1C-1
Voting Technology	Premier Accuvote[a]	1.8	2.1	−.9
	Sequoia Optech Insight	.02	1.5	1.9
	ES&S M100	.7	.3	2.0
	Accuvote (AV)	1.7	1.8	−.9
	Insight (AV)	.9	.6	.5
	M100 (AV)	−.4	.09	−.4

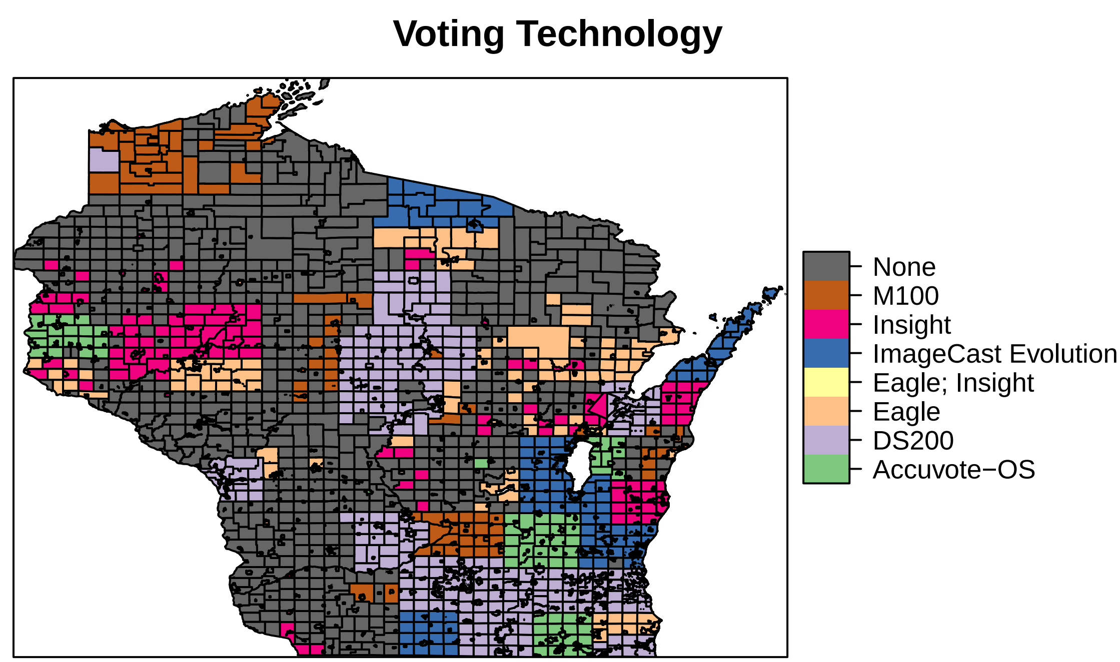
Note: t-statistics for differences between pairs of coefficients. (AV) denotes tests for AV (absentee) observations. Reference categories in the multinomial regression models are `T0C0`, Premier Accuvote Voting Technology.

[a] "Premier Accuvote" is the Intercept term.

[b] Additional regressors are HRC proportion, turnout, active voter proportion and county vote population.

