`eforensics` Analysis of the Turkish 2023 Presidential Election*

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Abstract

I use polling station data to estimate `eforensics`-frauds to measure the magnitude of malevolent distortions of electors' intentions—frauds—in the 2023 elections for president and legislature in the Republic of Türkiye ("Turkey"). I compare these elections in some respects to earlier ones. I find there are extensive and ample `eforensics`-frauds, but whether the `eforensics`-frauds result from malevolent distortions of electors intentions is not a simple matter to determine. Strategic behavior and lost votes are among the plausible alternative explanations.

The May 14, 2023, election in the Republic of Türkiye ("Turkey") is of interest for election forensics at least because the first round of the presidential election failed to produce a candidate with more than half the votes, so the election went to a second round. Also both rounds of the election are of interest because of `eforensics` (Mebane 2022, 2023) results I have for several previous elections in Turkey. I use polling station data to estimate the `eforensics` model (Ferrari, Mebane, McAlister and Wu 2019) to measure the magnitude of `eforensics`-fraudulent votes. Vote and elector count data come from the *Yüksek Seçim Kurulu* (YSK, the Supreme Election Council).[1]

Office for Democratic Institutions and Human Rights (2023) identifies a variety of flaws in the election process. `eforensics` is designed to measure what I call *realized frauds*, as opposed to the *procedural frauds* that many other approaches are intended to detect. A realized fraud is a malevolent distortion of votes that makes the outcome of the election not match electors' intentions. `eforensics` measures the number of `eforensics`-fraudulent votes at each polling station. `eforensics` is valid for measuring realized frauds but not perfect. See Mebane (2022), which actually dates from July 2021, for a preliminary discussion of the technology and how to interpret its results. The current analysis reflects features of `eforensics` I've learned since writing Mebane (2022).

`eforensics` operationalizes the idea that `eforensics`-frauds occur when one candidate gains votes by a combination of manufacturing votes from abstentions and stealing votes from opposing candidates (Mebane 2022). The Bayesian specification of `eforensics` allows posterior means and credible intervals for counts of `eforensics`-fraudulent votes to be determined both for the entire election and for individual polling stations. The model requires that some ballot alternative be designated the "leader," which is the alternative that the model allows to benefit from added `eforensics`-fraudulent votes. The candidate

[1] On May 19, 2023, Ahmet Aykac provided the version of the 2023 presidential election first round counts I analyze here. He provided 2023 parliamentary election polling station counts on May 19 and 20, 2023. On June 19, 2023, he provided the version of the 2023 presidential election second round counts I analyze. Preston Due helped me download data for the 2017 constitutional referendum from the YSK website. Data from the 2015 parliamentary elections come from Rob Barry and Tom McGinty. Earlier parliamentary election polling station counts come from someone who wishes not to be named.

with the most votes in each election is this designated leader candidate: eforensics-fraudulent votes can add to the votes for Erdoğan for president.

The most important feature of eforensics to keep in mind when considering eforensics estimates is that eforensics likely responds both to bad acts such as vote-buying, intimidation, violence and disinformation and to strategic elector behavior, and the estimates can be distorted by lost votes. A challenge for eforensics is to be able to identify which eforensics-fraudulent votes reflect malevolent distortions (bad acts) and which stem from strategic behavior by electors (eligible voters) (see Mebane 2022, 2023). Also votes lost asymmetrically from opposition (the set of non-leader alternatives) can appear to be eforensics-fraudulent votes for the leader. The eforensics model is a finite-mixture model that distinguishes "no frauds" from "incremental frauds" and "extreme frauds": extreme frauds are larger. Analysis like that described in Mebane (2022) suggests that often strategic behavior produces positive incremental frauds estimated eforensics-fraudulent vote counts, but usually strategic behavior is not associated with positive extreme frauds estimated counts. So incremental frauds estimates are generally more ambiguous than are extreme frauds estimates. The eforensics estimation used here employs four Monte Carlo Markov Chain (MCMC) chains (Mebane 2022, 2023). Votes being lost especially among non-leader candidates often induces posterior multimodality for the mixture probability parameters in the MCMC chains (Mebane 2023).

For convenience in interpreting subsequent maps Figure 11 in the Appendix presents a map of the regions of Turkey.

1 2017 Constitutional Referendum

To introduce a few of the complexities of eforensics analysis of Turkish elections consider briefly the 2017 constitutional referendum. In that election data from the YSK[2] show there

[2] The YSK data, obtained April 28–30, 2017, include 173895 polling stations but only 173327 have complete elector and vote count values with a positive number of reported votes cast.

were 56,669,068 electors[3] and 49,651,009 votes cast, with 25,075,936 voting "Yes," 23,715,116 voting "No" and 859,957 votes labeled "invalid":[4] I include the "invalid" votes among the votes cast because given compulsory voting rules[5] in Turkey I assume the invalid votes include blank votes, which is different from electors not participating at all. Figure 1 shows scatterplots, histograms and empirical densities for turnout and leader ("Yes") vote proportions. Figure 1(a) plots the original data while Figure 1(b) plots the data after removing region fixed effects. The latter plot represents the data as they are being treated in the `eforensics` estimates reported in Table 1, because that specification of the model includes region fixed effects for turnout and vote choice.

The key feature of Figure 1(b) is that the points in the scatterplot are clumpy.[6] Such clumpiness is a symptom of frauds, strategic behaviors or lost votes—or all of these—having occurred. In a referendum with only "Yes" or "No" as the valid alternatives wasted-vote strategies cannot occur, but there may be strategically informed mobilizations to vote or to boycott. Boycotts would produce lost votes, and it is easy to imagine that opponents of the referendum would be more likely to boycott it than supporters would be. Lost votes might also occur due to malevolent efforts to intimidate electors or otherwise suppress votes.

[3]The count of electors for each polling station is the larger of `registered voters` (*secmen sayisi*) and `voters` (*oy sayısı(seçime katılım)*).

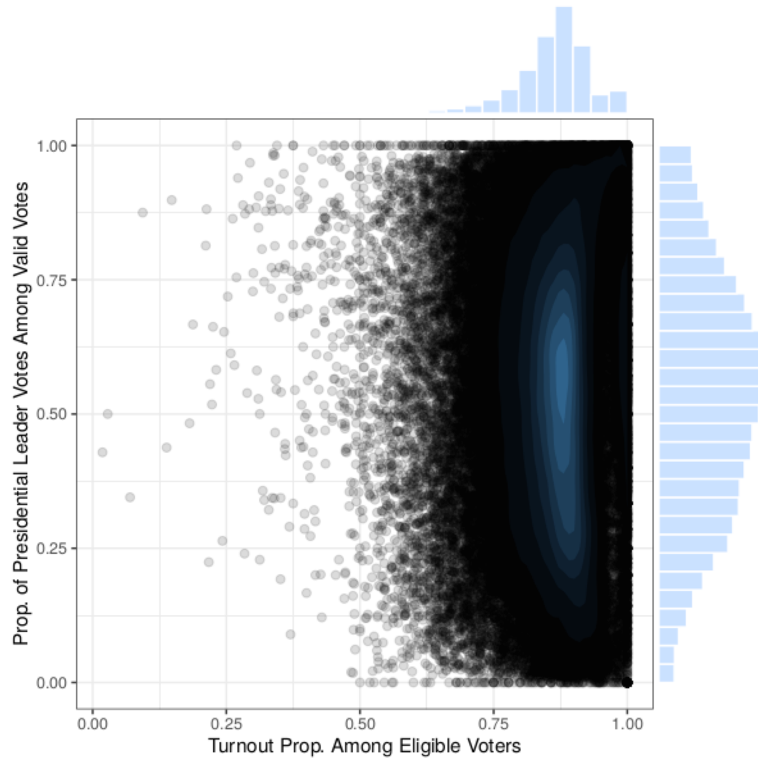
[4]Votes labeled "invalid" are those labeled in Turkish as *gecersiz oy sayısı*.

[5]On compulsory voting in Turkey see `https://www.idea.int/data-tools/country-view/287/40`.

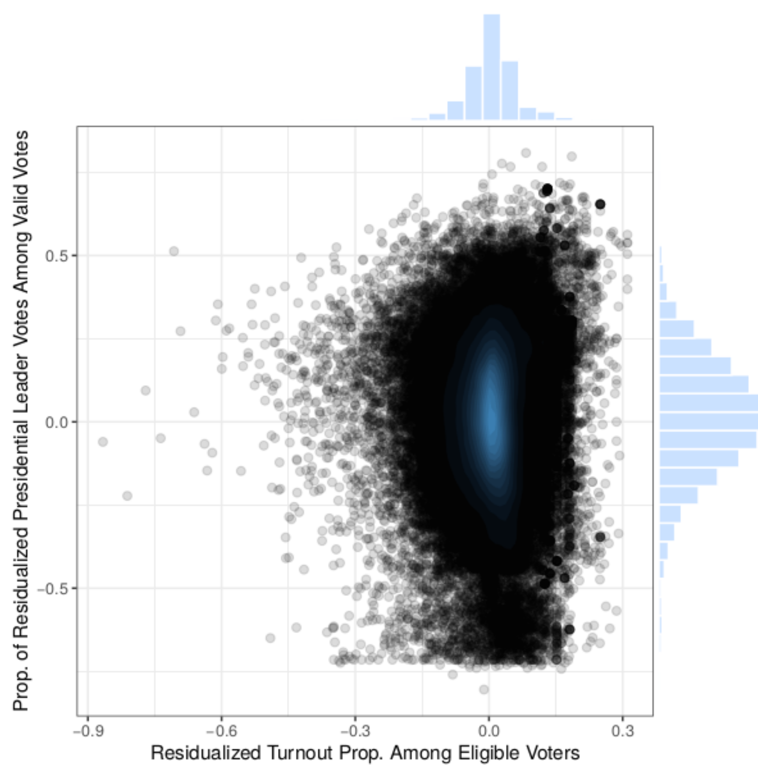
[6]Clumps are somewhat easier to see if the image is expanded by at least 2x magnification.

Figure 1: `eforensics`-plots: Turkey 2017

(a) original data



(b) region-residualized data



Note: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions. For `eforensics` estimates see Table 1.

In fact the `eforensics` estimates reported in Table 1 show strong signs that there were lost votes.[7] Both of the diagnostic statistics for posterior MCMC multimodality in the mixture probability parameters give clear signals: for π_1 and π_2 the dip test for the null hypothesis of unimodality (Hartigan and Hartigan 1985) over all MCMC chains has a p-value of approximately zero; and again for π_1 and π_2 the differences between the largest and smallest chain-specific posterior means are about as large as they can be. All of the polling stations classified as `eforensics`-fraudulent have extreme frauds, for which the most immediately relevant mixture probability is π_3, which is diagnosed as having a unimodal posterior distribution.

Nonetheless it is unclear how lost votes might affect the model's estimates particularly of the frauds magnitudes and of the parameters that control those magnitudes (δ_{M0} and δ_{S0}). I cannot say for sure how many of the $F_w = 473874.6$ estimated `eforensics`-fraudulent votes (posterior mean) are due to malevolent distortions of electors intentions and how many are due to voluntary abstentions ("boycotts") by referendum opponents. In any case F_w is less than the difference of 1,360,820 between the totals of votes cast for "Yes" or for "No," so that simply removing the posterior mean of the estimated total number of `eforensics`-fraudulent votes from the count of "Yes" votes would not have changed which alternative had the most votes nor dropped the proportion "Yes" of valid votes below .5:

$$\frac{25075936 - 473874.6}{49651009 - 859957} = .5042\,.$$

The proportion also remains above .5 if in addition to removing the `eforensics`-fraudulent votes from "Yes" also manufactured votes are removed from votes cast:

$$\frac{25075936 - 473874.6}{49651009 - 859957 - 131005.8} = .5056\,.$$

[7] Note that the estimates in Table 1 differ from those reported in Mebane (2022) because the specification used in Mebane (2022) does not include "invalid votes" as votes cast.

The proportion drops below .5 if in addition the stolen votes are added to "No":

$$\frac{25075936 - 473874.6 - (473874.6 - 131005.8)}{49651009 - 859957 - 131005.8} = .49854\,.$$

The proportion is greater than .5 if all votes are retained and "invalid" votes are included in votes cast (25075936/49651009 = .5050) but not if `eforensics`-fraudulent votes are removed from "Yes" while "invalid" votes are included in votes cast:
(25075936 − 473874.6)/49651009 = .4955 (cf. Klimek, Jiménez, Hidalgo, Hinteregger and Thurner 2018).

