Election Forensics: Statistics, Recounts and Fraud[*]

Walter R. Mebane, Jr.[†]

April 10, 2007

Abstract

I use data from the 2006 federal election in Mexico to compare the impression conveyed by tests based on the second digits of reported vote counts to the impression conveyed by a manual recount done for a nonrandom sample of the ballots cast for president. The patterns identified by the 2BL tests match classical ideas about how local political machines operate: in municipalities whose mayors are affiliated with two of the three major party coalitions, the party's candidates do better in voting for president, senate and deputy than expected according to a natural voting baseline. For the three parties that were not competitive in the presidential election, the second-digit tests strongly suggest vote counts were affected either by massive intimidation or by widespread strategic voting. The manual recount did not detect any such patterns, and the changes produced by the recount are unrelated to the vote counts' second digits. Second-digit tests can detect election phenomena that have nothing to do with tabulation errors.

[†]Professor, Department of Government, Cornell University. 217 White Hall, Ithaca, NY 14853–7901 (Phone: 607-255-2868; Fax: 607-255-4530; E-mail: wrm1@cornell.edu).

Introduction

Manual recounts are frequently proposed as a method for validating vote tabulations. In the United States, the idea that a small random sample of ballots should always be recounted manually has recently become increasingly popular. A recent report finds that "currently, more than a dozen states require post-election random manual audits" (electionline.org 2007). Legislation currently pending before the U.S. House of Representatives proposes rules to mandate such random recounts throughout the country (H.R. 811, the "Voter Confidence and Increased Accessibility Act of 2007"). These proposals are being considered not so much with the idea that manual recounts will detect routine tabulation errors but rather with the idea that a system of random recounts will deter election fraud. For instance, one argument for sampling precincts with probabilities proportional to the number of votes cast (or perhaps the number of registered voters) motivates such a sampling procedure as an optimal strategy given a particular type of rational adversary (Rivest 2007).

Clearly a regime of mandatory recounts cannot provide complete protection against all kinds of election fraud. There are many ways to (literally or figuratively) stuff ballot boxes, cause votes selectively to disappear or intimidate or mislead voters that will not produce tabulation errors. Such methods have an unfortunately rich and detailed history not only in the United States but in many places where elections have been conducted (e.g. Harris 1934; Gumbel 2005; Lehoucq 2003; Bjornlund 2004). In many cases election observers can document widespread administrative irregularities, violence or vote buying based on eyewitness accounts. But eyewitness accounts may be disputed or deemed not credible, and in any case eyewitness testimony may not provide a sufficient basis for determining either the prevalence of irregularities or how much the reported vote counts have been affected.

Statistical assessments based on the reported vote counts themselves may usefully supplement manual recounts, direct observation and other methods for trying to detect irregularities and deter fraud. I use the name *election forensics* to designate such statistical techniques (Mebane 2006a,b,c, 2007). Techniques I have particularly been involved in developing include methods for

robust estimation and outlier detection (Wand, Shotts, Sekhon, Mebane, Herron, and Brady 2001; Mebane and Sekhon 2004), for testing whether votes are randomly distributed among voting machines (Mebane 2006a) and for testing whether the second digits of vote counts follow a distribution specified by Benford's law (the 2BL test) (Mebane 2006b). Others have emphasized methods focusing on geographic differences in residual vote rates (Brady and Hui 2006) or on methods for analyzing the relationship between turnout and candidate vote shares (Myagkov, Ordeshook, and Shaikin forthcoming). Election forensics may also include simply mucking around in the available data using whatever statistical assessments or comparisons seem likely to help understanding how the vote counts were generated (e.g. Mebane and Dill 2007).

In this paper I use data from the 2006 federal election in Mexico to compare the impression conveyed by tests based on the second digits of reported vote counts to the impression conveyed by a manual recount done for a nonrandom sample of the ballots cast for president. The 2006 Mexican presidential election was close and highly controversial (European Union 2006). Five party coalitions sought votes in the election, and the winning candidate's margin of victory was 0.56 percent. The winning candidate was Felipe Calderón of the *Partido Acción Nacional* (PAN), and the candidate receiving the second largest number of votes was Andrés Manuel López Obrador of the *Coalición por el Bien de Todos* (PBT).

PAN and PBT filed hundreds of challenges that alleged election day irregularities (European Union 2006, 42–43). As part of its response to the challenges, the election court ordered a manual recount of the ballots in 11,839 of the election's 130,788 polling stations or *casillas* (European Union 2006, 46). The recount included casillas from 26 of Mexico's 32 states. The selected casillas were not a random sample but were casillas about which complaints had specifically been filed. The recount produced reductions in the vote counts for all the parties, with larger reductions for PAN than for PBT, but the reductions did not change the election outcome (European Union 2006, 47). European Union observers concluded that, based on the areas where they were able to observe the recount, "discrepancies found during the re-count process were the result of unintentional human error, did not reveal any pattern that might raise suspicions of fraud, and

were probably a consequence of the long hours of work during election day, pressure of time and, in some cases, the low educational level of the polling station members in more remote areas" (European Union 2006, 3). The recount results did not, however, persuade the principal losing candidate that he had lost fairly.