Figure 1: Some of the machines used in Wisconsin and Michigan elections

Figure 2: Wisconsin Technologies by Municipality



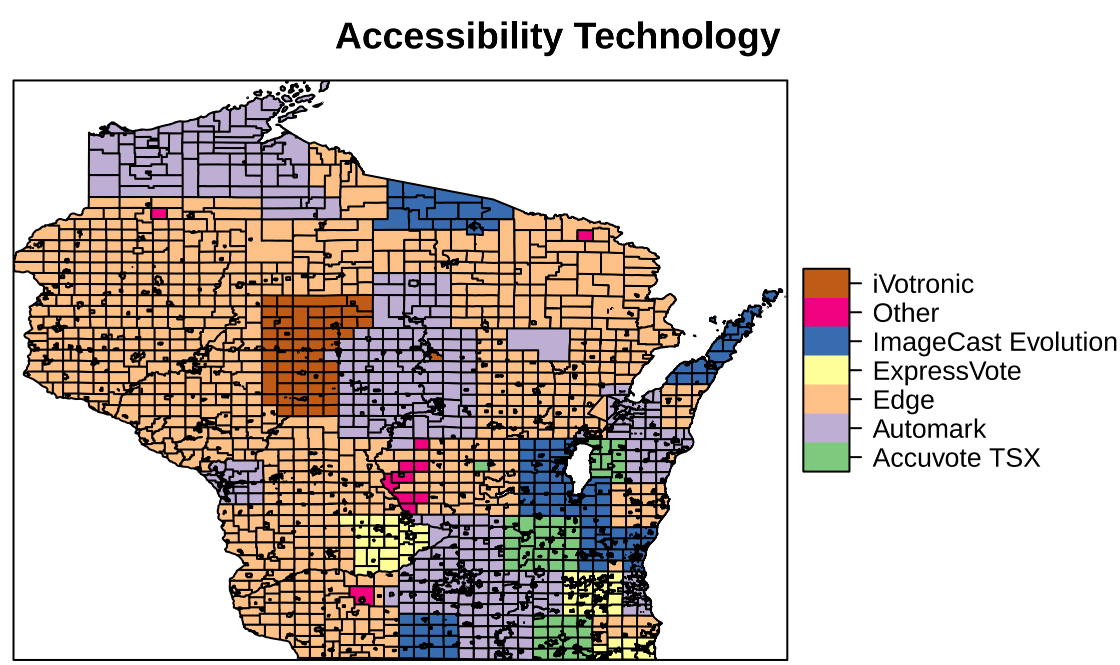
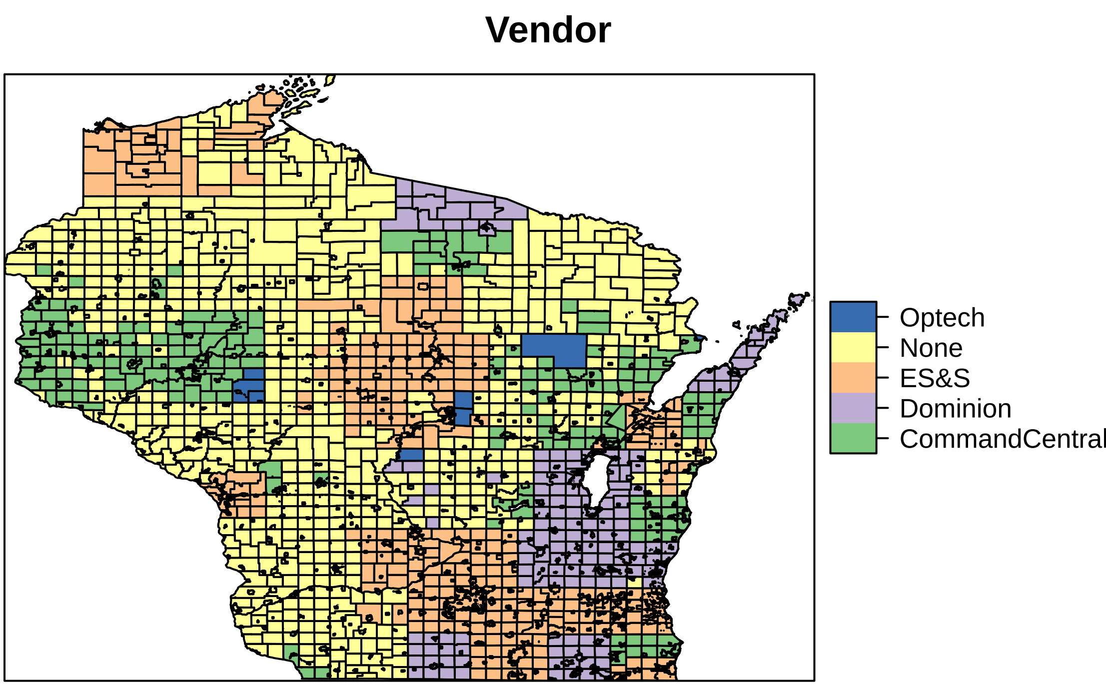


Figure 3: Wisconsin Vendors by Municipality



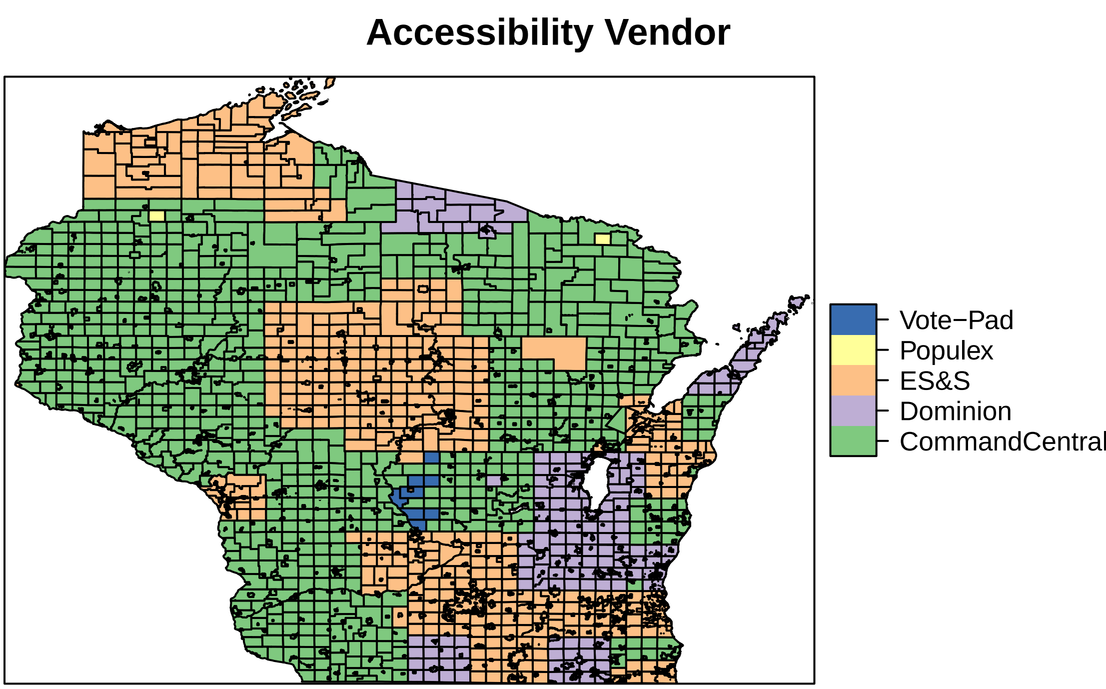


Figure 4: Michigan Technologies by City and Township