Figure 2 maps the proportions of votes that are `eforensics`-fraudulent by town.[8] Posterior means of polling station `eforensics`-frauds and observed totals of leader vote or of votes cast are summed by town then used to compute proportions. Using W_j to denote leader votes in town j, V_j to denote votes cast, F_{tj} to denote manufactured votes and $(F_{wj} - F_{tj})$ to denote stolen votes, for each town j Figure 2(a) shows F_{wj}/W_j, Figure 2(b) shows F_{tj}/W_j, Figure 2(c) shows $(F_{wj} - F_{tj})/W_j$, and Figure 2(d) shows F_{tj}/V_j.[9] The proportions are colored in terms of their relative magnitudes for each kind of proportion: blue means a town has zero `eforensics`-fraudulent votes; green means that the proportion of leader votes or of votes cast that are `eforensics`-fraudulent exceeds the median value for the referent ratio across all towns; red means the proportion exceeds the third quartile value. Noteworthy is that `eforensics`-frauds are scant in Istanbul and Izmir, but are more prevalent in the north central and south eastern parts of the country.

[8]The indivdual towns are somewhat easier to see if the image is expanded by at least 3x magnification.
[9]The median, third quartile and maximum values of the town proportions are as follows: F_{wj}/W_j, .00958, .0371, .443; F_{tj}/W_j, .00239, .00972, .131; $(F_{wj} - F_{tj})/W_j$, .00694, .0269, .314; F_{tj}/V_j, .00126, .00642, .126. The minimum and first quartile values are zero.

Table 1: 2017 Referendum Election `eforensics` Estimates, Region Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.840	.478	.958
	π_2	Incremental Fraud	.117	1.56e-08	.478
	π_3	Extreme Fraud	.0435	.0420	.0449
turnout	β_0	(Intercept)	1.87	1.76	1.93
vote choice	γ_0	(Intercept)	$-.0212$	$-.253$	.0677
incremental frauds	ρ_{M0}	(Intercept)	$-.492$	$-.653$	$-.127$
	ρ_{S0}	(Intercept)	$-.858$	-1.14	$-.734$
extreme frauds	δ_{M0}	(Intercept)	-2.34	-2.68	-1.43
	δ_{S0}	(Intercept)	-2.57	-3.45	-1.55

posterior multimodality diagnostics:
all-chains dip test p-values $D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = .997$.[c]
posterior means difference $M(\pi_1) = .466$; $M(\pi_2) = .465$; $M(\pi_3) = .00139$.[d]

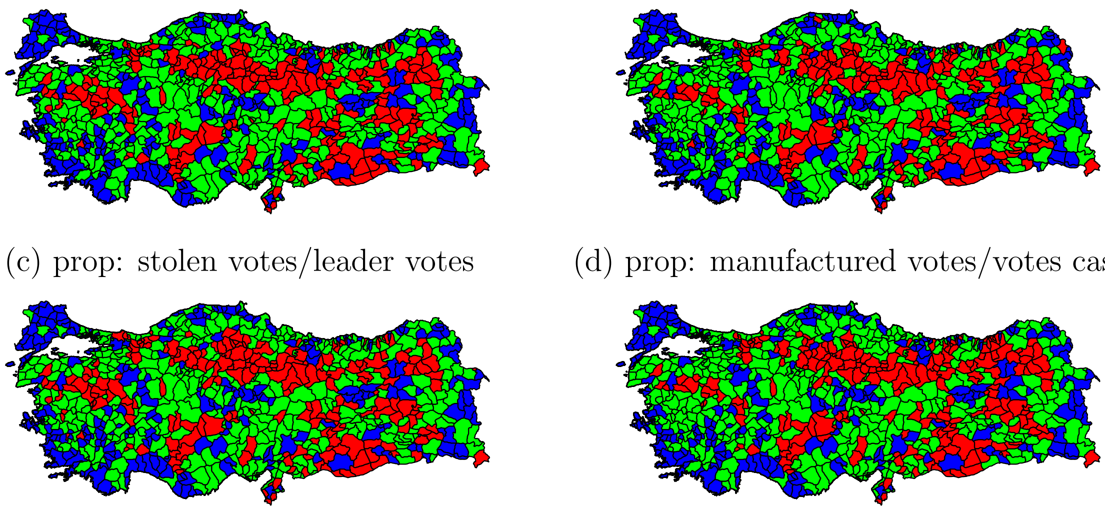
units `eforensics`-fraudulent: (0 incremental, 7251 extreme, 166076 not fraudulent)
manufactured votes $F_t = 131005.8$ $[112290.0, 178292.6]$[e]
total fraudulent votes $F_w = 473874.6$ $[439513.4, 546312.2]$[e]

`eforensics`-Fraudulent Polling Station and Vote Counts[f] by Station Type

	Polling Station Type			
count	abroad	customs	prison	village
polling stations	3202	3708	388	166029
`eforensics`-fraudulent polling stations	270	209	2	6770
`eforensics`-fraudulent votes[g]	38343.1	4230.9	128.8	536912.7
manufactured votes[g]	480.7	1026.8	41.2	157786.7
stolen votes[g]	37862.3	3204.1	87.6	379126.0

Note: selected `eforensics` model parameter estimates (posterior means and highest posterior density credible intervals). N_i is the maximum for each i of `registered.voters` and `Voters`. Region fixed effects for turnout and vote choice are not shown. $n = 171352$ polling station units. Electors, valid votes and votes for the leader: $\sum_{i=1}^{n} N_i = 56678224$; $\sum_{i=1}^{n} V_i = 49651009$; $\sum_{i=1}^{n} W_i = 25075936$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis (Hartigan and Hartigan 1985) over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval]. [f] posterior means. [g] sums of posterior means of polling stations of each type.

Figure 2: 2017 Constitutional Referendum `eforensics`-fraudulent Votes Proportions



(a) prop: `eforensics`-frauds/leader votes

(b) prop: manufactured votes/leader votes

(c) prop: stolen votes/leader votes

(d) prop: manufactured votes/votes cast

Note: town maps of the `eforensics`-frauds proportions of either leader votes (a,b,c) or of votes cast (d). Blue means a town has zero `eforensics`-fraudulent votes. Green means that the proportion of leader votes or of votes cast that are `eforensics`-fraudulent exceeds the median value across all towns for the referent ratio. Red means the proportion exceeds the third quartile value.

2 2023 President Election

In the 2023 election for president the first round did not produce any candidate with more than fifty percent of the votes, so election day for a second round occurred on May 28, 2023. In that second round Erdoğan reportedly won with the most votes.

2.1 2023 President Election round 1

In the first round of the election data from the YSK[10] show the counts of electors and votes shown in Table 2. Notice that although the number of "invalid" votes in the 2023 president

[10]The YSK data include 191883 polling stations but only 191863 have complete elector and vote count values with a positive number of reported votes cast.

first round is greater than the number “invalid” in the 2017 constitutional referendum, as a proportion of electors the amount “invalid” is only slightly higher in 2023 than in 2017: 1021326/60735325 = .0168 versus 859957/56669068 = .0152. Turnout (votes cast/electors) is about the same in both elections: 53934143/60735325=.888 and 49651009/56669068=.876.

Table 2: 2023 President Election Round 1 Vote and Elector Totals

Contest	Candidate or Feature	Count
President		
	Recep Tayyip Erdoğan	26071379
	Muharrem İnce	226855
	Kemal Kılıçdaroğlu	23819000
	Sinan Oğan	2795583
	invalid	1021326
Eligible Voters and Cast Votes		
	Electors	60735325
	Cast Votes	53934143

Note: number of voters by candidate.

Figure 3 shows scatterplots, histograms and empirical densities for turnout and leader (Erdoğan) vote proportions. Figure 3(a) plots the original data while Figure 3(b) plots the data after removing province fixed effects. The latter plot represents the data as they are being treated in the `eforensics` estimates reported in Table 3, because that specification of the model includes province fixed effects for turnout and vote choice. As in Figure 1(b), the key feature of Figure 3(b) is that the points in the scatterplot are clumpy.

As in Table 1, the `eforensics` estimates for the president first round reported in Table 3 show strong signs that there were lost votes. Both of the diagnostic statistics for mixture probability parameter posterior MCMC multimodality give clear signals: for all three mixture probabilities the dip test for the null hypothesis of unimodality (Hartigan and Hartigan 1985) over all MCMC chains has a p-value of approximately zero, and the differences between the largest and smallest chain-specific posterior means are about as large as they can be.

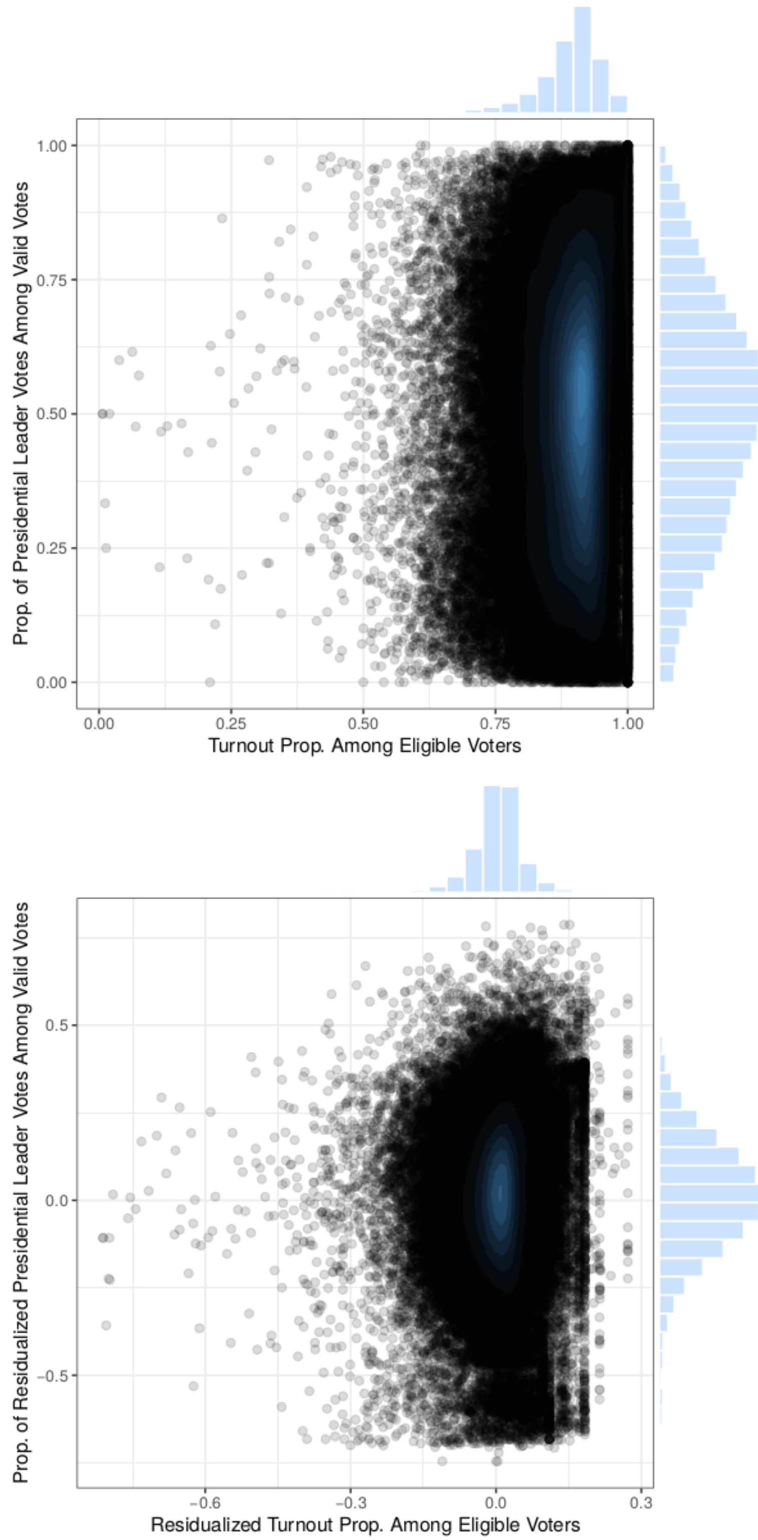
If lost votes are the reason for most of the posterior MCMC multimodalities, then as for the 2017 elections it is unclear how much the lost votes result from malevolent distortions and how much from other reasons such as natural disasters (Office for Democratic Institutions and Human Rights 2023). The multimodality may also be due to certain features of the model specification used for the current estimation.[11]

A lower proportion of polling stations have incremental frauds than π_2 might suggest: $29039/191863 = .1513 < .314$; but the 95% HPD interval for π_2—$[.0210, .441]$—is wide and does include the proportion. In contrast to the 2017 election, in the 2023 election more than five times as many polling stations have incremental frauds than have extreme frauds.