In particular in this paper I ask whether there are distortions in the vote counts that are associated with the party identity of the mayor (*Presidente Municipal*) of each municipality. The official spreadsheet files that report the vote counts for each casilla locate each casilla in one of 2,422 municipalities (*municipios*). Each municipality may contain several towns and villages, but for the municipality itself there is a government and each such government has an elected mayor. Except in a number of municipalities in the state of Oaxaca, each mayor is affiliated with either a single party or a coalition of parties.[1]

While the mayor has no official role in administering the federal election, the mayor's party coalition likely corresponds to the locally dominant partisan organization. There are many ways such organizational capacity may produce distortions in the electoral results. For instance, the European Union reports that at some casillas "some polling station staff members did not turn up, and had to be replaced by substitutes or voters in line" (European Union 2006, 37). PBT activists suggested that in some cases replacements were not haphazard but instead were planned to make sure the polling staff were controlled by one party's supporters. For instance, email I received from an activist (Miguel Kelley) on September 6, 2006, described circumstances in the state of Guanajuato as follows (Kelley 2006):

```
I took an election observer from Global Exchange to
Leon, Guanajuato, where we interviewed (one by one) a
number of teachers who indicated how the teacher's
union (whose leaders formed the "Nueva Alianza" party)
forced them to join the party and support the party's
efforts.  In many communities, the local teacher is
traditionally the first in line to vote – and the
```

---

[1] According to data from the *Sistema Nacional de Información Municipal* (SNIM) (Instituto Nacional para el Federalismo y el Desarrollo Municipal 2006), at the time of the 2006 federal election 421 of the 570 municipalities in Oaxaca had mayors selected via an indigenous method called *uso y costumbre* that does not involve affiliation with a political party. For one discussion of this electoral method, see Eisenstadt (2007).

> election rules state the, when a poll worker doesn't
> show up, the first person in line is selected to become
> a poll worker. If the missing worker is the "presidente
> de casilla", the first in line joins the "mesa" and
> then the group elects a "presidente". And the teacher,
> generally the most educated and/or respected person in
> the "mesa", becomes the "presidente de casilla". To top
> things off, the teacher's union paid many "presidentes
> de casilla" NOT to show up – so that their substitution
> could take place.
>
> Elba Esther Gordillo, for years the president of the
> SNTE (teacher's union), has formed a strong political
> team within the union. The union receives substantial
> federal funding, is the largest and most powerful union
> in Mexico, and has negociated many privileges. One is
> that 10% of the teachers are always free from teaching
> duties – the time is presumably to be used as a
> sabbatical. That has allowed the SNTE to form their
> political team. And the SNTE has often been responsible
> for training the poll workers. So they have the people,
> the time and the intimate knowledge of the elections
> process. It is fair to say that in this election they
> generated nearly 4M votes for Calderon. Their
> instructions to the membership were simple – vote for
> Nueva Alianza's candidate for senator and "diputado",
> but vote for Calderon for President.

Kelley's message describes not only alleged manipulation of the polling station staff, with the implicit suggestion that the staff could then do something to change the way the votes were counted, but also coercion directed at a particular set of voters. Note that the *Nueva Alianza* (NA) party was formed as a splinter from the longtime ruling party, the *Partido Revolucionario Institucional* (PRI). Both NA and PRI fielded candidates in the 2006 presidential election. PRI formed a coalition called *Alianza por México* (APM) with the *Partido Verde Ecologista de México*. Coercion to support NA might be expected to affect the vote counts recorded for APM.

If a party's having a presence at the casilla was an advantage for that party, then the fifth party presenting candidates in the 2006 election, *Alternativa Socialdemócrata y Campesina* (ASDC), is likely to have been especially vulnerable. ASDC had representatives among the polling station

staff (the *mesa*) only in 5.7 percent of the casillas, and ASDC had party members officially observing the election day proceedings only in 3.2 percent of the casillas (Instituto Federal Electoral 2006, Cuadros 1–2).

2BL Test Statistics

I use three kinds of statistics to implement tests of whether vote counts reported for the 2006 Mexican federal election have the 2BL distribution. One is the Pearson chi-squared statistic used in Mebane (2006b). To define this statistic, let q_j denote the expected relative frequency, according to Benford's law, with which the second significant digit is j.[2] Let n_j be the number of times the second digit is j among the vote counts being considered, and define $N = \sum_{j=0}^{9} n_j$. The statistic for a 2BL test is

$$X_{2BL}^2 = \sum_{j=0}^{9} \frac{(n_j - Nq_j)^2}{Nq_j} \, .$$

This statistic may be compared to the χ^2-distribution with 9 degrees of freedom (χ_9^2).

The second kind of test supports asking whether the distribution of second digits differs from the nominal 2BL distribution in a way that depends on observed conditioning factors. When the conditioning variables are discrete, it is straightforward to partition the vote counts into sets based on the variables and then compute X_{2BL}^2 separately for each set. Indeed, Mebane (2006b) does this when computing the 2BL test statistic separately for different candidates or different geographic jurisdictions, using the false discovery rate (FDR) method (Benjamini and Hochberg 1995) to handle the fact that many hypotheses are being considered.

