LMI Stability-Constrained Identification for Composite Adaptive Internal Model Control

Zeng Qiu, Jing Sun, *Fellow, IEEE*, Mrdjan Jankovic, *Fellow, IEEE*, and Mario Santillo

***Abstract*—Internal model control (IMC), which explicitly incorporates a plant model and a plant inverse model as its components, has an intuitive control structure and simple tuning procedure. Within the IMC structure, we propose composite adaptive IMC (CAIMC) which simultaneously identifies the plant and the plant inverse to minimize modeling errors and further reduce the tracking error. In this paper, the design procedure of CAIMC is generalized to an n-th-order SISO plant. The main challenge in the generalization is to find an identification algorithm for an n-th order system that satisfies the stability constraint, while assuring closed-loop stability. In the literature, stability-constrained identification has been formulated as a convex programming problem by re-parameterizing the constraint as a linear matrix inequality, but boundedness and continuity of the estimated parameters, which are critical for closed-loop stability of an adaptive control algorithm, are not guaranteed. We propose a modified stability-constrained identification method with established boundedness and continuity properties. Closed-loop stability and asymptotic performance of CAIMC are then established under proper conditions. The effectiveness of the proposed algorithm is demonstrated with an example.**

***Index Terms*—Adaptive control, LMIs, system identification, optimization.**

NOMENCLATURE

$\mathcal{R}, \mathcal{R}^i$	Real numbers, $i \times 1$ real vectors.
$I_i, 0_{i\times j}$	$i \times i$ identity matrix, $i \times j$ zero matrix.
$[\cdot]^T, \lambda(\cdot)$	Transpose, eigenvalues of a matrix.
$\mathrm{vec}(\cdot)$	A column vector from vectorization of a matrix.
e, e_M, e_Q	Tracking error, modeling error, and inverse modeling error.
k, m, n	Integers.
l	IMC feedback, $l = r - y + y_M$.
r	Reference.
u, y, y_M	Plant input, plant output, model output.
v^*	Optimal cost at the optimal solutions.
z	Observation of a parametric model.
A	The state-space matrix of a transfer function.
G, G_f	Plant, m-th-order filter.
H, H^*	$n \times 1$ vector and its optimal solution.
J	Quadratic cost function.
M, Q	Model, inverse model.
P, P^*	$n \times n$ Lyapunov matrix and its optimal solution.
$R^*, \hat{R}$	Actual denominator and its estimate.
T_s	Sampling time period of the estimation.
$Z^*, \hat{Z}$	Actual nominator and its estimate.
γ	Weighting gain of the regularizing term.
$\epsilon, \hat{\epsilon}$	Estimation error of the unconstrained and the stability-constrained estimation.
$\theta^*, \theta, \hat{\theta}$	Actual parameter vector, its unconstrained estimate, and its stability-constrained estimate.
ϕ	Regressor of a parametric model.
χ, χ^*	Optimization variables and optimal solutions.
Γ	A positive-definite matrix that serves as an adaption gain.
Λ	Hurwitz polynomial.

Manuscript received May 26, 2018; revised November 19, 2018; accepted February 23, 2019. Date of publication April 9, 2019; date of current version December 3, 2019. This project is funded by the Ford Motor Company. Recommended by Associate Editor W. X. Zheng. *(Corresponding author: Zeng Qiu.)*

Z. Qiu, M. Jankovic, and M. Santillo are with the Ford Motor Company, Dearborn, MI 48124 USA (e-mail: cqiu1@ford.com; mjankov1@ford.com; msantil3@ford.com).

J. Sun is with the Department of Naval Architecture and Marine Engineering, University of Michigan, Ann Arbor, MI 48109 USA (e-mail: jingsun@umich.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TAC.2019.2909682

I. INTRODUCTION

INTERNAL model control (IMC) is an intuitive control structure with a simple tuning procedure [1]. As shown in Fig. 1, IMC incorporates the plant model M as an explicit part of the controller. The controller Q can be chosen as the approximate inverse of M, augmented with a filter to make it causal [2]–[5]. Combining the IMC structure with adaptive control leads to adaptive IMC (AIMC), which can handle unknown or slowly varying parameters in the plant and its operating environment. For standard AIMC, the plant is identified and the inverse is derived by inverting the estimated plant. Comprehensive studies on AIMC have been carried out in [6]–[9], and there exist many successful applications [10]–[12]. Simultaneous identification of the plant and the plant inverse is very tempting: intuitively, IMC performs better with more accurate plant and plant inverse models. By identifying the plant inverse directly, as opposed to calculating the inverse model from the identified plant model, one can have a more accurate representation of the plant inverse dynamics. This motivated composite AIMC (CAIMC) for a first-order plant as presented in [13], [14], where "composite" means that the plant and the plant inverse are identified simultaneously, is shown in Fig. 2. In this paper, CAIMC is generalized for n-th-order SISO plants.

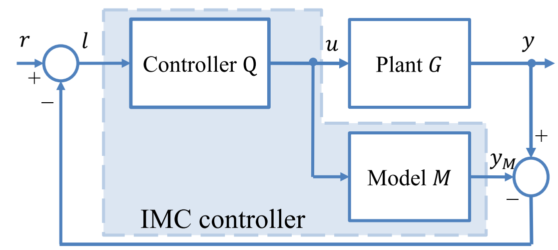

Fig. 1. IMC structure [13].

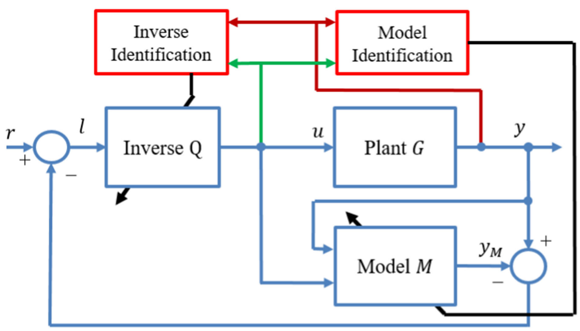

Fig. 2. CAIMC illustration.

One fundamental assumption of the IMC design is that both the plant and the plant inverse model are stable. Stability of first-order and second-order transfer functions yield linear constraints, whereas for a general n-th-order transfer function ($n > 2$), the stability constraint yields nonlinear and nonconvex constraints with nonsmooth boundaries in the original parameter space. The main challenge in generalizing CAIMC is to develop an effective constrained identification algorithm with the following requirements:

R1) It can meet the stability requirement, which translates to nonconvex constraints with nonsmooth boundaries in the parameter space.