The estimated total number of `eforensics`-fraudulent votes (posterior mean $F_w = 1000425.6$) is smaller than the difference of $26071379 - 23819000 = 2252379$ between Erdoğan and second-place finisher Kılıçdaroğlu. In this case it is important to notice the extremely wide 99.5% credible interval for that total: $F_w \in [197731.4, 1334308.0]$. Kılıçdaroğlu remains behind even if stolen votes are added to his observed votes: $26071379 - 23819000 - (1000425.6 - 223726.7) = 1475680$. Because of the high degree of posterior MCMC multimodality, it is unlikely that the posterior mean is a good summary for the most representative value of F_w or of the other `eforensics`-fraud magnitude estimates. I'll return to this matter in a few weeks once I've had a chance to tweak the model specification: (6/9/2023) THE TWEAKED ANALYSIS IS DISCUSSED IN SECTION 2.2.

[11] In about three weeks I'll have estimates that use region instead of province fixed effects and that include geographic fixed effects for frauds magnitudes. In at least one election I've analyzed, the latter kind of tweak eliminated posterior multimodality, albeit milder multimodality than occurs in the current case.

Figure 3: `eforensics`-plots: 2023 President Round 1



(a) original data

(b) province-residualized data

Note: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions. For `eforensics` estimates see Table 3.

Table 3: 2023 President Election, Round 1, `eforensics` Estimates, Province Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.661	.530	.964
	π_2	Incremental Fraud	.314	.0210	.441
	π_3	Extreme Fraud	.0253	.0145	.0321
turnout	β_0	(Intercept)	1.91	1.73	1.97
vote choice	γ_0	(Intercept)	$-.236$	$-.315$	$-.0326$
incremental frauds	ρ_{M0}	(Intercept)	$-.379$	$-.786$	$-.114$
	ρ_{S0}	(Intercept)	$-.807$	$-.907$	$-.692$
extreme frauds	δ_{M0}	(Intercept)	-1.48	-2.30	$-.417$
	δ_{S0}	(Intercept)	-1.68	-2.19	$-.619$

posterior multimodality diagnostics:
all-chains dip test p-values $D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = 0$.[c]
posterior means difference $M(\pi_1) = .43$; $M(\pi_2) = .413$; $M(\pi_3) = .0166$.[d]

units `eforensics`-fraudulent: (29039 incremental, 5336 extreme, 157488 not fraudulent)
manufactured votes $F_t = 223726.7$ $[55373.8, 388497.8]^e$
incremental manufactured $F_t = 144832.1$ $[3687.2, 288555.5]^e$
extreme manufactured $F_t = 78894.6$ $[51443.3, 99990.2]^e$
total `eforensics`-fraudulent votes $F_w = 1000425.6$ $[197731.4, 1334308.0]^e$
incremental total $F_w = 690416.4$ $[13411.9, 999331.9]^e$
extreme total $F_w = 310009.2$ $[183533.9, 369993.5]^e$

Note: selected `eforensics` model parameter estimates (posterior means and highest posterior density credible intervals). N_i is the maximum for each i of `registered.voters` and `Voters`. Province fixed effects for turnout and vote choice are not shown. $n = 191863$ polling station units. Electors, valid votes and votes for the leader: $\sum_{i=1}^{n} N_i = 60732242$; $\sum_{i=1}^{n} V_i = 53934143$; $\sum_{i=1}^{n} W_i = 26071379$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis (Hartigan and Hartigan 1985) over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval]. [f] posterior means.

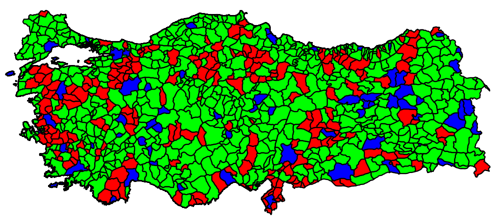
Meanwhile Figure 4 maps the proportions of votes that are `eforensics`-fraudulent by town. As in Figure 2, posterior means of polling station `eforensics`-frauds and observed totals of leader vote or of votes cast are summed by town then used to compute proportions.[12] As previously the proportions are colored in terms of their relative magnitudes for each kind of proportion: blue means a town has zero `eforensics`-fraudulent votes; green means that the proportion of leader votes or of votes cast that are `eforensics`-fraudulent exceeds the median value for the referent ratio across all towns; red means the proportion exceeds the third quartile value. Unlike in 2017, in the 2023 president first round `eforensics`-frauds occur both in Istanbul and in Izmir as they do throughout most of the country.

While the number and proportion of `eforensics`-frauds and of `eforensics`-fraudulent votes in the 2023 president election may appear to be large, even if they appear not to have determined the election outcome, it is important to keep in mind that they may not all represent malevolent distortions of elector intentions. See the discussion at the end of section 3.

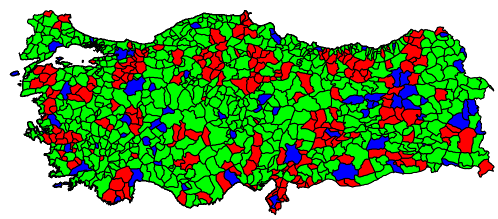
[12]The first quartile, median, third quartile and maximum values of the town proportions are as follows: F_{wj}/W_j, .0233, .0546, .0974, .408; F_{tj}/W_j, .00496, .0115, .0209, .118; $(F_{wj} - F_{tj})/W_j$, .0178, .0420, .0736, .289; F_{tj}/V_j, .00223, .00623, .0123, .101. The minimum values are zero.

Figure 4: 2023 President Round 1 `eforensics`-fraudulent Votes Proportions

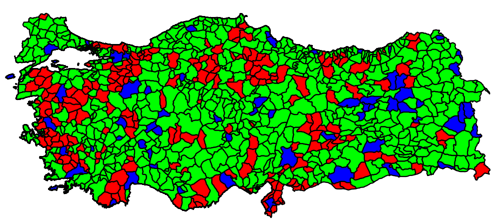
(a) prop: `eforensics`-frauds/leader votes



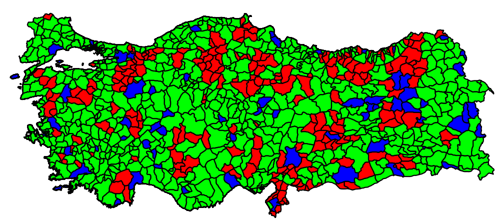
(b) prop: manufactured votes/leader votes



(c) prop: stolen votes/leader votes



(d) prop: manufactured votes/votes cast



Note: town maps of the `eforensics`-frauds proportions of either leader votes (a,b,c) or of votes cast (d). Blue means a town has zero `eforensics`-fraudulent votes. Green means that the proportion of leader votes or of votes cast that are `eforensics`-fraudulent exceeds the median value across all towns for the referent ratio. Red means the proportion exceeds the third quartile value.

2.2 2023 President Election First Round Model with `eforensics`-Frauds Magnitudes Fixed Effects

Because of the poor performance of the `eforensics` estimates reported in Table 3 for the president first round—posterior MCMC multimodality is excessive—I consider a specification that adds fixed effects for the frauds magnitudes parameters (ρ_{Mk}, ρ_{Sk}, δ_{Mk} and δ_{Sk} for regions $k = 0, \ldots, 86$).[13] The `eforensics` estimates reported in Table 4 that include such fixed effects do not convey any signs of lost votes via multimodality diagnostics, but the estimates have another unusual feature: estimates for π_1 and π_2 are essentially the same, and the number of polling stations classified as having incremental frauds (115490) greatly exceeds the number of polling stations that have no frauds (70864). For discussion of how such estimates arise and more about what they may imply, see the discussion in Section 3, which has `eforensics` estimates with similar features for the legislative election. Because the estimates in Table 4 lack posterior MCMC multimodalities they are to be preferred to those reported in Table 3.

With region fixed effects included for turnout, vote choice and `eforensics`-frauds magnitude parameters, the estimates reported in Table 4 now show that the total of the estimated `eforensics`-fraudulent votes exceeds the difference of $26071379 - 23819000 = 2252379$ between Erdoğan and second-place finisher Kılıçdaroğlu. Simply removing the posterior mean of the estimated total number of

[13] Regions in the model reported in Table 4 and along the x-axis in Figures 5 and 6 correspond to the following numbers: 0 ADANA, 1 ADIYAMAN, 2 AFYONKARAHİSAR, 3 AKSARAY, 4 AMASYA, 5 ANKARA 1, 6 ANKARA 2, 7 ANKARA 3, 8 ANTALYA, 9 ARDAHAN, 10 ARTVİN, 11 AYDIN, 12 AĞRI, 13 BALIKESİR, 14 BARTIN, 15 BATMAN, 16 BAYBURT, 17 BOLU, 18 BURDUR, 19 BURSA 1, 20 BURSA 2, 21 BİLECİK, 22 BİNGÖL, 23 BİTLİS, 24 DENİZLİ, 25 DÜZCE, 26 DİYARBAKIR, 27 EDİRNE, 28 ELAZIĞ, 29 ERZURUM, 30 ERZİNCAN, 31 ESKİEHİR, 32 GAZİANTEP, 33 GÜMÜHANE, 34 GİRESUN, 35 HAKKARİ, 36 HATAY, 37 ISPARTA, 38 IĞDIR, 39 KAHRAMANMARAŞ, 40 KARABÜK, 41 KARAMAN, 42 KARS, 43 KASTAMONU, 44 KAYSERİ, 45 KIRIKKALE, 46 KIRKLARELİ, 47 KIRŞEHİR, 48 KOCAELİ, 49 KONYA, 50 KÜTAHYA, 51 KİLİS, 52 MALATYA, 53 MANİSA, 54 MARDİN, 55 MERSİN, 56 MUĞLA, 57 MUŞ, 58 NEVŞEHİR, 59 NİĞDE, 60 ORDU, 61 OSMANİYE, 62 RİZE, 63 SAKARYA, 64 SAMSUN, 65 SİNOP, 66 SİVAS, 67 SİİRT, 68 TEKİRDAĞ, 69 TOKAT, 70 TRABZON, 71 TUNCELİ, 72 UŞAK, 73 VAN, 74 YALOVA, 75 YOZGAT, 76 ZONGULDAK, 77 ÇANAKKALE, 78 ÇANKIRI, 79 ÇORUM, 80 İSTANBUL 1, 81 İSTANBUL 2, 82 İSTANBUL 3, 83 İZMİR 1, 84 İZMİR 2, 85 ŞANLIURFA, 86 ŞIRNAK.

eforensics-fraudulent votes from the count of Erdoğan votes changes which alternative has the most votes: $2252379 - 4135522.7 = -1883144$. If manufactured votes are removed from votes cast the proportion of votes for Erdoğan remains below fifty percent:

$$\frac{26071379 - 4135522.7}{53934143 - 722326.6} = .412\,.$$

With (equation (1a)) or without (equation (1b)) manufactured votes removed, if stolen votes are added to Kılıçdaroğlu's votes then Kılıçdaroğlu has more than fifty percent:

$$\frac{23819000 + (4135522.7 - 722326.6)}{53934143 - 722326.6} = .512 \tag{1a}$$

$$\frac{23819000 + (4135522.7 - 722326.6)}{53934143} = .505\,. \tag{1b}$$

At least because including additional sets of fixed effects changes the implications from eforensics estimation, it is appropriate to examine these effects. Figure 5 shows the fixed effects from the model of Table 4 for turnout and for vote choice: these do not differ substantially from corresponding fixed effects estimated from the model of Table 3. Figure 6 shows the fixed effects from the model of Table 4 for the eforensics-frauds magnitudes parameters. Most likely the discrepantly low incremental stolen parameters for İZMİR 1 and 2 in Figure 6(b) and the discrepantly high values for ŞANLIURFA for extreme manufactured and stolen parameters in Figure 6(c,d) are the reasons the specification reported in Table 3 performed so poorly: the Normal-prior random effects in equations (2c–d) in Mebane (2022) could not well model those effects.