But when a conditioning variable is not discrete, the partitioning approach is not feasible. In this case I use a ten-category multinomial logit model, with one category for each of the ten digits $j \in \{0, \ldots, 9\}$. For observation i the covariates are $x_i = (1, x_{1i}, \ldots, x_{Ki})'$, and for coefficients

---

[2]The expected frequencies are (rounded) $(q_0, \ldots, q_9) = (.120, .114, .109, .104, .100, .097, .093, .090, .088, .085)$.

$b_{.j} = (b_{0j}, b_{1j}, \ldots, b_{Kj})'$ we have linear predictors

$$
z_{ji} = \begin{cases} x_i'b_{.j}, & j = 0, \ldots, 8, \\ 0, & j = 9, \end{cases}
$$

and the probability that the second digit in vote count i is j is

$$
p_{ji} = \exp(z_{ji}) / \sum_{j=0}^{9} \exp(z_{ji}) .
$$

If the distribution of the second digits does not depend on the conditioning variables then we should have $b_{kj} = 0$ for $k \in \{1, \ldots, K\}$, and if the second digits are 2BL-distributed we should observe $b_{0j} = \log(q_j/q_9)$. I use maximum likelihood to estimate the model. Using $\hat{V}_k$ to denote the subset of the estimated variance-covariance matrix of the parameter estimates that corresponds to the parameters $b_{k.} = (b_{k0}, \ldots, b_{k8})'$, $k \in \{0, \ldots, K\}$, a Wald test statistic for the hypothesis $b_{kj} = 0$ for all $j \in \{0, \ldots, 8\}$ is

$$
W_k = b_{k.}' \hat{V}_k^{-1} b_{k.} .
$$

A test statistic for the hypothesis $b_{0.} = b_B \equiv (\log(q_0/q_9), \ldots, \log(q_8/q_9))'$ is

$$
W_{2BL} = (b_{0.} - b_B)' \hat{V}_0^{-1} (b_{0.} - b_B) .
$$

In each case the reference distribution for a single test is χ_9^2.

The third kind of test adapts an idea used in Grendar, Judge, and Schechter (2007), comparing the arithmetic mean of the second digits to the mean value expected if the digits are 2BL-distributed. Grendar et al.'s analysis focuses on the first significant digit and is intended to identify what they describe as generalized Benford distributions. Their analysis is motivated especially by Rodriguez (2004), who uses a particular family of mixtures of uniform distributions

to define a family of distributions for first significant digits. The Benford's law distribution for first digits is a member of this family.[3] Grendar et al. use a family of divergence measures to define an exponential family of distributions based on the mean of the first digits. Data that do not conform to Benford's law, these authors suggest, may have first digits that match a member of these related families.

In Mebane (2006b) I argue that vote counts in general do not have digits that match Benford's law at all. For instance, the distribution of the first digits of vote counts is undetermined. Mebane (2006b) demonstrates a pair of naturalistic models that produce simulated vote counts with second digits but not first digits that are distributed roughly as specified by Benford's law. Nonetheless we can use the mean of the second digits to test how closely the digits match the 2BL distribution. Given 2BL-distributed counts, the value expected for the second-digit mean is (rounded) $\sum_{j=0}^{9} jq_j = 4.187$. I use a basic nonparametric bootstrap (Davison and Hinkley 1997, 71) to test whether the observed second digit means deviate significantly from that expected mean.

Votes in the 2006 Mexican Federal Election by Municipality Party Affiliation

To study how vote counts recorded in the 2006 Mexican federal election vary in relation to the mayor of each municipality, the first step is to match the federal election data with the corresponding data about municipalities. The source for election data is the *Instituto Federal Electoral* (IFE), which posts spreadsheets containing casilla-level vote counts at its website.[4] The principal source for the party of each municipality's mayor is SNIM (Instituto Nacional para el Federalismo y el Desarrollo Municipal 2006).[5] The SNIM data list a party name in cases where the municipality mayor is affiliated with a single party, but for 201 of the 2440 municipalities in

---

[3]Janvresse and de la Rue (2004) prove that Benford's law uniquely characterizes the digits of values produced by observing a value from each of a collection of intervals, each interval having a random upper bound and each value being uniformly distributed on its interval conditional on the interval's upper bound. Of course vote counts are integers, not real numbers, so such arguments based on mixtures of uniform distributions do not directly apply to them.

[4]The IFE URL is `http://www.ife.org.mx`. The title for the set of spreadsheets is *las Bases de Datos de Cómputos Distritales 2006*. There is one spreadsheet each for the votes cast for *Presidente de los Estados Unidos Mexicanos*, *Diputados por los Principios de Mayoría Relativa y Representación Proporcional* and *Senadores por los Principios de Mayoría Relativa y Representación Proporcional*. The exact URL for the spreadsheets varies.

[5]The party affiliations are listed in a file named `Nueva.dbf` (size 5049961 bytes, timestamp Aug 29, 2006).

the file the data indicate only that the mayor is affiliated with a coalition. The SNIM data do not indicate which parties are included in each coalition. To identify the members of each municipality coalition, I used information organized by CIDAC,[6] in a few instances supplemented by information from the *Instituto de Mercadotecnia y Opinion* (IMO).[7]

The SNIM and IFE databases do not use the same numeric codes to identify municipalities. Also, the SNIM data do not include the municipality names but instead the name of the municipality *cabecera* (the municipality seat of government).[8] The municipality cabecera is usually but not always the same as the electoral cabecera designated by the IFE. In many cases, however, the municipality cabecera name is the same as the IFE municipality name. Where such correspondences do not occur, I used lists found on the web[9] to match up most of the records. In a few instances matches were made based on visual inspection of adjacent sets of geographic maps.