R2) It can produce a reasonable approximation for the unconstrained identification estimates.

R3) It is computationally efficient.

R4) It assures closed-loop stability of the control system.

The projection method is not applicable for nonconvex constraints with nonsmooth boundaries [15]. Several approaches for handling stability constraints were established in the context of subspace identification [16]. In [17] and [18], a regularization term was added to the least-squares cost function. The amount of regularization for assuring stability was determined by solving a generalized eigenvalue problem. In [19], the constrained identification problem was cast into a constrained optimization problem, which was reformulated as a convex optimization problem with linear matrix inequality (LMI) constraints. The LMIs were formulated by re-parameterizing the stability constraints. Such convex programming can be solved efficiently with interior-point methods [20]. In [21], the approach of [19] was generalized, which allowed the eigenvalues of the estimates to lie in any convex region in the complex plane. However, none of the above methods have established boundedness and continuity properties for the estimated parameters, which are critical for closed-loop stability of an adaptive control system.

The contribution of this paper is twofold: First, we present a stability-constrained identification algorithm that adds a regularization term to the approach proposed in [19], [21]. Uniqueness, boundedness, and continuity properties of the optimal solutions are established, which do not exist in the literature. The technique is general, in the sense that it is applicable to any adaptive control algorithm where stability constraint should be considered, and closed-loop stability can be established. Second, CAIMC is generalized for n-th-order SISO plants using the proposed stability-constrained identification algorithm, and closed-loop stability and asymptotic performance are established. The generalized CAIMC was originally presented as a conference paper [22] with the stringent assumption that the plant had relative degree zero and the stability constraint was not handled. In this paper, both issues are addressed.

This paper is organized as follows: In Section II, the standard identification technique is introduced and the stability-constrained identification problem is formulated as a convex programming problem based on techniques from [19] and [21]. Then, the modified stability-constrained identification is presented with a simulation, and properties for the identified parameters are established. In Section III, the design process of CAIMC for an n-th-order plant with relative degree $m \geq 0$ is presented. In Section IV, the closed-loop stability and asymptotic performance are established for the ideal case when there are no unmodeled dynamics, and the effect of unmodeled dynamics is discussed. In Section V, CAIMC is applied to a third-order linear time invariant (LTI) plant. Section VI draws the conclusions.

II. Stability-Constrained Identification

In this section, we present a stability-constrained identification algorithm that satisfies the requirements (R1)–(R4), and establish its boundedness and continuity properties. The preliminaries, including the linear parametric model and the unconstrained normalized adaptive law, are first introduced in Sections II-A and II-B. They are based on [15].

A. Linear Parametric Model

An n-th-order linear dynamic model with relative degree $m \geq 0$ can be assumed to have the general form of

$$y^{(n)} + a^*_{n-1}y^{(n-1)} + \cdots + a^*_0 y = b^*_{n-m}u^{(n-m)} + b^*_{n-m-1}u^{(n-m-1)} + \cdots + b^*_0 u \quad (1)$$

where u and y are the plant input and output, respectively. Assume that the parameter vector $\theta^* = [b^*_0, b^*_1, \ldots, b^*_{n-m}, a^*_0, a^*_1, \ldots, a^*_{n-1}]^T$ is unknown, one can obtain a linear parametric model

$$z = \theta^{*T}\phi, \quad (2)$$

where the signal z and ϕ are defined as

$$z = \left\{\frac{s^n}{\Lambda}\right\} y,$$

$$\phi = \left[\left\{\frac{1}{\Lambda}\right\} u, \ldots, \left\{\frac{s^{n-m}}{\Lambda}\right\} u, -\left\{\frac{1}{\Lambda}\right\} y, \ldots, -\left\{\frac{s^{n-1}}{\Lambda}\right\} y\right]^T.$$

z and ϕ are referred to as the observation and the regressor, respectively. Throughout the paper, $\{\cdot\}$ represents the dynamic operator, whose transfer function is $(\cdot)$. $\Lambda = s^n + \lambda_{n-1}s^{n-1} + \cdots + \lambda_0$ is chosen as a Hurwitz polynomial, $\frac{1}{\Lambda}$ is introduced to avoid using derivatives in the identification. θ^* can be estimated using standard adaptive laws, such as the least-squares algorithm or gradient method.

B. Unconstrained Normalized Adaptive Law

The normalized gradient algorithm is presented here to obtain θ, the estimate of the unknown parameter vector θ^* in (2). The algorithm identifies θ by minimizing certain performance cost. For computational and robustness reasons, the identified parameters $\theta(k)$ can be updated at specific time instants kT_s, where T_s is the sampling time. Here, we consider a quadratic cost function of ϵ: $J(\theta) = \frac{\epsilon^2 m^2}{2}$, where

$$\epsilon(t) = \frac{z(t) - \theta^T(k)\phi(t)}{m(t)^2}, \quad \forall t \in [kT_s, (k+1)T_s) \tag{3}$$

is the normalized estimation error, and $m(t)^2 = 1 + \phi(t)^T\phi(t)$ is the normalizing term. Applying the gradient method, one can obtain

$$\theta(k+1) = \theta(k) + \Gamma \int_{kT_s}^{(k+1)T_s} \epsilon(t)\phi(t)dt \tag{4}$$

where $\Gamma = \Gamma^T$ is a positive-definite matrix affecting how rapidly θ converges.

Lemma 1: Let $2 - T_s\lambda_{\max}(\Gamma) \geq c$ for some $c > 0$. The adaptive law (4) for a parametric model (2) guarantees that

1) $\theta \in l_\infty$.[1]
2) $\Delta\theta \in l_2$,[2] where $\Delta\theta(k) = \theta(k+1) - \theta(k)$.
3) $\epsilon, \epsilon m \in \mathcal{L}_2 \cap \mathcal{L}_\infty$.[3]
4) If $m, \phi \in \mathcal{L}_\infty$ and ϕ is persistently exciting (PE),[4] then $\theta(k) \to \theta^*$ as $k \to \infty$ exponentially fast.

The normalized gradient algorithm (4) minimizes the cost function $J(\theta)$ with no constraints, i.e., it allows θ to lie anywhere in $\mathcal{R}^{2n-m+1}$. In the closed-loop stability analysis of Section IV, one of the sufficient conditions for establishing closed-loop stability is the frozen-time stability of M and Q, namely their denominators have to be Hurwitz at each sample time. Therefore, an algorithm to constrain the stability of an n-th-order polynomial is required. As the order of the polynomial increases, however, the stability constraint becomes nonconvex with nonsmooth boundaries, which causes problems with standard techniques such as the projection algorithm. To handle the stability constraint, LMI is exploited.