Table 4: 2023 President Election, Round 1, `eforensics` Estimates, Region Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.485	.485	.486
	π_2	Incremental Fraud	.485	.484	.486
	π_3	Extreme Fraud	.0296	.0284	.031
turnout	β_0	(Intercept)	1.93	1.86	1.97
vote choice	γ_0	(Intercept)	$-.422$	$-.462$	$-.398$
incremental frauds	ρ_{M0}	(Intercept)	$-.693$	$-.874$	$-.302$
	ρ_{S0}	(Intercept)	$-.590$	$-.737$	$-.533$
extreme frauds	δ_{M0}	(Intercept)	-1.18	-1.48	$-.921$
	δ_{S0}	(Intercept)	-2.35	-2.67	-1.99

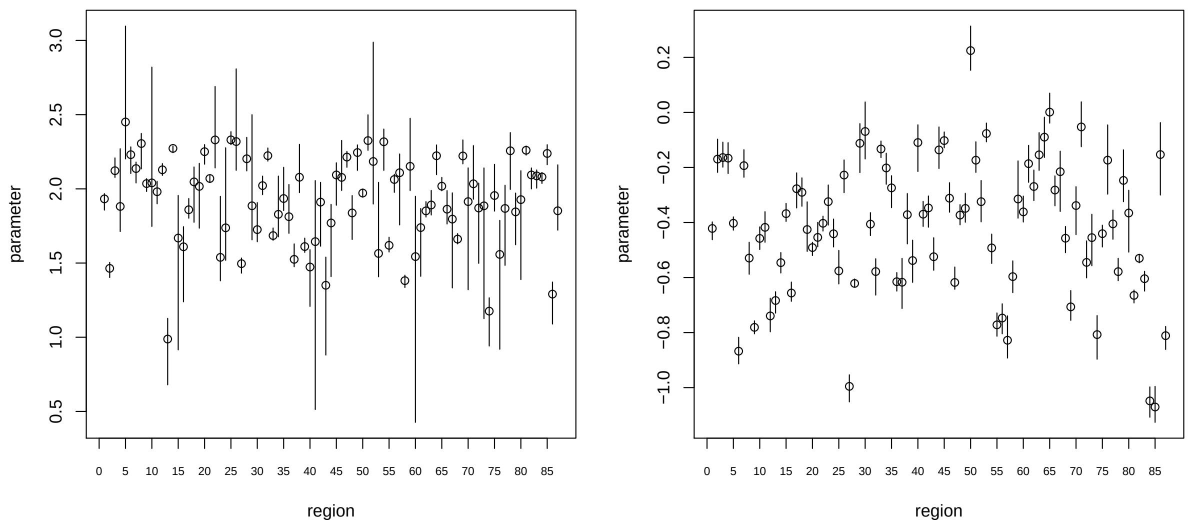
posterior multimodality diagnostics:
all-chains dip test p-values $D(\pi_1) = 1$; $D(\pi_2) = 1$; $D(\pi_3) = 1$.[c]
posterior means difference $M(\pi_1) = .000738$; $M(\pi_2) = .000629$; $M(\pi_3) = .00137$.[d]

units `eforensics`-fraudulent: (115490 incremental, 5509 extreme, 70864 not fraudulent)
manufactured votes $F_t = 722326.6$ $[579333.5, 1098266.8]$[e]
  incremental manufactured $F_t = 622028.9$ $[487886.5, 980133.8]$[e]
  extreme manufactured $F_t = 100297.7$ $[90202.5, 118283.3]$[e]
total `eforensics`-fraudulent votes $F_w = 4135522.7$ $[3958111.9, 4242876.1]$[e]
  incremental total $F_w = 3733031.9$ $[3563109.1, 3839450.1]$[e]
  extreme total $F_w = 402490.8$ $[383401.5, 414671.2]$[e]

Note: selected `eforensics` model parameter estimates (posterior means and highest posterior density credible intervals). N_i is the maximum for each i of `registered.voters` and `Voters`. Province fixed effects for turnout, vote choice and `eforensics`-frauds magnitudes are not shown (see Figures 5 and 6). $n = 191863$ polling station units. Electors, valid votes and votes for the leader: $\sum_{i=1}^{n} N_i = 60732242$; $\sum_{i=1}^{n} V_i = 53934143$; $\sum_{i=1}^{n} W_i = 26071379$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis (Hartigan and Hartigan 1985) over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval]. [f] posterior means.

Figure 5: 2023 President round 1: Turnout and Vote Choice Fixed Effect Parameters

(a) turnout (β_0 to β_{86}) (b) vote choice (γ_0 to γ_{86})



Note: fixed effects parameters (posterior means and 95% HPD intervals) for turnout (β_0 to β_{86}) and vote choice (γ_0 to γ_{86}) parameters in the `eforensics` model reported in Table 4. See note 13 for the regions that correspond to the "region" numbers along the x-axis in each plot.

As I discuss further at the end of Section 3, the incremental frauds estimated by the model of Table 4 should be viewed as partially stimulated by elector strategic behavior: all the incremental frauds magnitude parameters shown in Figure 6(a,b) are negative.[14] Estimates from many other countries' elections suggest that when `eforensics`-frauds are being stimulated by strategic behavior then the incremental frauds magnitude parameters are negative. Incremental fraud manufactured and stolen votes may include some proportion of `eforensics`-frauds that result from malevolent distortions of elector intentions and some proportion that stems from strategic behavior. We cannot say what proportion is produced by which cause.

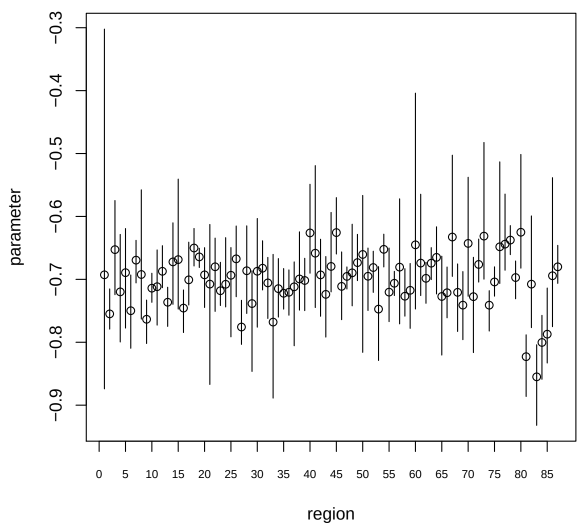
The vote counts reported in Table 2 are in line with what one should see if strategic coordination prompted most votes to go to the top two finishers while the remaining

[14]In Table 3 estimates for both ρ_{M0} and ρ_{S0} are negative.

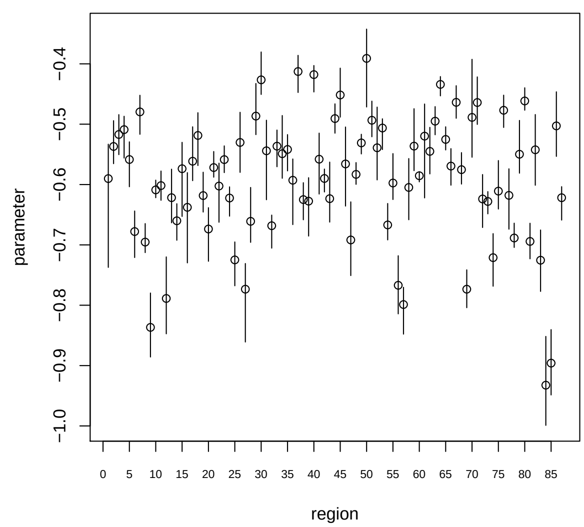
candidates obtained a very small residual of the votes. The incremental frauds magnitude parameters are compatible with such strategic behavior, indeed the estimates for the president election are more in line with such an interpretation than are ρ_{M0} and ρ_{S0} for the 2023 legislative election reported in Section 3 due to the 95% HPD interval for ρ_{M0} in Table 9 not being strictly negative.

Figure 6: 2023 President round 1: `eforensics`-frauds Magnitude Fixed Effect Parameters

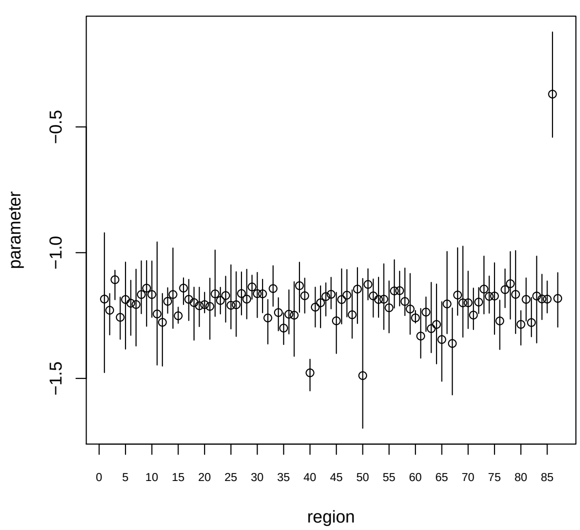
(a) incremental manufactured
ρ_{M0} to ρ_{M86}



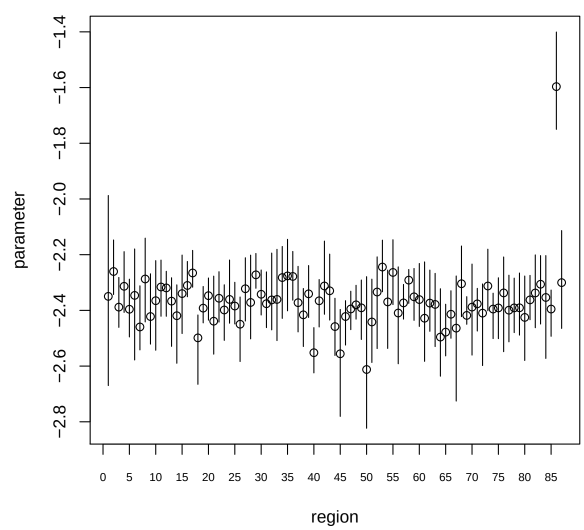
(b) incremental stolen
ρ_{S0} to ρ_{S86}



(c) extreme manufactured
δ_{M0} to δ_{M86}



(d) extreme stolen
δ_{S0} to δ_{S86}



Note: active fixed effects parameters (posterior means and 95% HPD intervals) for frauds magnitude (ρ_{M0} to ρ_{M86}, ρ_{S0} to ρ_{S86}, δ_{M0} to δ_{M86} and δ_{S0} to δ_{S86}) parameters in the `eforensics` model reported in Table 4. See note 13 for the regions that correspond to the "region" numbers along the x-axis in each plot.

2.3 2023 President Election round 2

In the 2023 election for president the second round occurred on May 28, 2023, with Erdoğan receiving more votes than Kılıçdaroğlu and winning the election with a margin of 1962502 votes, as detailed in Table 5. Comparing the first and second rounds (Tables 2 and 5) noteworthy is that there are more votes cast (including invalid votes) in the first round (53934143) than in the second round (52093374), even though our data report a higher number of electors in the second round (60837492) than in the first round (60735325).