Among all the SNIM municipalities that match an IFE municipality there are 28 distinct parties with which the mayor was affiliated either directly or as part of a local coalition at the time of the 2006 federal election.[10] Municipality parties align by name with three of the coalitions standing for the federal election, namely PAN, APM and PBT.[11] Table 1 shows the number of municipalities having such affiliations. In some cases members of a municipality's governing coalition align with two federal election coalitions, either PAN and PBT or APM and PBT. Table 1 tabulates these separately.

*** Table 1 about here ***

In line with the argument in Mebane (2006b) that 2BL-distributed digits should not be expected for casilla-level vote counts, because of the "REDWL" phenomenon, but may occur for

---

[6]URL `http://www.cidac.org/`. See the link labeled *Base de datos de elecciones locales 1980-2006*.

[7]URL `http://www.imocorp.com.mx/`. Phone calls to officials in each municipality were required to resolve contradictions among SNIM, CIDAC and IMO regarding a few municipalities in Coahuila, Chiapas, Sinaloa and Sonora.

[8]The municipality cabecera names appear in a file named `prinloc.dbf` (size 763743 bytes, timestamp Aug 29, 2006).

[9]E.g., one useful source is `http://www.answers.com/topic/municipalities-of-zacatecas` along with similar sites (linked to that URL) for other states.

[10]This count does not include the Oaxaca mayors selected by *uso y costumbre*.

[11]One municipality, Sahuaripa in the state of Sonora, shows a coalition between PRI Sonora and Nueva Alianza. In Table 1 this municipality is included among the APM municipalities.

the vote counts for *secciones* or sections, I focus on the latter. Table 1 shows the number of secciones in each set of party-affiliated municipalities. Secciones include all the voters registered in relatively small, contiguous geographic areas. They correspond to the administrative units called precincts in the United States. A casilla contains no more than 750 voters from a seccion, with each casilla containing all the voters in the seccion with names in a contiguous range of the alphabet. For various reasons the number of votes cast in the casillas for a given seccion is not exactly the same. Mebane (2006b) describes the casilla pattern in relation to the seccion's voters as an example of "roughly equal division with leftovers" (REDWL) and shows that such a pattern induces departures from 2BL in the subdivided vote totals even when the undivided totals are 2BL-distributed.

In addition to the vote counts for president, I also examine the votes for senator (*Senadores*) and deputy (*Diputados*). The Mexican legislature is elected partly using a plurality rule (*Mayoría Relativa*) in single-member districts and partly using proportional representation (*Representación Proporcional*) within five large regional districts. The totals used to award legislative seats include the same votes counted for the plurality seats plus additional votes cast by people not voting at their home casilla on election day. I analyze the latter vote counts separately from the plurality vote counts.

Tests using the X^2_{2BL} statistic reveal that the vote counts' second digits depart substantially from the 2BL distribution. As can be seen in Table 2, the departures among the presidential vote counts follow a number of interesting patterns. Controlling the FDR across all 90 statistics reported in Table 2, the smallest X^2_{2BL} value for which the 2BL hypothesis is rejected for a test level of $\alpha = .05$ is the value $X^2_{2BL} = 22.5$ that occurs for the presidential APM vote with PAN-PBT-affiliated mayors. Using this FDR approach with test level of $\alpha = .05$, all X^2_{2BL} values in Table 2 larger than 22.5 may be considered statistically significant.

<div align="center">*** Table 2 and about here ***</div>

First, for the PAN and PBT vote counts, there is a coincidence between the mayor's party and significant X^2_{2BL} statistics. X^2_{2BL} is significant for presidential PAN votes only among secciones

in municipalities with a PAN mayor, and X^2_{2BL} is significant for presidential PBT votes only where there is either a PBT-affiliated mayor or a mayor not affiliated with one of the presidential coalitions. This pattern is largely reproduced for PAN's senate and deputy vote counts. For the PBT deputy vote counts, X^2_{2BL} is also significant where there is a PBT-affiliated mayor. For PBT the pattern varies somewhat for the other offices. X^2_{2BL} is significant for PBT senate votes only where there is an Other mayor, and for PBT deputy votes X^2_{2BL} is significant where there is either a PBT or a PAN mayor.

For none of the offices do the APM vote counts have a significant X^2_{2BL} statistic where there is an APM-affiliated mayor, but in several other instances the APM vote counts depart significantly from the 2BL-distribution. In particular, the presidential APM vote counts have significant X^2_{2BL} statistics for every type of mayor except the APM-affiliated mayors. There are not comparably significant X^2_{2BL} statistics for the senate and deputy APM vote counts. For those counts, X^2_{2BL} is significant only when there is either a PAN mayor or an Other mayor.