C. Stability-Constrained Identification

We seek Lyapunov inequality to represent the stability constraint: For plant (1) whose estimated transfer function has denominator $s^n + \sum_{i=0}^{n-1}\hat{a}_i s^i$, the corresponding controllable canonical form of its state-space realization has $A = \begin{bmatrix} -\hat{\theta}_A^T \\ I_{n-1} \quad 0 \end{bmatrix}$, where $\hat{\theta}_A = [\hat{a}_{n-1}, \ldots, \hat{a}_0]^T \in \mathcal{R}^n$. The stability condition can be expressed as the nonemptiness of the set defined by $\mathcal{P} = \{P | P \succ 0, AP + PA^T \prec 0\}$.

Let $\theta_A = [a_{n-1}, \ldots, a_0]^T \in \mathcal{R}^n$ be the estimates of $[a^*_{n-1}, \ldots, a^*_0]^T$ from the gradient algorithm (4). In this section, an optimization problem is formulated to find a stable $\hat{\theta}_A \in \mathcal{R}^n$ that best approximates the unconstrained parameter θ_A.

A natural formulation of the optimization problem is to minimize the quadratic error between θ_A and $\hat{\theta}_A$, subject to $\hat{\theta}_A$ satisfying the stability constraint:

$$\underset{\hat{\theta}_A, P}{\text{minimize}} \ \|\theta_A - \hat{\theta}_A\|_2^2,$$

$$\text{subject to } P \succ 0 \text{ and} \begin{bmatrix} -\hat{\theta}_A^T P \\ [\,I_{n-1} \quad 0\,]P \end{bmatrix} + \begin{bmatrix} -\hat{\theta}_A^T P \\ [\,I_{n-1} \quad 0\,]P \end{bmatrix}^T \prec 0. \tag{5}$$

However, $\hat{\theta}_A^T P$ in (5) introduces a nonconvex bilinear matrix inequality (BMI). The BMI optimization problem (5) can be solved with global approaches such as branch and bound, but it is computationally expensive [20]. To reduce the computational complexity, the BMI optimization problem is reformulated as an LMI optimization problem.

1) Stability-Constrained Identification in the Literature: Using the technique as presented in [19] and [21], define a new variable

$$H = \hat{\theta}_A^T P \in \mathcal{R}^n. \tag{6}$$

A weighting matrix P is added to the quadratic cost function $\|\theta_A - \hat{\theta}_A\|_2^2$, and (5) becomes

$$\underset{P,H}{\text{minimize}} \ \|\theta_A^T P - H\|_2^2,$$

$$\text{subject to } P \succ 0 \text{ and } \begin{bmatrix} -H \\ [\,I_{n-1} \quad 0\,]P \end{bmatrix} + \begin{bmatrix} -H \\ [\,I_{n-1} \quad 0\,]P \end{bmatrix}^T \prec 0. \tag{7}$$

Note that with the reformulated cost function in terms of the redefined parameters P and H, (7) has an LMI constraint. While (7) is not equivalent to (5), the new optimization formulation replaces the BMI with an LMI, leading to a simpler problem amenable to many effective solvers. Equation (7) satisfies the requirements (R1)–(R3) in the introduction; however, it does not satisfy requirement (R4). It has no established properties as reported in the literature, while boundedness and continuity of the estimated parameters are crucial for closed-loop stability of an adaptive control system. Also note that when the cost $\|\theta_A^T P - H\|_2^2$ reaches its minimum, there may be infinitely many solutions for P and H, which can be arbitrarily large. The indefiniteness may pose computational difficulties.

2) Modified Stability-Constrained Identification: For uniqueness and boundedness of the optimal solution P^* and H^*, a regularizing term $\gamma\|[\text{vec}(P), H]\|_2^2$ is added to the cost function in (7) to make the cost strictly convex, where $\gamma > 0$ is a small constant. The constraints in (7) are tightened by

[1] A discrete signal $x \in l_\infty$ when $\sup_{k\geq 1}|x(k)|$ is finite.

[2] A discrete signal $x \in l_2$ when $(\sum_{k=1}^{\infty} x(k)^2)^{\frac{1}{2}}$ is finite.

[3] A continuous signal $x \in \mathcal{L}_2$ when $(\int_0^\infty x(t)^2 dt)^{\frac{1}{2}}$ is finite. $x \in \mathcal{L}_\infty$ when $\sup_{t\geq 0}|x(t)|$ is finite.

[4] A piece-wise continuous signal vector ϕ is PE if there exist constants $\alpha_0, \alpha_1, T_0 > 0$, such that $\alpha_0 I \leq \frac{1}{T_0}\int_t^{t+T_0} \phi(\tau)\phi^T(\tau)d\tau \leq \alpha_1 I, \forall t \geq 0$.

$P \succeq \epsilon_0 I$ and $PA^T + AP \preceq -\epsilon_0 I$ to assure the existence of the optimal solution, where $\epsilon_0 > 0$ is a small constant. These modifications transform (7) to:

$$\underset{P,H}{\text{minimize}} \ \|\theta_A^T P - H\|_2^2 + \gamma\|[\text{vec}(P), H]\|_2^2$$

subject to $P \succeq \epsilon_0 I$ and

$$\begin{bmatrix} -H \\ [I_{n-1} \quad 0]P \end{bmatrix} + \begin{bmatrix} -H \\ [I_{n-1} \quad 0]P \end{bmatrix}^T \preceq -\epsilon_0 I. \quad (8)$$

Solving the optimization problem returns the optimal P^* and H^*. $\hat{\theta}_A$ can be calculated as $\hat{\theta}_A = (H^* P^{*-1})^T$ according to (6). Equation (8) can be solved efficiently with interior point methods, for which there are many mature tools available [23].

Given the parametric model (2), the implementation of the constrained identification is summarized as follows:

Algorithm 1: At the k-th Sample:

1) Use the unconstrained adaptive law (4) to calculate $\theta(k)$.
2) Calculate the stability-constrained $\hat{\theta}(k)$ from $\theta(k)$.
   - Solve the convex optimization problem (8) for the optimal solution P^* and H^*, with $\theta_A = \theta_A(k)$, where $\theta_A(k)$ is the parameter vector of the transfer function denominator parameters in $\theta(k)$.
   - Compute the constrained parameter vector $\hat{\theta}_A(k) = (H^* P^{*-1})^T$.
   - $\hat{\theta}(k)$, which is obtained by substituting $\theta_A(k)$ in the unconstrained parameters vector $\theta(k)$ with $\hat{\theta}_A(k)$, is used for the control signal calculation.