Table 5: 2023 President Election Round 2 Vote and Elector Totals

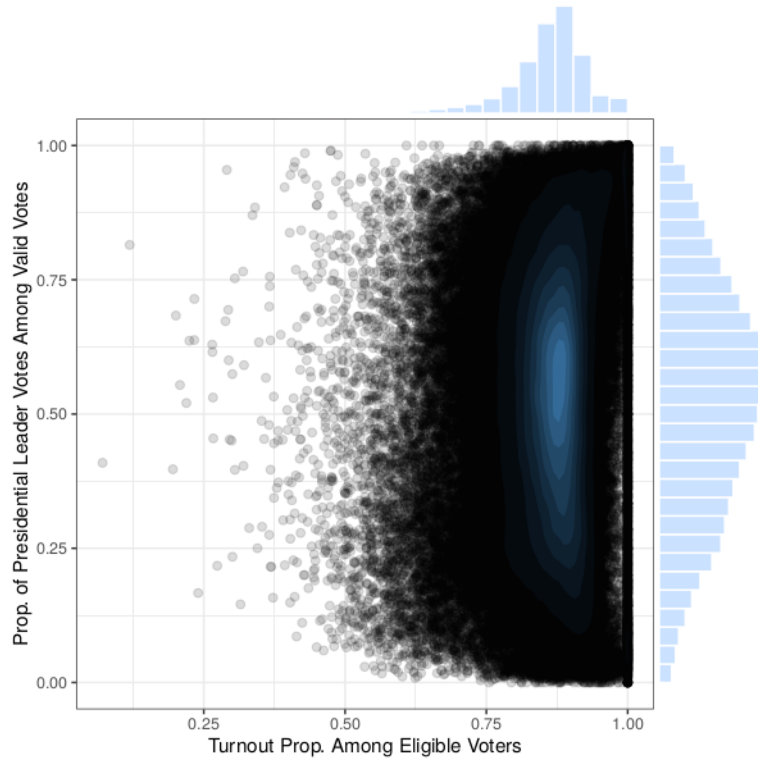
Contest	Candidate or Feature	Count
President		
	Recep Tayyip Erdoğan	26690529
	Kemal Kılıçdaroğlu	24728027
	invalid	674818
Eligible Voters and Cast Votes		
	Electors	60837492
	Cast Votes	52093374

Note: number of voters by candidate.

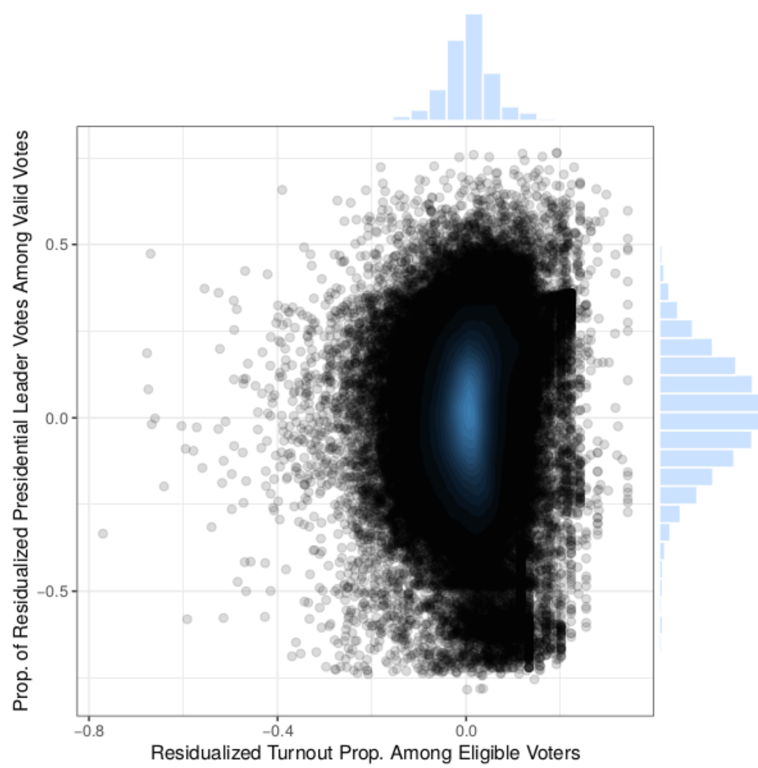
Figure 7(a) reinforces what the cast votes total in Table 5 reports, which is that despite the existence of a compulsory voting requirement not everyone participates. Indeed comparing Figure 7(a) to Figure 3(a) shows that the tail of polling stations with low turnout is greater in the second round than in the first round: the proportion of polling stations with turnout below .6 is .00294 in round 1 but .008162 in round 2.

Figure 7: `eforensics`-plots: 2023 President Round 2

(a) original data



(b) province-residualized data



Note: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions. For `eforensics` estimates see Table 6.

Even with region fixed effects included not only for the turnout and vote choice parameters but also for the frauds magnitudes parameters, as in the specification used to produce the first round estimates reported in Table 4, the second round `eforensics` estimates reported in Table 6 exhibit posterior MCMC multimodality diagnostics that provide strong signals that there are lost votes: $D(\pi_2) = 0$ and $M(\pi_2) = .343$. That such signals occur in the second round but not in the first round when the same kinds of region fixed effects are used in both `eforensics` model specifications reinforces the message from the just-discussed apparent decline in the number and proportion of votes cast: not only did participation decline between election rounds, but the decline probably occurred asymmetrically more among electors who were inclined to have supported one of the two candidates had those electors had their votes recorded (for further discussion of some of the nuances of `eforensics` and lost votes see Mebane 2023). Notwithstanding the evidence that there are lost votes, both Erdoğan and Kılıçdaroğlu have higher proportions of the votes cast in the second round (.512 and .475) than in the first round (.483 and .442), with Kılıçdaroğlu gaining proportionally more votes (.0331 versus .0290).

The `eforensics` estimates reported for the second round in Table 6 show that the estimated total of the `eforensics`-fraudulent votes has a posterior mean that is less than the margin of 1962502 votes between Erdoğan and Kılıçdaroğlu, but the margin is less than the upper bound of the 99.5% credible interval: $F_w = 1286255.9\ [468231.8, 1978951.3]$; if estimated `eforensics`-fraudulent votes are subtracted from the observed vote total for leader Erdoğan, and nothing else changes, then there is some chance that Kılıçdaroğlu has more votes than does Erdoğan in the second round. So the `eforensics` estimates for the second round do not suggest that the election had the wrong outcome as strongly as do the estimates for the first round (recall Table 4), but such a possibility is included if the statistical uncertainty in the estimates is appropriately taken into account.

Table 6: 2023 President Election, Round 2, `eforensics` Estimates, Region Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.658	.523	.882
	π_2	Incremental Fraud	.308	.0846	.443
	π_3	Extreme Fraud	.0340	.033	.0349
turnout	β_0	(Intercept)	1.54	1.21	1.75
vote choice	γ_0	(Intercept)	$-.130$	$-.244$	.0212
incremental frauds	ρ_{M0}	(Intercept)	$-.490$	$-.971$	.0111
	ρ_{S0}	(Intercept)	$-.662$	$-.812$	$-.525$
extreme frauds	δ_{M0}	(Intercept)	-1.14	-1.58	$-.554$
	δ_{S0}	(Intercept)	-1.66	-2.05	-1.22

posterior multimodality diagnostics:
all-chains dip test p-values $D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = 1$.[c]
posterior means difference $M(\pi_1) = .343$; $M(\pi_2) = .343$; $M(\pi_3) = .000581$.[d]

units `eforensics`-fraudulent: (34761 incremental, 34761 extreme, 150645 not fraudulent)
manufactured votes $F_t = 473335.3$ $[146542.4, 846808.0]$[e]
incremental manufactured $F_t = 311639.8$ $[22090.1, 647193.7]$[e]
extreme manufactured $F_t = 161695.5$ $[124442.1, 200985.9]$[e]
total `eforensics`-fraudulent votes $F_w = 1286255.9$ $[468231.8, 1978951.3]$[e]
incremental total $F_w = 839263.6$ $[84142.6, 1488955.3]$[e]
extreme total $F_w = 446992.3$ $[384193.1, 491731.2]$[e]

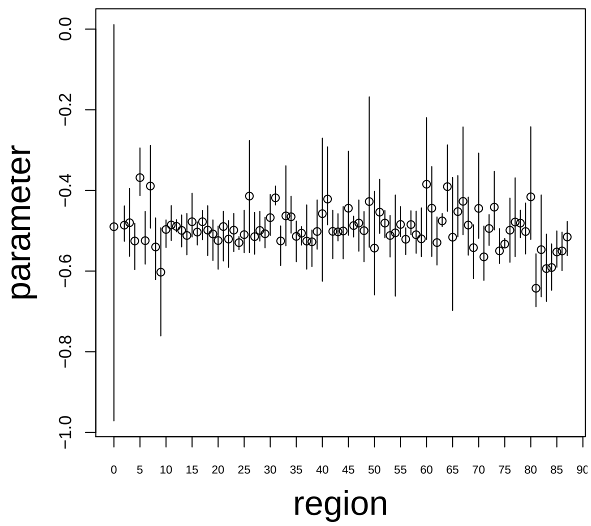
Note: selected `eforensics` model parameter estimates (posterior means and highest posterior density credible intervals). N_i is the maximum for each i of `registered.voters` and `Voters`. Province fixed effects for turnout, vote choice and `eforensics`-frauds magnitudes are not shown (see Figure 8). $n = 192214$ polling station units. Electors, valid votes and votes for the leader: $\sum_{i=1}^{n} N_i = 60837492$; $\sum_{i=1}^{n} V_i = 52093374$; $\sum_{i=1}^{n} W_i = 26690529$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis (Hartigan and Hartigan 1985) over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval]. [f] posterior means.

Figure 8 displays the frauds magnitude fixed effects that are "active" in the sense that the effect's region includes at least one polling station that is classified by the model of Table 6 as `eforensics`-fraudulent. Comparing these fixed effects to the analogous active fixed effects displayed in Figure 6, we see that the second round estimated `eforensics`-frauds differ from those in the first round. While `eforensics`-frauds occur in almost all regions in the first round, `eforensics`-frauds are even more widespread in the second round. Even though in the first round there are more `eforensics`-fraudulent polling stations and `eforensics`-fraudulent votes (recall Table 4), in the second round the polling stations that are `eforensics`-fraudulent are somewhat more widely dispersed across the entire country: in the second round every region includes at least one polling station that has incremental `eforensics`-frauds and at least one that has extreme `eforensics`-frauds.[15]

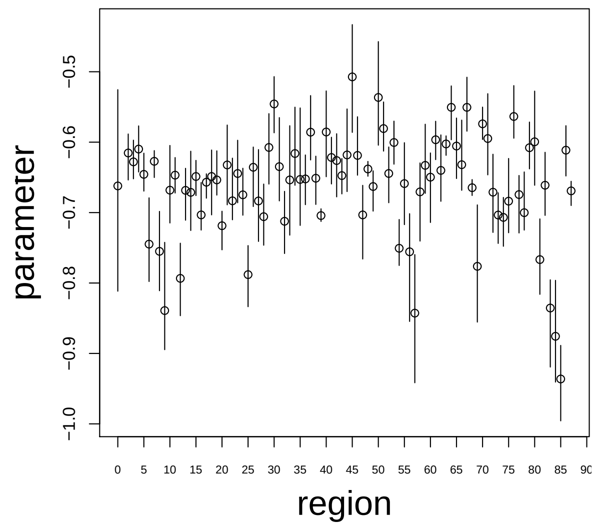
[15]The region that appears with especially high extreme frauds fixed effects in Figures 8(c,d) is ŞANLIURFA.