Differences between the presidential vote counts and the vote counts for the other two offices are even more stark for the NA vote counts. All the X^2_{2BL} statistics for NA presidential vote counts are significant. In part this result reflects the fact that the NA presidential vote counts tend to be small. The bottom part of Table 2 shows that the NA candidate received at least ten votes in only 3,740 of the 17,721 secciones. Moreover the median vote count among the secciones with at least ten votes for NA is 14. On the other hand, as the email shown above from Kelley indicates, there is reason to believe these small counts are the consequence of an electoral strategy originating with the leader of the NA party. Among the NA vote counts for senate and deputy there is only one significant X^2_{2BL} value, and that is for the NA deputy vote where there is an APM-affiliated mayor.

There are numerous significant X^2_{2BL} values for ASDC. For ASDC presidential vote counts there is a significant X^2_{2BL} value only where there is an APM-affiliated mayor, but for ASDC senate vote counts X^2_{2BL} is significant in very case except where there is an Other mayor, and for ASDC deputy vote counts X^2_{2BL} is significant in every case except where there is a PBT-affiliated

mayor or an Other mayor.

Tests using the X^2_{2BL} statistics with the senate and deputy vote counts from only the *Representación Proporcional* casillas produce very different results, with few significant X^2_{2BL} values. Table 3 reports these results. Controlling the FDR across all 150 statistics reported in Tables 2 and 3, the smallest X^2_{2BL} value for which the 2BL hypothesis is rejected for a test level of $\alpha = .05$ is now the value $X^2_{2BL} = 21.8$ that occurs for the proportional representation deputy PAN vote with PBT-affiliated mayors. Only two other X^2_{2BL} values in Table 3 are significant, and both occur where there are APM-affiliated mayors: the value for senate PBT vote counts and the value for deputy ASDC vote counts.

*** Table 3 and about here ***

Tests using the means of the vote counts' second digits provide more information about how the digits' distributions differ from the 2BL-distribution. Table 4 reports 95% bootstrap confidence intervals for the difference between the presidential and plurality seccion vote counts' second digit means and $\sum_{j=0}^{9} jq_j = 4.187$, the mean value expected according to 2BL. The number of these intervals that indicate significant differences from the expected mean value is much greater than the number of significant X^2_{2BL} values for the same vote counts. This is so even if we read Table 2 naively and do not impose FDR control, so that values of X^2_{2BL} greater than 16.7 (the critical value for a simple test at level $\alpha = .05$ using χ^2_9) are considered significant. Such a comparison is fair because the intervals in Table 4 are not adjusted to control the FDR. A significant difference is not necessarily a large one, however. As it is not yet clear how big a difference should be in order to be considered substantively large, I focus on differences that have an absolute magnitude of at least 0.05. These differences largely coincide with the X^2_{2BL} values that are significant with FDR control. Exceptions include the presidential ASDC vote counts with APM-PBT-affiliated mayors and the senate PBT vote counts with PAN-PBT-affiliated mayors (large mean difference but not a significant X^2_{2BL} value), and the senate and deputy APM vote counts with PAN mayors (a significant X^2_{2BL} value but not a large mean difference).

*** Table 4 and about here ***

11

The mean differences point to reliable patterns of vote count increases and decreases. When PAN and PBT-affiliated mayors are significantly associated with large departures from 2BL, the mean differences are positive. The suggestion is that vote counts are higher in those cases than they would be otherwise. But the large mean differences for the APM and NA vote counts are all negative, suggesting that the vote counts are lower in those cases than they might otherwise be. The large mean differences for the ASDC vote counts are also all negative. It is only a slight stretch to summarize the patterns by saying that in places where there are significant departures from 2BL, PAN and PBT seem to be gaining votes while APM, NA and ASDC are losing votes. It is plausible to say votes seem to be going from APM, NA and ASDC to either PAN or PBT.

Vote Count Digits and the Recount

To assess the relationship between the 2BL tests and the partial recount of the presidential votes, we need to shift the analysis from seccion vote counts to vote counts at the level of casillas. The recount examined only selected casillas, and the casillas that were not selected in a seccion were not recounted. Having to shift to casilla-level vote counts is undesirable in light of the REDWL phenomenon, but in this case we have little choice.

As it happens, in the current case the shift to a casilla-level analysis appears to be innocuous. Among the recounted casillas there is no relationship between the results of the recount and the second digits in the vote counts. I use two different specifications of the ten-category multinomial logit model method to estimate the relationship. In one specification there are five regressor variables defined as the difference between the recounted vote totals for the party coalitions and the original vote totals. The other specification uses a single indicator variable that has the value 1 if the recount changed the vote count for any party. Both models also include as regressors five dummy variables to distinguish the six sets of municipality mayor party affiliations. The statistic W_k is not significant for the recount-related regressors in either specification. Whatever the 2BL tests are picking up is not to be the kind of thing the recount is catching, and vice versa.

12

Fraud, Intimidation or Strategic Voting?

The patterns identified by the 2BL tests in part match classical ideas about how local political machines operate. In municipalities where PAN and PBT-affiliated mayors were present, federal candidates affiliated with PAN and PBT, respectively, seem to have tended to gain more votes. If the 2BL digit pattern is a marker for natural voting patterns, then these gains are unnatural. Vote counts for PAN and PBT candidates in the respective municipalities depart significantly from the 2BL-distribution, more so for PAN than for PBT.

But the patterns may not always fit with machine expectations. Municipalities with APM-affiliated mayors do not exhibit a similar pattern of seemingly higher votes for APM candidates.