D. Modified Stability-Constrained Identification Simulation

This section demonstrates the effectiveness of Algorithm 1 for constraining the identified system in the stable region. Consider a third-order stable LTI plant

$$y = \left\{ \frac{\theta_{b2}^* s^2 + \theta_{b1}^* s + \theta_{b0}^*}{s^3 + \theta_{a2}^* s^2 + \theta_{a1}^* s + \theta_{a0}^*} \right\} u \quad (9)$$

where $\theta_a^* = [2, 2, 3]^T, \theta_b^* = [2, 2, 1]^T$ are assumed to be unknown. A third-order plant is adopted because it has a nonconvex stability region, and the standard projection algorithm does not apply to such a constraint. White noises of variance 1 are added to the plant input and output with a first-order noise model. The signal-to-noise ratio is 10. The time constant of the noise model is uniformly distributed between 0 and 0.5.

Algorithm 1 is applied to estimate the unknown parameters: at each sample time, θ is calculated from the unconstrained adaptive law (4), then $\hat{\theta}$ is calculated from θ considering the stability constraints. Five thousand identifications are performed with different initial values and different noise model time constant. Each identification is performed with 100 sample times. The three poles of the final identified system are shown in Fig. 3.

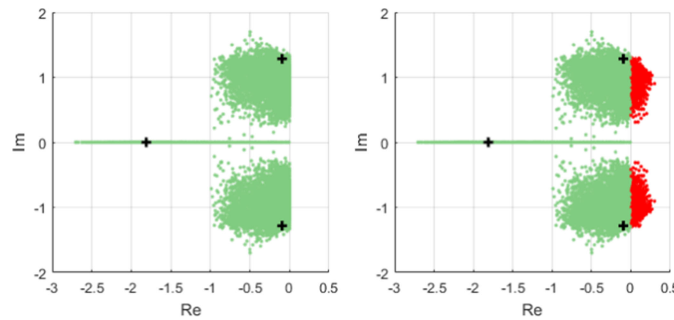

Fig. 3. Poles of the identified systems from Algorithm 1 (left) and unconstrained adaptive law (4) (right).

The actual poles are marked by "+." The estimated poles on the open left half plane (OLHP) are marked in green, and the ones on the right half plane (RHP) are marked in red. Algorithm 1 keeps the identified system poles in the stable region.

E. Modified Stability-Constrained Identification Analysis

The continuity and boundedness properties of the identified parameter $\hat{\theta}$ are crucial for closed-loop stability of the adaptive control system. In this section, we establish these properties by using tools from the optimization field to analyze the optimization problem (8).

When $\gamma > 0$, the cost function in (8) is strictly convex. Since the feasible set of (8) is nonempty, closed, and convex, from Lemma 5 in Appendix A, there exists a unique optimal solution. Let $f(\chi, \theta_A, \gamma)$ represent the cost function of (8), where $\chi = [\text{vec}(P), H]$. Let $P^*(\theta_A, \gamma)$ and $H^*(\theta_A, \gamma)$ represent the optimal solution function. $\hat{\theta}_A(\theta_A, \gamma) = (H^*(\theta_A, \gamma) P^*(\theta_A, \gamma)^{-1})^T$ is the constrained parameter vector.

Lemma 2: $\hat{\theta}_A(\theta_A, \gamma)$ has the following properties:

1) $\hat{\theta}_A(\theta_A, \gamma)$ is Lipschitz continuous with respect to θ_A and γ when $\gamma > 0$.
2) When θ_A is stable, $\lim_{\gamma \to 0} \hat{\theta}_A(\theta_A, \gamma) = \theta_A$ and $\hat{\theta}_A(\theta_A, 0) = \theta_A$, i.e., $\hat{\theta}_A(\theta_A, \gamma)$ is Lipschitz continuous with respect to γ when $\gamma \geq 0$.

Proof: i) The Lipschitz continuity of $P^*(\theta_A, \gamma)$ and $H^*(\theta_A, \gamma)$ can be proven using Lemma 7 of Appendix A: The second-order growth condition holds because the cost function $f(\chi, \theta_A, \gamma)$ is a strictly convex quadratic function of χ when $\gamma > 0$. Consider the difference between $f(\chi, \theta_{A0}, \gamma)$ and $f(\chi, \theta_A, \gamma)$, namely $\|\theta_A^T P - H\|_2^2 - \|\theta_{A0}^T P - H\|_2^2$. It is Lipschitz continuous with respect to P and H, modulus $c\|\theta_A - \theta_{A0}\|$ for some $c > 0$ for bounded P, H, and θ_A.

Applying Lemma 7, $\|\chi^*(\theta_A, \gamma) - \chi^*(\theta_{A0}, \gamma)\| \leq c\|\theta_A - \theta_{A0}\|$ for some $c > 0$. Therefore, the optimal solutions $P^*(\theta_A, \gamma)$ and $H^*(\theta_A, \gamma)$ are Lipschitz continuous with respect to θ_A for $\gamma > 0$. Since $\hat{\theta}_A(\theta_A, \gamma) = (H^*(\theta_A, \gamma) P^*(\theta_A, \gamma)^{-1})^T$, $P^*(\theta_A, \gamma) \succeq \epsilon_0 I$, $\hat{\theta}_A(\theta_A, \gamma)$ is Lipschitz continuous with respect to θ_A for $\gamma > 0$.

Similarly, $\hat{\theta}_A(\theta_A, \gamma)$ is Lipschitz continuous with respect to γ with $\gamma > 0$.

ii) Let $v^*(\theta_A, \gamma)$ represent the optimal value of the cost function. When $\gamma = 0$, and θ_A is stable, the optimal cost value $v^*(\theta_A, 0) = 0$. Therefore, $\theta_A^T P^*(\theta_A, 0) - H^*(\theta_A, 0) = 0$, and $\hat{\theta}_A(\theta_A, 0) = \theta_A$.