Figure 8: 2023 President round 2: `eforensics`-frauds Magnitude Fixed Effect Parameters

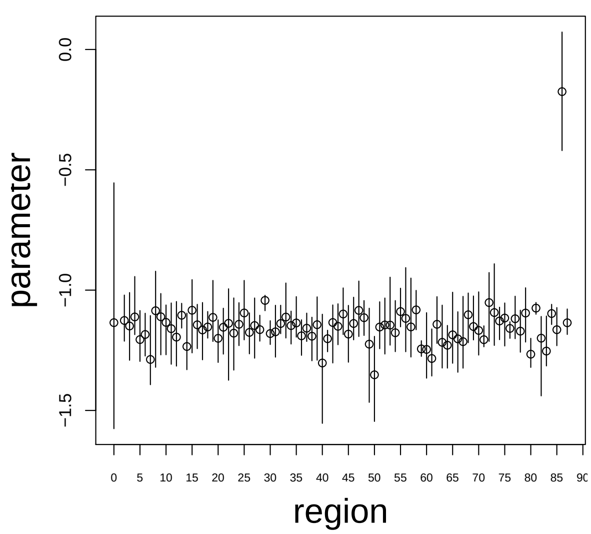
(a) incremental manufactured
ρ_{M0} to ρ_{M86}



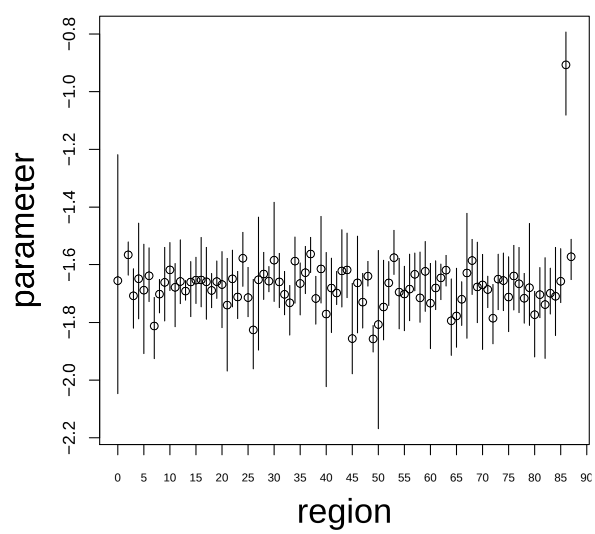
(b) incremental stolen
ρ_{S0} to ρ_{S86}



(c) extreme manufactured
δ_{M0} to δ_{M86}



(d) extreme stolen
δ_{S0} to δ_{S86}



Note: active fixed effects parameters (posterior means and 95% HPD intervals) for frauds magnitude (ρ_{M0} to ρ_{M86}, ρ_{S0} to ρ_{S86}, δ_{M0} to δ_{M86} and δ_{S0} to δ_{S86}) parameters in the `eforensics` model reported in Table 6. See note 13 for the regions that correspond to the "region" numbers along the x-axis in each plot.

3 2023 Legislative Election

In the 2023 legislative election at least the named entities shown in Table 7 received votes: the displayed names appear in the YSK data I have available to analyze;[16] the names with the smallest vote totals appear to be independent individual candidates, while the other names are for political parties. The vote totals accumulate votes received throughout the country. Some parties like AKP, which has the largest vote total, compete essentially everywhere throughout the country, while other parties have more limited scope. In the elections seats in the legislature are allocated "by the D'Hondt method, a party-list proportional representation system, from 87 electoral districts which represent the 81 administrative provinces of Turkey (Istanbul and Ankara are divided into three electoral districts whereas İzmir and Bursa are divided into two each because of its large populations)."[17] The "regions" I use in the current analysis are the "electoral districts" referenced in the preceding statement. In addition to passing a 7% threshold, "parties must be officially organised in at least half of provinces (41 or more) and in at least a third of districts in those provinces, and must nominate two candidates in 41 or more provinces, in order to be entitled to seats."[18] Other election rules address alliances between parties and other details. The various details of the election rules pave the way for electors to benefit from behaving strategically, that is, by attending to their expectations about what other electors are likely to do.

Figure 9 shows scatterplots, histograms and empirical densities for turnout and leader vote proportions. The leader for the plots and for the `eforensics` analysis reported below is the party that has the most votes in each region. Table 8 lists these parties along with the votes the party received in each region according to the YSK data. Figure 9(a) plots the original data while Figure 9(b) plots the data after removing region fixed effects. The

[16]The YSK data include 191885 polling stations but only 191875 have complete elector and vote count values with a positive number of reported votes cast.

[17]`https://en.wikipedia.org/wiki/Grand_National_Assembly_of_Turkey`

[18]`https://en.wikipedia.org/wiki/2018_Turkish_parliamentary_election`

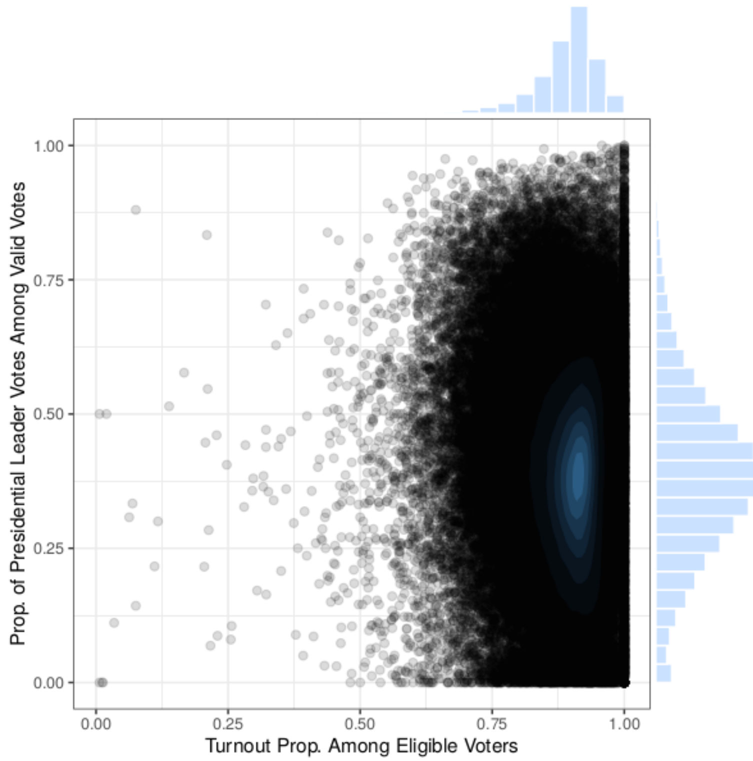
Table 7: 2023 Legislative: Parties and National Vote Totals

party	votes	party	votes
Adalet Ve Kalkınma Partisi (AKP)	18361192	Millet Ittifakı	174191
Cumhuriyet Halk Partisi (CHP)	13221404	Genç Parti	111089
Milliyetçi Hareket Partisi	5218537	Adalet Partisi	107036
Iyi Parti	5151006	Sol Parti	73887
Yeşil Sol Parti (YSP)	4604045	Ana Vatan Partisi	64927
Yeniden Refah Partisi	1495203	Türkiye Komünist Partisi	60601
Zafer Partisi	1193045	Vatan Partisi	54529
Türkiye Işçi Partisi	901993	Halkın Kurtuluş Partisi	31463
Büyük Birlik Partisi	524576	Türkiye Komünist Hareketi	17209
Memleket Partisi	487657	Ata Ittifakı	5024
Cum Hur Ittifakı	284951	Emek Özgürlük Ittifakı	4906
Bağımsız 10 Aday Toplamı	226739	Sosyalist Güçbirliği	2420
Diğer 6 Parti Toplamı	192070		

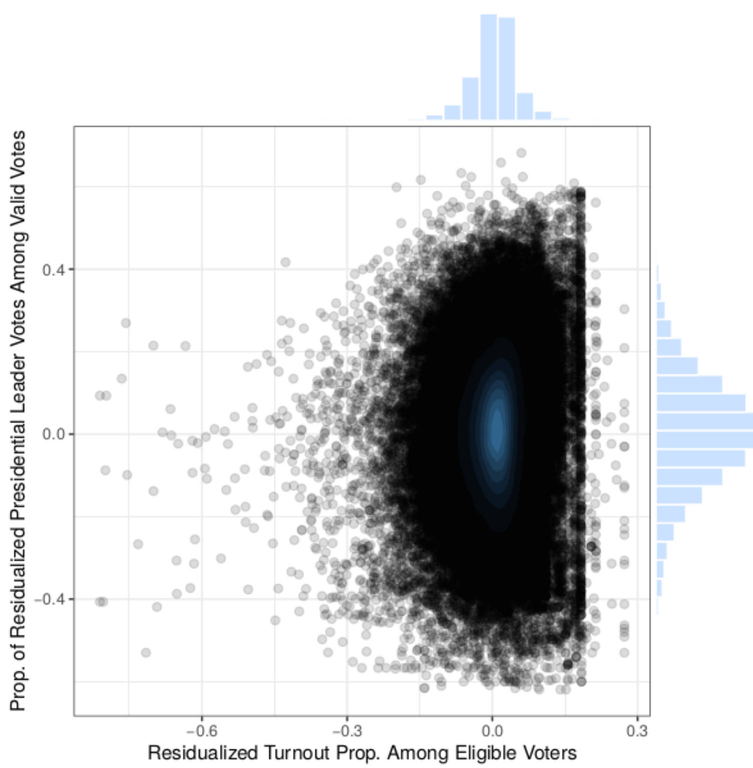
latter plot represents the data as they are being treated in the `eforensics` estimates reported in Table 9, because that specification of the model includes region fixed effects for turnout and vote choice. As in Figure 3(b), the key feature of Figure 9(b) is that the points in the scatterplot are clumpy.

Figure 9: `eforensics`-plots: 2023 Legislative

(a) original data



(b) region-residualized data



Note: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions. For `eforensics` estimates see Table 9.

Table 8: 2023 Legislative: Region Leader Parties

region	party	votes	region	party	votes	region	party	votes
ADANA	AKP	412304	DÜZCE	AKP	130470	KONYA	AKP	673639
ADIYAMAN	AKP	151965	EDİRNE	CHP	110546	KÜTAHYA	AKP	178157
AFYONKARAHİSAR	AKP	205730	ELAZIĞ	AKP	141651	MALATYA	AKP	190082
AĞRI	YSP	118264	ERZİNCAN	AKP	57388	MANİSA	AKP	304602
AKSARAY	AKP	108377	ERZURUM	AKP	183547	MARDİN	YSP	242245
AMASYA	AKP	89925	ESKİEHİR	CHP	208981	MERSİN	CHP	294739
ANKARA 1	CHP	576376	GAZİANTEP	AKP	502307	MUĞLA	CHP	266042
ANKARA 2	AKP	472945	GİRESUN	AKP	127376	MUŞ	YSP	94524
ANKARA 3	AKP	418596	GÜMÜHANE	AKP	36671	NEVŞEHİR	AKP	77378
ANTALYA	CHP	535045	HAKKARİ	YSP	94846	NİĞDE	AKP	73250
ARDAHAN	AKP	18501	HATAY	AKP	285957	ORDU	AKP	223112
ARTVİN	AKP	40304	IĞDIR	YSP	45314	OSMANİYE	AKP	106933
AYDIN	CHP	272580	ISPARTA	AKP	90844	RİZE	AKP	123264
BALIKESİR	AKP	299426	İSTANBUL 1	AKP	1280924	SAKARYA	AKP	330506
BARTIN	AKP	47448	İSTANBUL 2	AKP	1248981	SAMSUN	AKP	372383
BATMAN	YSP	192439	İSTANBUL 3	AKP	1035807	ŞANLIURFA	AKP	405797
BAYBURT	AKP	29233	İZMİR 1	CHP	624013	SİİRT	YSP	76603
BİLECİK	AKP	54896	İZMİR 2	CHP	624466	SİNOP	AKP	60843
BİNGÖL	AKP	55362	KAHRAMANMARAŞ	AKP	296230	ŞIRNAK	YSP	169861
BİTLİS	YSP	70567	KARABÜK	AKP	58442	SİVAS	AKP	161232
BOLU	AKP	76229	KARAMAN	AKP	65995	TEKİRDAĞ	CHP	274606
BURDUR	AKP	64311	KARS	YSP	40175	TOKAT	AKP	143116
BURSA 1	AKP	383852	KASTAMONU	AKP	112604	TRABZON	AKP	252698
BURSA 2	AKP	479180	KAYSERİ	AKP	366991	TUNCELİ	YSP	24565
ÇANAKKALE	CHP	137810	KİLİS	AKP	32669	UŞAK	AKP	87633
ÇANKIRI	AKP	51426	KIRIKKALE	AKP	61299	VAN	YSP	286401
ÇORUM	AKP	139987	KIRKLARELİ	CHP	118660	YALOVA	AKP	57854
DENİZLİ	AKP	235740	KIRŞEHİR	AKP	56591	YOZGAT	AKP	109408
DİYARBAKIR	YSP	491169	KOCAELİ	AKP	530992	ZONGULDAK	AKP	154942

The `eforensics` estimates for the legislative election reported in Table 9 do not convey any signs of lost votes via multimodality MCMC diagnostics, but the estimates feature another feature that is unusual but not unexampled among elections I have analyzed: estimates for π_1 and π_2 are essentially the same, and the number of polling stations classified as having incremental frauds (122609) greatly exceeds the number of polling stations that have no frauds (68433). Mechanically this result can occur because the prior for the mixture probabilities (Mebane 2022, 5) means π_1 must be weakly larger than either of the other two mixture probabilities,[19] but via the Metropolis-Hastings algorithm (Mebane 2022, 7) the `eforensics` estimator can produce values $Z_i = 2$ more frequently than π_2 would suggest. Estimates with this characteristic appear for elections in a few other countries, and as I'll show in section 3.1 they also appear for other Turkish elections.