Before counting the lack of significant 2BL test statistics for APM candidates in APM-affiliated municipalities as indicating a striking exception to local machine rule, it is important to consider the background. In most of the non-APM municipalities, the presidential vote counts for APM seem to have been significantly reduced. With the possibly important exception of the APM plurality vote counts for deputy, the APM vote counts for senate and deputy were not similarly reduced. It is unclear why a local machine that could significantly manipulate presidential vote counts would not also manipulate the counts for the federal legislative offices. So it seems unlikely that the reduction in APM's presidential vote counts is a symptom of widespread local machine action at APM's expense. Instead there seems to have been a general movement of votes, originating elsewhere. Against such a background, perhaps we should be impressed by APM-affiliated mayors' apparent ability to hold the line against APM defections in their municipalities.

The message I quoted from Miguel Kelley may suggest that the apparent movement away from APM should be credited to NA leadership and NA muscle. The 2BL test results support the claim that presidential votes that would have been cast for NA were shifted away from NA. NA deputy candidates received just under 1.9 million votes, while the NA presidential candidate received just over 400,000 votes. That leaves 2.5 millions votes to account for to get to the

"nearly 4M votes" Kelley claimed the NA was responsible for shifting. Could those votes have come from APM?

It is well beyond the scope of a few tests on the vote counts' second digits to decide that question, but it is worthwhile to notice how close the presidential election was and to mention strategic voting. Many voters whose first choice for president was the APM candidate may have noted that candidate's unlikely prospects, observed the closeness of the race between PAN and PBT and decided to abandon the APM candidate in favor of their second choice. How such calculations may have interacted with the party affiliations of the municipality mayors is a topic worth thinking about and investigating.

In any case, such a conundrum—massive intimidation or widespread strategic voting?—is brought to the forefront by the 2BL tests but completely invisible to the manual recount. While the 2BL tests cannot answer the question, the manual recount does not even ask it. This example very strongly illustrates how 2BL tests can supplement manual recounts. The 2BL tests detect election phenomena that have nothing to do with tabulation errors. The second digits of the vote counts are in the present case completely unrelated to the changes the manual recount produced in the vote count tabulations. While the manual recount arguably did not turn up any suspicious patterns, the 2BL tests identify substantial distortions. The case for including 2BL tests in the toolkit for election forensics grows stronger.

References

Benjamini, Yoav and Yosef Hochberg. 1995. "Controlling the False Discovery Rate: A Practical and Powerful Approach to Multiple Testing." *Journal of the Royal Statistical Society, Series B* 57 (1): 289–300.

Bjornlund, Eric C. 2004. *Beyond Free and Fair: Monitoring Elections and Building Democracy*. Washington, DC: Woodrow Wilson Center Press.

Brady, Henry E. and Iris Hui. 2006. "Accuracy and Security in Voting Systems." Unpublished MS.

Davison, A. C. and D. V. Hinkley. 1997. *Bootstrap Methods and their Application*. New York: Cambridge University Press.

Eisenstadt, Todd A. 2007. "Usos Y Costumbres and Postelectoral Conflicts in Oaxaca, Mexico, 1995-2004." *Latin American Research Review* 42 (1): 52–77.

electionline.org. 2007. "Case Study: Auditing the Vote." Electionline.org Briefing number 17.

European Union. 2006. "Final Report: Mexico Presidential and Parliamentary Elections, 2 July 2006." European Union Election Observation Mission, Mexico 2006, 23 November 2006, Mexico City/Brussels.

Grendar, Marian, George Judge, and Laura Schechter. 2007. "An Empirical Non-Parametric Likelihood Family of Data-Based Benford-Like Distributions." *Physica A* forthcoming.

Gumbel, Andrew. 2005. *Steal This Vote*. New York: Nation Books.

Harris, Joseph P. 1934. *Election Administration in the United States*. Washington, DC: Brookings Institution.

Instituto Federal Electoral. 2006. "Reporte de Representantes de Casilla." URL http://www.ife.org.mx/docs/Internet/IFE_Home/CENTRAL/Contenidos_Centrales/estaticos/ Informes/representantes_PPfinal.pdf (accessed April 5, 2007).

Instituto Nacional para el Federalismo y el Desarrollo Municipal. 2006. *Sistema Nacional de Información Municipal, versión 7*. Instituto Nacional para el Federalismo y el Desarrollo Municipal Mexico City. CD-ROM.

Janvresse, Élise and Thierry de la Rue. 2004. "From Uniform Distributions to Benford's Law." *Journal of Applied Probability* 41: 1203–1210.

Kelley, Miguel. 2006. "Email: Subject: Re: benford contra representantes." Message sent September 6, 2006.

Lehoucq, Fabrice. 2003. "Electoral Fraud: Causes, Types, and Consequences." *Annual Review of Political Science* 6 (June): 233–256.

Mebane, Walter R., Jr. 2006a. "Detecting Attempted Election Theft: Vote Counts, Voting Machines and Benford's Law." Paper prepared for the 2006 Annual Meeting of the Midwest Political Science Association, Chicago, IL, April 20–23.

Mebane, Walter R., Jr. 2006b. "Election Forensics: Vote Counts and Benford's Law." Paper prepared for the 2006 Summer Meeting of the Political Methodology Society, UC-Davis, July 20–22.