From Lemma 6 of Appendix A, $\lim_{\gamma\to 0} v^*(\theta_A, \gamma) \leq v^*(\theta_A, 0) = 0$. $v^*(\theta_A, \gamma)$ is a nonnegative quadratic function. Thus,

$$\lim_{\gamma\to 0} v^*(\theta_A, \gamma) = \lim_{\gamma\to 0}(\|\theta_A^T P^*(\theta_A, \gamma) - H^*(\theta_A, \gamma)\|_2^2 + \gamma\|[\mathrm{vec}(P^*(\theta_A, \gamma)), H^*(\theta_A, \gamma)]\|_2^2) = 0$$

which implies

$$\lim_{\gamma\to 0}(\theta_A^T P^*(\theta_A, \gamma) - H^*(\theta_A, \gamma)) = 0,$$

$$\lim_{\gamma\to 0}(\theta_A^T - H^*(\theta_A, \gamma)P^*(\theta_A, \gamma)^{-1}) = \lim_{\gamma\to 0}(\theta_A^T - \hat{\theta}_A^T(\theta_A, \gamma)) = 0.$$

Therefore, when θ_A is stable, $\lim_{\gamma\to 0} \hat{\theta}_A(\theta_A, \gamma) = \theta_A$. ■

Note from Algorithm 1 that $\hat{\theta}(k)$ is a function of $\theta(k)$. In the following theorem, the boundedness of $\hat{\theta}$ is established from Lemmas 1 and 2. Define the estimation error

$$\hat{\epsilon}(t) = \frac{z(t) - \hat{\theta}(k)^T \phi(t)}{m(t)^2}, \quad \forall t \in [kT_s, (k+1)T_s). \tag{10}$$

Boundedness of $\hat{\epsilon}$ is also established as it is also critical to closed-loop stability of an adaptive control system.

Theorem 3: Algorithm 1 for a parametric model (2) guarantees that

1) $\hat{\theta} \in l_\infty$.
2) $\Delta\hat{\theta} \in l_2$, where $\Delta\hat{\theta}(k) = \hat{\theta}(k+1) - \hat{\theta}(k)$.
3) $\hat{\epsilon}, \hat{\epsilon}m \in \mathcal{L}_\infty$.
4) If $\theta(k)$ is stable $\forall k > k_c$, where k_c is a finite number, $\lim_{\gamma\to 0} \hat{\epsilon} = \epsilon$, $\lim_{\gamma\to 0} \hat{\epsilon}m = \epsilon m$.

Proof: i) From Lemma 1, $\theta \in l_\infty$. From Lemma 5 in Appendix A, $P^*, H^* \in l_\infty$ for each fixed γ. Since $\hat{\theta}_A = (H^* P^{*-1})^T$, $P^* \succeq \epsilon_0 I$, we can conclude that $\hat{\theta}_A \in l_\infty$, and therefore $\hat{\theta} \in l_\infty$.

ii) From Lemma 1, $\Delta\theta_k \in l_2$. From Lemma 2 (i), $\hat{\theta}_A$ is Lipschitz continuous with respect to θ_A, i.e., $\exists c > 0$, $\|\Delta\hat{\theta}_{Ak}\| \leq c\|\Delta\theta_{Ak}\|$. Therefore, $\Delta\hat{\theta}_A \in l_2$, and $\Delta\hat{\theta} \in l_2$.

iii) $\hat{\epsilon}(t) = \epsilon(t) + \frac{(\theta(k) - \hat{\theta}(k))^T \phi(t)}{m(t)^2}$, $\forall t \in [kT_s, (k+1)T_s)$. From Lemma 1, $\epsilon \in \mathcal{L}_\infty$, $\frac{\phi}{m^2} \in \mathcal{L}_\infty$, $\theta \in l_\infty$, and from Theorem 3 (i), $\theta - \hat{\theta} \in l_\infty$. Therefore, $\hat{\epsilon} \in \mathcal{L}_\infty$. Similarly, $\hat{\epsilon}m \in \mathcal{L}_\infty$.

iv) $\hat{\epsilon}(t) = \epsilon(t) + \frac{(\theta(k) - \hat{\theta}(k))^T \phi(t)}{m(t)^2}$, $\forall t \in [kT_s, (k+1)T_s)$. From Lemma 2 (ii), when θ_A is stable, $\lim_{\gamma\to 0} \hat{\theta}_A = \theta_A$. Since $\frac{\phi(t)}{m(t)^2} \in \mathcal{L}_\infty$, $\lim_{\gamma\to 0} \hat{\epsilon} = \epsilon$. Similarly, $\lim_{\gamma\to 0} \hat{\epsilon}m = \epsilon m$. ■

III. CAIMC FOR AN n-TH ORDER SISO PLANT

A. Internal Model Control (IMC) and Its Tracking Error

IMC has an intuitive control structure as shown in Fig. 1, where G, M, and Q represent the plant, plant model, and plant inverse model, respectively. When the model matches the plant exactly, i.e., $M = G$, the IMC structure becomes open-loop. Q can be designed as an open-loop feedforward controller. For a minimum-phase plant, Q is designed as the inverse of the plant appended with a filter G_f to make it causal, namely $Q = G^{-1}G_f$. The bandwidth of the control system is defined by the bandwidth of G_f [1].

Ideally when $M = G$, we have $\{G_f\}r - y = 0$, where r is the reference signal and y is the plant output. Thus, we consider the tracking error $e = \{G_f\}r - y$, which can be expressed as

$$e = e_M + e_Q \tag{11}$$

where

$$e_M = \{G_f\}(y - y_M) \tag{12}$$

$$e_Q = \{G_f\}l - y. \tag{13}$$

y_M is the model response, and $l = r - y + y_M$ is the input to the approximate inverse Q as shown in Fig. 1. Equation (11) can be derived by noting that $e = \{G_f\}r - y = \{G_f\}(r - y + y_M) + \{G_f\}(y - y_M) - y = \{G_f\}(y - y_M) + (\{G_f\}l - y)$.

Note that e_M is the filtered difference between the plant and the model responses, and e_Q, on the other hand, is the difference between the filtered input to Q and the plant response, which reflects the inverse modeling error.

Using the triangle inequality, $|e| \leq |e_M| + |e_Q|$, the tracking error is upper-bounded by $|e_M| + |e_Q|$. This expression inspires and justifies the separate identifications of M and Q. Intuitively, e_M is related to the plant model estimation error ϵ_M, and e_Q is related to the inverse estimation error ϵ_Q, where ϵ_M and ϵ_Q are the estimation errors defined in (3) for the plant model and inverse parametric model, respectively. Recall that the gradient method (4) is based on minimizing the quadratic cost function of the estimation error ϵ. Therefore, the minimization of the quadratic cost function of ϵ_M and ϵ_Q will contribute to reducing the tracking error, e. Therefore, we propose CAIMC as shown in Fig. 2, where "composite" means that the model and the inverse are identified simultaneously.

B. Composite Adaptive Internal Model Control

Consider the SISO plant

$$y = \{G\}u \tag{14}$$

with order n and relative degree m. The control objective is that y tracks $\{G_f\}r$. G_f is an m-th-order filter introduced for the causality of the inverse, and it can be designed for the desired bandwidth of the closed-loop system.