The estimated total number of `eforensics`-fraudulent votes in the legislative election (posterior mean $F_w = 3777994.9$) is more than three times larger than the number observed in the contemporaneous election for president ($F_w = 1000425.6$). Almost three-quarters of the legislative `eforensics`-fraudulent votes are stolen ($F_w - F_t = 2787018$ and $2787018/3777994.9 = .738$). Figure 10 illustrates in which regions the number of `eforensics`-fraudulent votes exceeds the margin in the region, the margin being the difference between the votes for the leader and the votes for the second-place party.[20] Table 10 lists the elector, vote and `eforensics`-frauds values for the 36 exceeding regions.

[19] This prior is intended to discourage label switching.

[20] To match the numbers used to represent regions in the figure, a list of the regions with matching numbers follows: 1 ADANA, 2 ADIYAMAN, 3 AFYONKARAHİSAR, 4 AĞRI, 5 AKSARAY, 6 AMASYA, 7 ANKARA 1, 8 ANKARA 2, 9 ANKARA 3, 10 ANTALYA, 11 ARDAHAN, 12 ARTVİN, 13 AYDIN, 14 BALIKESİR, 15 BARTIN, 16 BATMAN, 17 BAYBURT, 18 BİLECİK, 19 BİNGÖL, 20 BİTLİS, 21 BOLU, 22 BURDUR, 23 BURSA 1, 24 BURSA 2, 25 ÇANAKKALE, 26 ÇANKIRI, 27 ÇORUM, 28 DENİZLİ, 29 DİYARBAKIR, 30 DÜZCE, 31 EDİRNE, 32 ELAZIĞ, 33 ERZİNCAN, 34 ERZURUM, 35 ESKİEHİR, 36 GAZİANTEP, 37 GİRESUN, 38 GÜMÜHANE, 39 HAKKARİ, 40 HATAY, 41 IĞDIR, 42 ISPARTA, 43 İSTANBUL 1, 44 İSTANBUL 2, 45 İSTANBUL 3, 46 İZMİR 1, 47 İZMİR 2, 48 KAHRAMANMARAŞ, 49 KARABÜK, 50 KARAMAN, 51 KARS, 52 KASTAMONU, 53 KAYSERİ, 54 KİLİS, 55 KIRIKKALE, 56 KIRKLARELİ, 57 KIRŞEHİR, 58 KOCAELİ, 59 KONYA, 60 KÜTAHYA, 61 MALATYA, 62 MANİSA, 63 MARDİN, 64 MERSİN, 65 MUĞLA, 66 MUŞ, 67 NEVŞEHİR, 68 NİĞDE, 69 ORDU, 70 OSMANİYE, 71 RİZE, 72 SAKARYA, 73 SAMSUN, 74 ŞANLIURFA, 75 SİİRT, 76 SİNOP, 77 ŞIRNAK, 78 SİVAS, 79 TEKİRDAĞ, 80 TOKAT, 81 TRABZON, 82 TUNCELİ, 83 UŞAK, 84 VAN, 85 YALOVA, 86 YOZGAT, 87 ZONGULDAK.

Table 9: 2023 Legislative Election, `eforensics` Estimates, Region Fixed Effects

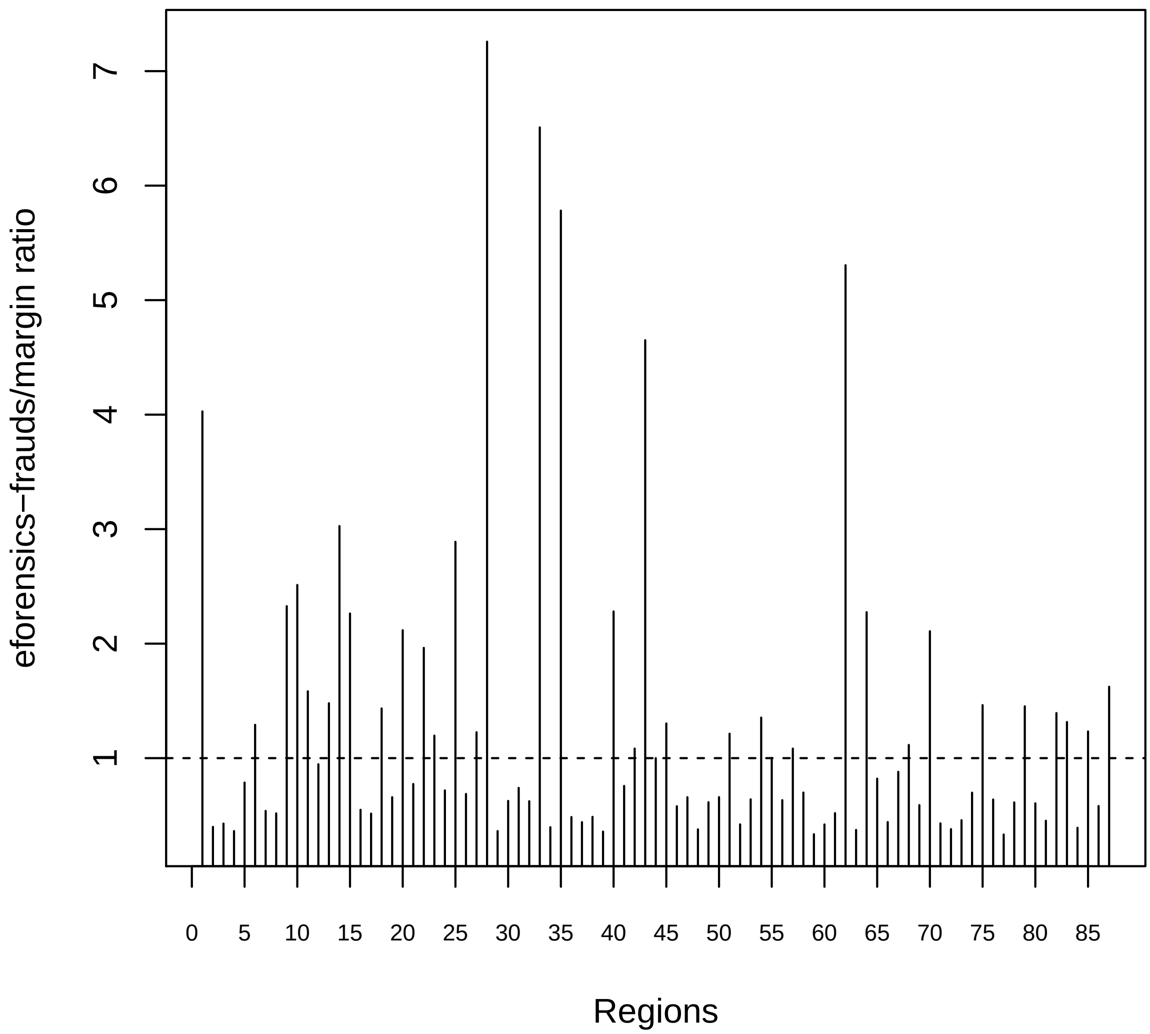
Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.498	.497	.498
	π_2	Incremental Fraud	.498	.497	.498
	π_3	Extreme Fraud	.00460	.00417	.00506
turnout	β_0	(Intercept)	1.73	1.53	1.99
vote choice	γ_0	(Intercept)	$-.866$	$-.885$	$-.838$
incremental frauds	ρ_{M0}	(Intercept)	$-.357$	$-.929$	.000908
	ρ_{S0}	(Intercept)	-1.08	-1.18	$-.963$
extreme frauds	δ_{M0}	(Intercept)	$-.708$	-1.23	$-.477$
	δ_{S0}	(Intercept)	-1.07	-1.52	$-.546$

posterior multimodality diagnostics:
all-chains dip test p-values $D(\pi_1) = .895$; $D(\pi_2) = .754$; $D(\pi_3) = 1.$[c]
posterior means difference $M(\pi_1) = .000183$; $M(\pi_2) = .000178$; $M(\pi_3) = .000358.$[d]

units `eforensics`-fraudulent: (122609 incremental, 833 extreme, 68433 not fraudulent)
manufactured votes $F_t = 990976.9$ $[521836.3, 1385848.4]$[e]
incremental manufactured $F_t = 972524.9$ $[506032.1, 1363873.5]$[e]
extreme manufactured $F_t = 18452.0$ $[15749.4, 22216.2]$[e]
total `eforensics`-fraudulent votes $F_w = 3777994.9$ $[3495691.5, 4039603.7]$[e]
incremental total $F_w = 3709843.9$ $[3433777.6, 3970061.3]$[e]
extreme total $F_w = 68151.0$ $[61495.2, 73286.9]$[e]

Note: selected `eforensics` model parameter estimates (posterior means and highest posterior density credible intervals). N_i is the maximum for each i of `registered.voters` and `Voters`. Region fixed effects for turnout and vote choice are not shown. $n = 191875$ polling station units. Electors, valid votes and votes for the leader: $\sum_{i=1}^{n} N_i = 60731989$; $\sum_{i=1}^{n} V_i = 53934513$; $\sum_{i=1}^{n} W_i = 20617139$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis (Hartigan and Hartigan 1985) over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval]. [f] posterior means. [g] sums of posterior means of polling stations of each type.

Figure 10: 2023 Legislative: Ratio between `eforensics`-frauds and Region Margin



Note: ratio of `eforensics`-fraudulent votes from the specification of Table 9 to the margin (additive difference) between the votes for the first-place and second-place parties in each region.
Regions for which `eforensics`-fraudulent votes that exceed the margin are 1 ADANA, 6 AMASYA, 9 ANKARA 3, 10 ANTALYA, 11 ARDAHAN, 13 AYDIN, 14 BALIKESİR, 15 BARTIN, 18 BİLECİK, 20 BİTLİS, 22 BURDUR, 23 BURSA 1, 25 ÇANAKKALE, 27 ÇORUM, 28 DENİZLİ, 33 ERZİNCAN, 35 ESKİEHİR, 40 HATAY, 42 ISPARTA, 43 İSTANBUL 1, 44 İSTANBUL 2, 45 İSTANBUL 3, 51 KARS, 54 KİLİS, 55 KIRIKKALE, 57 KIRŞEHİR, 62 MANİSA, 64 MERSİN, 68 NİĞDE, 70 OSMANİYE, 75 SİİRT, 79 TEKİRDAĞ, 82 TUNCELİ, 83 UŞAK, 85 YALOVA, 87 ZONGULDAK.