Mebane, Walter R., Jr. 2006c. "Election Forensics: The Second-digit Benford's Law Test and Recent American Presidential Elections." Paper prepared for the Election Fraud Conference, Salt Lake City, Utah, September 29–30, 2006.

Mebane, Walter R., Jr. 2007. "Evaluating Voting Systems To Improve and Verify Accuracy." Paper presented at the 2007 Annual Meeting of the American Association for the Advancement of Science, San Francisco, CA, February 16, 2007, and at the Bay Area Methods Meeting, Berkeley, March 2, 2007.

Mebane, Walter R., Jr. and David L. Dill. 2007. "Factors Associated with the Excessive CD-13 Undervote in the 2006 General Election in Sarasota County, Florida." Unpublished MS.

Mebane, Walter R., Jr. and Jasjeet S. Sekhon. 2004. "Robust Estimation and Outlier Detection for Overdispersed Multinomial Models of Count Data." *American Journal of Political Science* 48 (Apr.): 392–411.

Myagkov, Misha, Peter C. Ordeshook, and Dimitry Shaikin. forthcoming. "Estimating the Trail of Votes in Russia's Elections and the Likelihood of Fraud." In R. Michael Alvarez, Thad E. Hall, and Susan D. Hyde, editors, *The Art and Science of Studying Election Fraud: Detection, Prevention, and Consequences*, .Washington, DC: Brookings Institution.

Rivest, Ronald L. 2007. "On Auditing Elections When Precincts Have Different Sizes." Unpublished MS.

Rodriguez, Ricardo J. 2004. "First Significant Digit Patterns From Mixtures of Uniform Distributions." *American Statistician* 58 (Feb.): 64–71.

Wand, Jonathan, Kenneth Shotts, Jasjeet S. Sekhon, Walter R. Mebane, Jr., Michael Herron, and Henry E. Brady. 2001. "The Butterfly Did It: The Aberrant Vote for Buchanan in Palm Beach County, Florida." *American Political Science Review* 95 (Dec.): 793–810.

Table 1: Municipality Party Affiliations as of the Mexican 2006 Federal Election

Municipality Party Coalition Membership

	PAN	APM	PBT	PAN-PBT	APM-PBT	Other
municipalities	534	782	396	50	56	1014
secciones	17,721	19,192	10,534	1,666	2,556	13,020

Notes: Each municipality's party affiliation is determined by matching the members of the mayor's coalition to the parties and coalitions presenting candidates in the 2006 federal election. The number of municipalities is the number appearing in the IFE data. The number of secciones is the number used for voting in the presidential election.

Table 2: Mexican 2006 Federal Election: 2BL Test Statistics by Municipality Party

X^2_{2BL}	Party Voted	PAN	APM	PBT	PAN-PBT	APM-PBT	Other
		Municipality Party Coalition Membership					
President	PAN	60.3	7.2	10.2	8.3	10.2	17.4
	APM	44.9	10.5	59.8	22.5	24.8	18.9
	PBT	10.4	3.4	50.5	10.4	12.4	34.7
	NA	387.9	339.8	269.2	76.7	84.7	167.7
	ASDC	4.6	42.9	14.6	33.3	14.7	16.1
Senator	PAN	43.7	7.1	14.9	7.8	15.6	13.6
	APM	23.9	9.8	15.8	15.5	8.7	86.1
	PBT	12.8	10.9	14.6	14.6	19.0	44.0
	NA	10.6	16.6	18.0	14.8	10.6	18.3
	ASDC	131.1	182.0	24.5	111.2	63.4	5.2
Deputy	PAN	56.4	14.9	24.1	4.4	10.8	17.0
	APM	30.6	9.3	10.7	9.8	18.5	89.6
	PBT	25.4	11.0	29.0	18.4	11.9	12.5
	NA	10.2	30.0	11.4	11.7	7.9	9.1
	ASDC	133.9	167.7	16.1	59.6	92.7	6.4

N	Party Voted	PAN	APM	PBT	PAN-PBT	APM-PBT	Other
		Municipality Party Coalition Membership					
President	PAN	17,667	18,341	9,584	1,627	2,459	12,812
	APM	17,620	19,084	10,304	1,663	2,539	12,705
	PBT	17,243	18,570	10,436	1,595	2,412	12,887
	NA	3,740	3,258	2,183	312	582	2,228
	ASDC	10,957	8,173	5,162	680	1,287	9,105
Senator	PAN	17,664	18,294	9,537	1,624	2,451	12,547
	APM	17,642	19,121	10,341	1,665	2,542	12,681
	PBT	17,025	18,229	10,425	1,566	2,374	12,569
	NA	11,941	10,293	6,508	1,111	1,846	9,751
	ASDC	7,663	5,573	4,476	250	771	7,697
Deputy	PAN	17,662	18,310	9,554	1,623	2,449	12,542
	APM	17,632	19,107	10,331	1,664	2,541	12,682
	PBT	17,012	18,297	10,419	1,565	2,364	12,575
	NA	12,429	11,459	6,626	1,056	1,794	9,869
	ASDC	8,071	5,587	4,643	236	795	7,925

Notes: Top of table: X^2_{2BL} statistics. Bottom of table: N of vote counts ≥ 10. Tests are based on seccion vote counts greater than 9 for the referent party. Each *casilla extraordinaria* used for presidential voting is treated as a separate seccion. The statistics for Senator and Deputy use only *Mayoría Relativa* vote counts.