The plant is modeled as

$$G = \frac{Z_M^*}{R_M^*} \tag{15}$$

where Z_M^* is an $(n–m)$-th-order Hurwitz polynomial, and R_M^* is an n-th-order Hurwitz polynomial with leading coefficient 1. The other coefficients of R_M^* and Z_M^* are unknown.

The approximate stable and proper inverse is modeled as

$$G^{-1}G_f = \frac{Z_Q^*}{R_Q^*} \tag{16}$$

where R_Q^* and Z_Q^* are n-th-order Hurwitz polynomials. The leading coefficients of R_Q^* is 1. The other coefficients of R_Q^* and Z_Q^* are unknown.

The following assumptions are made:

A1) Order n of the plant is known.

A2) The relative degree m of the plant is known.

A3) The plant is stable.

A4) The plant is minimum-phase.

Assumptions (A1) and (A2) are standard assumptions for indirect adaptive control. (A3) is a standard assumption for IMC. And (A4) is for the invertibility of the plant. The design procedure of CAIMC for the plant (14), following the certainty equivalence principle, is described in steps as follows:

CAIMC Design Procedure:

1) Formulate a plant model and a plant inverse model as (15) and (16).
2) Design m-th-order filter G_f, whose bandwidth corresponds to the desired closed-loop bandwidth.
3) Derive the parametric models of the proposed plant model and plant inverse model, and identify the unknown parameters using Algorithm 1 in Section II-C2.
4) Treat the identified plant and plant inverse as the true plant and plant inverse, and embed them into the model M and the inverse model Q in the IMC structure, respectively.

C. Design Detail of CAIMC

This section presents the detail of step 3) and step 4) of the CAIMC design procedure. For step 3), the parametric models for the plant model and the plant inverse model are presented. For step 4), the identified parameters are treated as if they were the real ones to implement M and Q in the IMC structure. In the sequel, $\hat{Z}_M$, $\hat{Z}_Q$, $\hat{R}_M$, and $\hat{R}_Q$ denote the estimates Z_M^*, Z_Q^*, R_M^*, and R_Q^* in (15) and (16), $G_f = \frac{1}{\Lambda_f}$, where Λ_f is an m-th-order Hurwitz polynomial.

Plant Model Parameterization and Implementation: The goal of the plant model parameterization is to define z_M and ϕ_M, such that a parametric model $z_M = \theta_M^{*T}\phi_M$ can be used to identify θ_M^*, the unknown parameters of (15). Since a parametric model in the form of (2) for a given physical process is not unique, we attempt to find the particular one such that $\hat{\epsilon}_M m_m^2$ is closely related to $e_M = \{G_f\}(y - y_M)$ as defined in (12), in light of the discussion in Section III-A, so that e_M is minimized in the model identification process.

With simple manipulation of $y = \{\frac{Z_M^*}{R_M^*}\}u$ according to (15), and introducing a regressor filter $\frac{1}{\Lambda_M}$,

$$y = \left\{\frac{\Lambda_M - R_M^*}{\Lambda_M}\right\} y + \left\{\frac{Z_M^*}{\Lambda_M}\right\} u \tag{17}$$

where Λ_M is an n-th order-Hurwitz polynomial that serves as a regressor filter. Define

$$z_M = y, \theta_M^* = [\theta_a^{*T}, \theta_b^{*T}]^T$$
$$\phi_M = \left[\left\{\frac{1}{\Lambda_M}\right\} y, \ldots, \left\{\frac{s^{n-1}}{\Lambda_M}\right\} y, \left\{\frac{1}{\Lambda_M}\right\} u, \ldots, \left\{\frac{s^{n-m}}{\Lambda_M}\right\} u\right]^T \tag{18}$$

where θ_a^* is the coefficient vector of $\Lambda_M - R_M^*$ and θ_b^* is the coefficient vector of Z_M^*. Equation (17) can be expressed as $z_M = \theta_M^{*T}\phi_M$. Without considering the stability constraint, the normalized estimation error ϵ_M is $\frac{z_M - \theta_M^T\phi_M}{m_M^2}$, where θ_M is the estimation of θ_M^*, and $m_M^2 = 1 + \phi_M^T\phi_M$ is the normalizing term. The normalized gradient algorithm (4) that minimizes $J(\theta_M) = \frac{\epsilon_M^2 m_M^2}{2}$ can be adopted to identify θ_M. Then, Algorithm 1 can be used to estimate $\hat{\theta}_M$.

The nominal model M can be implemented as

$$y_M = \hat{\theta}_M^T\phi_M \tag{19}$$

then,

$$\hat{\epsilon}_M m_M^2 = z_M - \hat{\theta}_M^T\phi_M = y - y_M. \tag{20}$$

Recall that from (12), $e_M = \{G_f\}(y - y_M) = \{G_f\}(\hat{\epsilon}_M m_M^2)$. In the following analysis, we show how minimizing $\hat{\epsilon}_M m_M^2$ reduces e_M.

With (3) and (10), we can express $\hat{\epsilon}_M m_M^2 = \epsilon_M m_M^2 + (\theta_M - \hat{\theta}_M)^T\phi_M$. Applying Lemma 1, $\epsilon_M m_M \in \mathcal{L}_{2e}$. It is later shown in Section IV that all the signals are bounded, so $m_M \in \mathcal{L}_\infty$, which yields $\epsilon_M m_M^2 \in \mathcal{L}_{2e}$. Since all the signals are bounded, $(\theta_M - \hat{\theta}_M)^T\phi_M \in \mathcal{L}_{2e}$. Applying Lemmas 11 and 12, $\|e_{Mt}\|_2 \le \sup_\omega |G_f(j\omega)| \|(\hat{\epsilon}_M m_M)_t^2\|_2$. Consequently, the estimation algorithm that minimizes the estimation error $\hat{\epsilon}_M m_M^2$ also reduces e_M.

Plant Inverse Model Parameterization and Implementation: Similar to the plant identification, the key step in the plant inverse identification is to define a parametric model that directly relates to the inverse modeling error so that $\hat{\epsilon}_Q m_Q^2 = e_Q$, where $e_Q = \{G_f\}l - y$ as defined in (13) so that e_Q is minimized in the inverse identification process.