Table 10: 2023 Legislative: Regions where `eforensics`-fraudulent Votes Exceed the Margin between First and Second

	electors	votes cast	leader votes	second votes	margin	`eforensics` -frauds
1 ADANA	1613435	1412854	412304	391499	20805	83806.6
6 AMASYA	256804	236480	89925	68270	21655	27956.0
9 ANKARA 3	1470114	1350474	418596	367623	50973	118630.4
10 ANTALYA	1905655	1702383	535045	467921	67124	168651.4
11 ARDAHAN	68072	56297	18501	15942	2559	4053.1
13 AYDIN	876902	788114	272580	213590	58990	87298.4
14 BALIKESİR	989801	902238	299426	275485	23941	72468.3
15 BARTIN	156137	137888	47448	41644	5804	13137.1
18 BİLECİK	167149	153631	54896	39736	15160	21740.1
20 BİTLİS	220856	180602	70567	63804	6763	14319.2
22 BURDUR	204315	185340	64311	55447	8864	17401.7
23 BURSA 1	1220630	1113906	383852	293734	90118	107853.4
25 ÇANAKKALE	436195	399579	137810	122734	15076	43557.8
27 ÇORUM	398845	361980	139987	109480	30507	37406.8
28 DENİZLİ	794820	728103	235740	224997	10743	77965.0
33 ERZİNCAN	170127	153435	57388	54666	2722	17717.3
35 ESKİEHİR	689158	625799	208981	199250	9731	56265.1
40 HATAY	1062595	883171	285957	244722	41235	94067.3
42 ISPARTA	328226	294308	90844	62696	28148	30494.4
43 İSTANBUL 1	4201450	3836427	1280924	1191382	89542	416351.3
44 İSTANBUL 2	3113426	2805354	1035807	724265	311542	311545.3
45 İSTANBUL 3	4043692	3639749	1248981	927216	321765	419300.3
51 KARS	185133	147255	40175	35033	5142	6240.6
54 KİLİS	97074	86118	32669	22586	10083	13653.0
55 KIRIKKALE	203073	180554	61299	46557	14742	14765.0
57 KIRŞEHİR	173988	152227	56591	43062	13529	14653.1
62 MANİSA	1106611	1017537	304602	288508	16094	85381.0
64 MERSİN	1178910	1054671	294739	259112	35627	81040.5
68 NİĞDE	258127	227798	73250	53064	20186	22512.6
70 OSMANİYE	386187	338727	106933	94709	12224	25774.7
75 SİİRT	202571	168094	76603	56602	20001	29264.6
79 TEKİRDAĞ	857623	776087	274606	227339	47267	68677.9
82 TUNCELİ	65878	57226	24565	17993	6572	9159.7
83 UŞAK	280503	255208	87633	71553	16080	21150.1
85 YALOVA	204818	178464	57854	49463	8391	10350.3
87 ZONGULDAK	457673	408629	154942	125458	29484	47882.5

While the large number and high proportion of `eforensics`-frauds and of `eforensics`-fraudulent votes certainly grab one's attention, it is important to recall that `eforensics` responds not only to bad acts and lost votes but also to strategic elector behavior. The rules for gaining seats from the legislative election provide strong reasons for candidates, parties and electors to coordinate in order to win seats. Complex patterns of wasted-vote considerations on top of mobilization incentives likely exist. The best interpretation of the 2023 legislative election `eforensics` estimates is that they heavily convey consequences of electors' strategic behavior.

Important indicators for this are the signs of the incremental frauds magnitude parameters ρ_{M0} and ρ_{S0}. About 98% of the stolen votes in Table 9 are incremental frauds. Estimates from many other countries' elections (e.g., Germany, see Mebane 2022) suggest that when `eforensics`-frauds are being stimulated by strategic behavior then ρ_{M0} and ρ_{S0} are both negative. In Table 9 ρ_{S0} is clearly negative, while ρ_{M0} is negative except that the upper bound of its 95% HPD interval is very slightly positive. A reasonable interpretation of that estimate is that incremental fraud manufactured votes may include some proportion of `eforensics`-frauds that result from malevolent distortions of elector intentions, while the incremental fraud manufactured votes also stem in part from strategic behavior. We cannot say what proportion is produced by which cause.

3.1 Previous Legislative Elections

`eforensics` estimates for the legislative elections of 1999, 2002, 2007, 2011 and 2015 find several with features like those in the 2023 elections, while others differ.[21]

Table 11 gathers the mixture probability parameter estimates from `eforensics` models for each election. In the models the leader is the party with the most votes in each region, and there are region fixed effects in the turnout and vote choice equations (equations (2a–b) in Mebane 2022), as in the specification reported in Table 9. The parameters in

[21] The 1999, 2002, 2007, 2011 and 2015 specifications do not include "invalid votes" among votes cast.

1999 and 2011 strongly resemble those for 2023 in that π_1 is roughly equal to π_2. Unlike 1999 and 2023, 2011 has a noticeably larger value of π_3. In other elections except for November 2015, π_1 is much greater than π_2; in November 2015 π_1 is somewhat greater. In all these years the 95% HPD intervals are wide.

Table 11: 1999–2015 `eforensics` Mixture Probability Estimates

		1999			2002	
Parameter	Mean	lo[a]	up[b]	Mean	lo[a]	up[b]
π_1	.499	.498	.500	.833	.665	.954
π_2	.498	.498	.499	.163	.0442	.330
π_3	.00291	.00256	.00345	.00384	.00207	.00510

		2007			2011	
Parameter	Mean	lo[a]	up[b]	Mean	lo[a]	up[b]
π_1	.647	.601	.685	.485	.484	.487
π_2	.332	.295	.378	.485	.484	.486
π_3	.0215	.0202	.0227	.0294	.0270	.0312

		June 2015			November 2015	
Parameter	Mean	lo[a]	up[b]	Mean	lo[a]	up[b]
π_1	.748	.508	.896	.574	.477	.861
π_2	.223	.0754	.464	.386	.110	.478
π_3	.0282	.0252	.0305	.0403	.0285	.0451

Table 12 shows that the wide HPD intervals stem from posterior MCMC multimodality in the mixture probabilities. The chain-specific posterior means differ substantially for every election except 1999 and 2011, and the all-chains dip tests are significant for all bu two of the mixture probabilities. So all the elections show symptoms of lost votes.

Table 13 shows that the number of `eforensics`-fraudulent polling stations is greater than the number of polling stations that have no frauds in both 1999 and 2011. As in 2023, it is the number of polling stations that have incremental frauds that is exceptionally large. At least in 2011, the way candidates tended to disassociate from party labels may have increased the degree to which electors had to coordinate independent of party labels. The number of polling stations that have extreme frauds increases over time except for the June 2015 election, in which the number decreases compared to the previous election in 2011.

Table 12: 1999–2015 `eforensics` Frauds Magnitude Parameter Estimates

election	all-chains dip test p-values
1999	$D(\pi_1) = 0$; $D(\pi_2) = .842$; $D(\pi_3) = 0$.
2002	$D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = 0$.
2007	$D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = .997$.
2011	$D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = 0$.
Jun 2015	$D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = 0$.
Nov 2015	$D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = 0$.

election	chain-specific posterior means difference
1999	$M(\pi_1) = .000368$; $M(\pi_2) = .000208$; $M(\pi_3) = .00055$.
2002	$M(\pi_1) = .275$; $M(\pi_2) = .272$; $M(\pi_3) = .00258$.
2007	$M(\pi_1) = .0621$; $M(\pi_2) = .0614$; $M(\pi_3) = .00144$.
2011	$M(\pi_1) = .00152$; $M(\pi_2) = .00163$; $M(\pi_3) = .00316$.
Jun 2015	$M(\pi_1) = .318$; $M(\pi_2) = .321$; $M(\pi_3) = .00316$.
Nov 2015	$M(\pi_1) = .367$; $M(\pi_2) = .367$; $M(\pi_3) = .00423$.

But in the November 2015 election, which was strongly affected by events that intimidated electors, the number of polling stations that have extreme frauds increases by almost fifty percent over the number in June.

Table 13: 1999–2015 Polling Station Units `eforensics`-fraudulent

election	all (n)	not fraudulent	incremental	extreme
1999	208487	68731	139134	622
2002	172045	168898	2505	642
2007	159005	111786	43501	3718
2011	199560	71274	122643	5643
Jun 2015	173850	162046	6696	5108
Nov 2015	174619	99533	67686	7400

Across elections Table 14 shows there is a notable change in the composition of `eforensics`-fraudulent votes. In 1999, 2002 and 2007 the incremental frauds' magnitudes parameters ρ_{M0} and ρ_{S0} are negative, which as discussed in section 3 is compatible with the votes from incremental frauds being a reflection of strategic behavior. But in 2011 and in both 2015 elections ρ_{M0} has a strongly indeterminate sign: more and perhaps many more of the votes from incremental frauds probably reflect malevolent distortions of electors' intentions. In 1999 and in June 2015 the signs of the extreme frauds stolen votes magnitude parameters δ_{S0} are indeterminate.

Table 14: 1999–2015 `eforensics` Frauds Magnitude Parameter Estimates

	1999			2002		
Parameter	Mean	lo[a]	up[b]	Mean	lo[a]	up[b]
ρ_{M0}	−.633	−.856	−.449	−.268	−.364	−.179
ρ_{S0}	−1.36	−1.49	−1.17	−.885	−1.19	−.532
δ_{M0}	−.132	−.211	−.0908	−.437	−.724	−.0463
δ_{S0}	−.0920	−.241	.0105	−.625	−1.21	−.0449

	2007			2011		
Parameter	Mean	lo[a]	up[b]	Mean	lo[a]	up[b]
ρ_{M0}	−.562	−.637	−.504	−.352	−.787	.209
ρ_{S0}	−.784	−.815	−.740	−.594	−.689	−.503
δ_{M0}	−1.63	−1.80	−1.48	−1.53	−1.82	−1.05
δ_{S0}	−2.00	−2.17	−1.80	−2.40	−3.33	−1.91

	June 2015			November 2015		
Parameter	Mean	lo[a]	up[b]	Mean	lo[a]	up[b]
ρ_{M0}	−.433	−.842	.177	−.569	−.839	.0863
ρ_{S0}	−.975	−1.15	−.846	−.380	−.480	−.132
δ_{M0}	−1.64	−2.33	−1.20	−2.58	−3.44	−.485
δ_{S0}	−.289	−.579	.133	−1.48	−1.87	−.940

In Table 15, which shows the breakdown of `eforensics`-fraudulent votes in each election, an important result to notice is that none of the 99.5% credible intervals have the feature seen in Table 3, where the intervals range over almost two orders of magnitude. Those very wide credible intervals for the 2023 president election are also exceptional compared to other elections I have analyzed. Perhaps the forthcoming tweaked model specifications will perform better.

While in Table 15 the 1999 and 2011 estimates match the 2023 legislative election in having high F_w values compared to most of the other elections, F_w for November 2015 is greater than F_w in 1999. Evidently, given the indeterminate sign of ρ_{M0}, the `eforensics` estimates suggest that malevolent distortions of electors' intentions in November 2015 were extreme, not only much worse than in June 2015 but generally bad among recent Turkish

Table 15: 1999–2015 `eforensics`-fraudulent Votes

	1999		2002	
	posterior mean	99.5% CI	posterior mean	99.5% CI
F_t	466624.9	[433491.6, 509282.7]	32860.5	[16481.4, 46735.4]
incremental F_t	457784.9	[425434.3, 500324.0]	19610.4	[7085.3, 30443.3]
extreme F_t	8840.1	[7958.3, 9935.5]	13250.2	[9316.9, 16606.7]
F_w	2360625.2	[2150160.6, 2927792.8]	102246.2	[57970.2, 128759.0]
incremental F_w	2301438.8	[2092808.5, 2861237.1]	54598.4	[22926.4, 75469.1]
extreme F_w	59186.3	[55315.8, 66949.2]	47647.8	[34511.7, 54975.1]

	2007		2011	
	posterior mean	99.5% CI	posterior mean	99.5% CI
F_t	278274.3	[227301.4, 310265.9]	934725.0	[610704.2, 1515992.0]
incremental F_t	218447.0	[170631.1, 249233.8]	839308.0	[527496.7, 1387330.3]
extreme F_t	59827.3	[56518.9, 61652.1]	95417.0	[81482.8, 129526.5]
F_w	1170142.8	[967314.5, 1320256.9]	3845723.1	[3596154.1, 4328248.4]
incremental F_w	901684.4	[707906.7, 1049890.7]	3466988.8	[3225030.9, 3907524.1]
extreme F_w	268458.3	[258243.4, 275349.6]	378734.4	[325068.9, 422370.2]

	June 2015		November 2015	
	posterior mean	99.5% CI	posterior mean	99.5% CI
F_t	202406.4	[128213.2, 317492.3]	555504.4	[356172.1, 625989.2]
incremental F_t	65535.7	[12320.0, 147947.8]	389028.7	[180256.3, 460730.0]
extreme F_t	136870.7	[115710.2, 170085.0]	166475.8	[155522.2, 186667.7]
F_w	595896.6	[476043.5, 725646.5]	2577705.9	[825080.6, 3204703.1]
incremental F_w	150996.8	[46388.2, 255468.2]	2022849.2	[418751.0, 2585143.6]
extreme F_w	444899.8	[411598.6, 477840.7]	554856.7	[405357.5, 623853.3]

elections.

4 Appendix

Figure 11: Map of Regions in Turkey

Image source: `http://www.invest.gov.tr/en-US/investmentguide/investorsguide/Pages/Incentives.aspx` (obtained December 2, 2015).

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