Table 3: Mexican 2006 Federal Election: 2BL Test Statistics by Municipality Party, Proportional Representation Votes

X^2_{2BL}	Party Voted	Municipality Party Coalition Membership					
		PAN	APM	PBT	PAN-PBT	APM-PBT	Other
Senator	PAN	9.0	8.1	9.3	11.2	5.5	4.8
	APM	9.5	8.6	8.5	7.2	9.4	8.1
	PBT	8.5	26.9	19.6	11.8	5.2	14.8
	NA	9.5	16.9	7.2	8.3	6.0	1.7
	ASDC	15.1	14.9	6.2	4.3	14.2	5.8
Deputy	PAN	13.3	10.4	21.8	9.8	5.2	10.8
	APM	9.4	11.7	6.4	11.0	11.8	7.1
	PBT	5.3	11.0	6.2	5.0	4.2	9.1
	NA	15.8	19.5	7.5	15.5	8.8	11.9
	ASDC	18.6	39.8	10.8	14.7	7.5	20.7

N	Party Voted	Municipality Party Coalition Membership					
		PAN	APM	PBT	PAN-PBT	APM-PBT	Other
Senator	PAN	233	245	127	20	35	125
	APM	231	243	127	20	33	125
	PBT	233	245	129	19	34	125
	NA	166	122	94	12	18	86
	ASDC	128	99	57	5	13	72
Deputy	PAN	233	240	126	20	33	124
	APM	231	241	126	20	33	125
	PBT	230	241	128	20	33	124
	NA	146	165	94	13	28	79
	ASDC	51	53	34	2	8	28

Notes: Top of table: X^2_{2BL} statistics. Bottom of table: N of vote counts ≥ 10. Tests are based on seccion vote counts greater than 9 for the referent party. The statistics use only vote counts from *Representación Proporcional* casillas.

Table 4: Mexican 2006 Federal Election: 2BL Mean-digit Test Statistics by Municipality Party

President

Party Voted	Municipality Party Coalition Membership					
	PAN	APM	PBT	PAN-PBT	APM-PBT	Other
PAN	$.11, .19$	$-.02, .06$	$-.01, .11$	$-.16, .13$	$-.11, .11$	$.01, .11$
APM	$-.17, -.08$	$-.10, -.02$	$-.26, -.15$	$-.37, -.10$	$-.37, -.14$	$-.11, -.01$
PBT	$-.07, .01$	$-.03, .05$	$.11, .23$	$-.08, .19$	$-.28, -.03$	$.09, .19$
NA	$-.99, -.82$	$-1.00, -.81$	$-1.10, -.88$	$-1.58, -.98$	$-1.27, -.83$	$-.82, -.59$
ASDC	$-.06, .05$	$-.26, -.13$	$-.07, .08$	$-.76, -.36$	$-.36, -.06$	$.04, .16$

Senate

Party Voted	Municipality Party Coalition Membership					
	PAN	APM	PBT	PAN-PBT	APM-PBT	Other
PAN	$.08, .17$	$-.04, .05$	$-.09, .03$	$-.26, .02$	$-.16, .05$	$-.02, .08$
APM	$-.03, .05$	$-.08, -.00$	$-.10, .01$	$-.33, -.05$	$-.18, .04$	$-.27, -.17$
PBT	$-.10, -.02$	$-.07, .02$	$.00, .12$	$.06, .35$	$-.16, .09$	$.10, .20$
NA	$-.10, .01$	$-.13, -.02$	$-.16, -.02$	$-.26, .08$	$-.19, .08$	$.03, .15$
ASDC	$-.43, -.30$	$-.59, -.44$	$-.25, -.07$	$-1.98, -1.33$	$-.97, -.56$	$-.01, .12$

Deputy

Party Voted	Municipality Party Coalition Membership					
	PAN	APM	PBT	PAN-PBT	APM-PBT	Other
PAN	$.10, .19$	$.00, .09$	$-.07, .04$	$-.22, .07$	$-.25, -.00$	$-.01, .09$
APM	$-.04, .04$	$-.01, .07$	$-.07, .03$	$-.20, .06$	$-.14, .08$	$-.27, -.17$
PBT	$-.14, -.05$	$-.06, .03$	$.07, .19$	$-.20, .08$	$.02, .25$	$.00, .11$
NA	$-.10, .00$	$-.17, -.06$	$-.13, .00$	$-.19, .16$	$-.15, .12$	$.01, .12$
ASDC	$-.43, -.30$	$-.56, -.42$	$-.22, -.05$	$-1.62, -.95$	$-1.11, -.72$	$-.03, .10$

Notes: 95% nonparametric bootstrap confidence intervals for the difference between the mean of the second digits of each respective category's seccion counts and the 2BL expected mean value $\sum_{j=0}^{9} j q_j = 4.187$. Seccion counts smaller than 10 are ignored. Each *casilla extraordinaria* used for presidential voting is treated as a separate seccion. The statistics for Senator and Deputy use only *Mayoría Relativa* vote counts.