From (14), $\{G_f\}u = \{G^{-1}G_f\}y$, and according to the inverse model (16), $\{\frac{1}{\Lambda_f}\}u = \{\frac{Z_Q^*}{R_Q^*}\}y$. With a simple manipulation, we can write

$$\left\{\frac{1}{\Lambda_f}\right\} u = \left\{\frac{\Lambda_Q - R_Q^*}{\Lambda_Q}\right\}\left\{\frac{1}{\Lambda_f}\right\} u + \left\{\frac{Z_Q^*}{\Lambda_Q}\right\} y$$

where Λ_Q is an n-th-order Hurwitz polynomial that serves as a regressor filter. If we design the inverse parametric model in the same way we designed the plant model parameterization, the associated signals of the parametric model $z_{Q0} = \theta_Q^{*T}\phi_{Q0}$ can be defined as

$$z_{Q0} = \left\{\frac{1}{\Lambda_f}\right\} u, \quad \theta_Q^* = [\theta_c^{*T}, \theta_d^{*T}]^T$$
$$\phi_{Q0} = \left[\left\{\frac{1}{\Lambda_Q}\right\} y, \ldots, \left\{\frac{s^n}{\Lambda_Q}\right\} y, \right.$$
$$\left. \times \left\{\frac{1}{\Lambda_Q \Lambda_f}\right\} u, \ldots, \left\{\frac{s^{n-1}}{\Lambda_Q \Lambda_f}\right\} u\right]^T \tag{21}$$

where θ_c^* is the coefficient vector of Z_Q^* whose dimension is $n+1$, and θ_d^* is the coefficient vector of $\Lambda_Q - R_Q^*$ whose dimension is n. If Q is implemented such that

$$u = \left\{(\Lambda_Q - \hat{R}_Q)\frac{1}{\Lambda_Q}\right\} u + \left\{\hat{Z}_Q \frac{1}{\Lambda_Q}\right\} l \tag{22}$$

then according to (21), the estimation error is

$$\begin{aligned} z_{Q0} - \hat{\theta}_Q^T \phi_{Q0} &= \left\{\hat{R}_Q \frac{1}{\Lambda_Q \Lambda_f}\right\} u - \left\{\hat{Z}_Q \frac{1}{\Lambda_Q}\right\} y \\ &= \left\{\frac{1}{\Lambda_f}\hat{R}_Q \frac{1}{\Lambda_Q}\right\} u - \left\{\hat{Z}_Q \frac{1}{\Lambda_Q}\right\} y + \epsilon_1 \\ &= \left\{\frac{1}{\Lambda_f}\hat{Z}_Q \frac{1}{\Lambda_Q}\right\} l - \left\{\hat{Z}_Q \frac{1}{\Lambda_Q}\right\} y + \epsilon_1 \\ &= \left\{\hat{Z}_Q \frac{1}{\Lambda_Q}\right\} \left(\left\{\frac{1}{\Lambda_f}\right\} l - y\right) + \epsilon_2 \\ &= \left\{\hat{Z}_Q \frac{1}{\Lambda_Q}\right\} e_Q + \epsilon_2 \end{aligned} \tag{23}$$

where the last three equations are derived from (13) and (22). ϵ_1 and ϵ_2 are the residual terms resulting from applying the Swapping Lemma in Appendix B. In order to have a more direct and simple relation between e_Q and $\hat{\epsilon}_Q$, we consider $\{X\}$, an operator whose transfer function is $(\hat{Z}_Q \frac{1}{\Lambda_Q})^{-1}$. Note that from (23),

$$\begin{aligned} e_Q &= \{X\}(z_{Q0} - \hat{\theta}_Q^T \phi_{Q0} - \epsilon_2) \\ &= \{X\} z_{Q0} - \{X\}\hat{\theta}_Q^T \phi_{Q0} - \{X\}\epsilon_2 \\ &= \{X\} z_{Q0} - \hat{\theta}_Q^T \{X\}\phi_{Q0} - \{X\}\epsilon_2 + \epsilon_3 \end{aligned} \tag{24}$$

where ϵ_3 is the residual term resulting from applying the Swapping Lemma in Appendix B. If we redefine the inverse parametric model such that the new observation is $\{X\} z_{Q0}$ and the new regressor is $\{X\}\phi_{Q0}$, then the new estimation error is $e_Q + \{X\}\epsilon_2 - \epsilon_3$. Since X depends on the identified parameters whose values at the current sample time are unavailable, z_Q and ϕ_Q are defined as

$$z_Q = \{X_{k-1}\} u, \quad \theta_Q^* = [\theta_c^{*T}, \theta_d^{*T}]^T$$
$$\phi_Q = \{X_{k-1}\} \left[\left\{\frac{1}{\Lambda_Q}\right\} y, \ldots, \left\{\frac{s^n}{\Lambda_Q}\right\} y, \right.$$
$$\left. \times \left\{\frac{1}{\Lambda_Q \Lambda_f}\right\} u, \ldots, \left\{\frac{s^{n-1}}{\Lambda_Q \Lambda_f}\right\} u\right]^T \tag{25}$$

where X_{k-1} is a transfer function with $\hat{Z}_Q$ having the parameters identified at the previous sample time $(k-1)$. Without the stability constraints, the normalized gradient algorithm (4) that minimizes $J(\theta_Q) = \frac{\epsilon_Q^2 m_Q^2}{2}$ is adopted to identify θ_Q, where $\epsilon_Q = \frac{z_Q - \theta_Q^T \phi_Q}{m_Q^2}, m_Q^2 = 1 + \phi_Q^T \phi_Q$ are the normalized estimation error and the normalizing term. Algorithm 1 can be used to estimate $\hat{\theta}_Q$. The inverse model used in CAIMC is still implemented as (22).

To establish the connection between e_Q and $\hat{\epsilon}_Q$, from (24) and (25), the inverse estimation error can be expressed as

$$\begin{aligned} \hat{\epsilon}_Q m_Q^2 &= z_Q - \hat{\theta}_Q^T \phi_Q = \{X_{k-1}\} z_{Q0} - \hat{\theta}_Q^T \{X_{k-1}\}\phi_{Q0} \\ &= \{X_{k-1}\}(z_{Q0} - \hat{\theta}_Q^T \phi_{Q0}) + \epsilon_4 \\ &= \{X_{k-1}\}\{X\}^{-1} e_Q + \epsilon_5 = e_Q + \epsilon_6 \end{aligned} \tag{26}$$

where ϵ_4, ϵ_5, and ϵ_6 are residues from swapping the dynamic operators. These residues are bounded by $\hat{\theta}_Q(k) - \hat{\theta}_Q(k-1)$. Therefore, for the inverse $\hat{\epsilon}_Q m_Q^2 \approx e_Q$ when the parameter adaptation is sufficiently slow.

D. CAIMC Summary

The CAIMC scheme is summarized as following:

Model:
Plant model: $\frac{Z_M^*}{R_M^*}$. Plant inverse model: $\frac{Z_Q^*}{R_Q^*}$.
Parametric Model:
Model: $z_M = \theta_M^{*T}\phi_M$ with z_M and ϕ_M defined in (18). $\theta_M^* = [\theta_a^{*T}, \theta_b^{*T}]^T$, where θ_a^* is the coefficient vector of $\Lambda_M - R_M^*$ and θ_b^* is the coefficient vector of Z_M^*. Inverse: $z_Q = \theta_Q^{*T}\phi_Q$ with z_Q and ϕ_Q defined in (25). $\theta_Q^* = [\theta_c^{*T}, \theta_d^{*T}]^T$, where θ_c^* is the coefficient vector of Z_Q^*, and θ_d^* is the coefficient vector of $\Lambda_Q - R_Q^*$.
Adaptive Law:
$\hat{\theta}_M$ and $\hat{\theta}_Q$ are identified by Algorithm 1 in Section II-C2. The stability constraints in (8) are imposed on $\hat{\theta}_a$, $\hat{\theta}_c$, and $\hat{\theta}_d$.
Control Law:
$u = \{(\Lambda_Q - \hat{R}_Q)\frac{1}{\Lambda_Q}\}u + \{\hat{Z}_Q \frac{1}{\Lambda_Q}\}l$ as in (22), where $l = r - y + y_M$, $y_M = \hat{\theta}_M^T \phi_M$ as in (19).

Remark 1: Design of the CAIMC algorithm includes the following:

1) G_f is designed based on the desired closed-loop bandwidth.
2) In the parametric model (2) and the unconstrained adaptive law (4), the regressor filter $\frac{1}{\Lambda}$, the initial condition $\theta(0)$, and the adaption gain Γ need to be calibrated. To deal with the unmodeled dynamics, generally, a deadzone is added to the estimation error ϵ for robust estimation. The calibration procedure for these parameters is well established [15].

3) In the modified stability-constrained identification (8), γ and ϵ_0 need to be chosen. As shown in Lemma 2 (ii), γ should be very small for $\hat{\theta}_A$ to be close to θ_A when θ_A is stable. ϵ_0 should be small as it is only used for tightening the stability constraint to assure the existence of the optimal solution. One can also choose γ to be a sequence $\{\gamma_k\}$ with $\gamma_k \to 0$ as $k \to \infty$. In this case, the rate of convergence for γ_k should be carefully chosen as they will affect the parameter convergence of the overall adaptive control system.

IV. Stability Proof of CAIMC in the Ideal Case

In this section, stability and asymptotic performance of the ideal n-th-order CAIMC is established.

Theorem 4: Consider the plant (14) subject to the CAIMC scheme. For any bounded reference r, all the signals in the closed-loop system are uniformly bounded. When the regressors are PE, the tracking error $e = \{G_f\}r - y$ converges to 0 as $\gamma \to 0$.

Proof: Given that Λ_M in (18) and Λ_Q in (25) are Hurwitz polynomials with the same order that serve as the regressor filters, we choose $\Lambda_M = \Lambda_Q = \Lambda$ with the coefficient vector $\theta_\lambda \in \mathcal{R}^n$ throughout the proof. Note that the same analysis can be carried out with arbitrary choice of Hurwitz Λ_M and Λ_Q at the expense of some additional algebra. Defining

$$y_f = \left\{\frac{1}{\Lambda}\right\} y, \quad u_f = \left\{\frac{1}{\Lambda\Lambda_f}\right\} u, \epsilon_f = \left\{\frac{1}{\Lambda}\right\} (\hat{\epsilon}_M m_M^2) \tag{27}$$

we establish signal boundedness in the following steps:

Step 1. Correlate u and y to the estimation error: Defining the augmented states x as $[y_f, y_f^{(1)}, \ldots, y_f^{(n-1)}, u_f, u_f^{(1)}, \ldots, u_f^{(n+m-1)}, \epsilon_f, \epsilon_f^{(1)}, \ldots, \epsilon_f^{(n-1)}]^T$, we have

$$\begin{aligned} \dot{x} &= A(t)x + b_1(t)\hat{\epsilon}_M m_M^2 + b_2\bar{r} \\ \begin{bmatrix} y \\ u \end{bmatrix} &= C(t)x + d_1(t)\hat{\epsilon}_M m_M^2 + d_2\bar{r} \end{aligned} \tag{28}$$

where

$$A(t) = \left[\begin{array}{c:c:c} \begin{matrix} 0_{(n-1)\times 1} | I_{n-1} \\ (\hat{\theta}_a - \theta_\lambda)^T \end{matrix} & \begin{matrix} 0_{(n-1)\times(n+m)} \\ \hat{\theta}_{Mf}^T | 0_{1\times m} \end{matrix} & 0_{n\times n} \\ \hdashline 0_{(n+m)\times n} & \begin{matrix} 0_{(n+m-1)\times 1} | I_{n+m-1} \\ -\hat{\theta}_{Qf}^T \end{matrix} & \begin{matrix} 0_{(n+m-1)\times n} \\ (\eta\theta_\lambda - \hat{\theta}_{cn})^T \end{matrix} \\ \hdashline 0_{n\times n} & 0_{n\times(n+m)} & \begin{matrix} 0_{(n-1)\times 1} | I_{n-1} \\ -\theta_\lambda^T \end{matrix} \end{array}\right]$$

$$b_1(t) = \left[\begin{array}{c} 0_{(n-1)\times 1} \\ 1 \\ \hdashline 0_{(n+m-1)\times 1} \\ -\eta \\ \hdashline 0_{(n-1)\times 1} \\ 1 \end{array}\right], \quad b_2(t) = \left[\begin{array}{c} 0_{n\times 1} \\ \hdashline 0_{(n+m-1)\times 1} \\ 1 \\ \hdashline 0_{n\times 1} \end{array}\right]$$

$$C(t) = \begin{bmatrix} \hat{\theta}_a^T & \hat{\theta}_{Mf}^T | 0_{1\times m} & 0_{1\times n} \\ 0_{(1\times n)} & \theta_{\lambda f}^T - \hat{\theta}_{Qf}^T & (\eta\theta_\lambda - \hat{\theta}_{cn})^T \end{bmatrix}$$

$$d_1(t) = \begin{bmatrix} 1 \\ -\eta \end{bmatrix}, d_2(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

where $\begin{bmatrix} \hat{\theta}_{cn} \\ \eta \end{bmatrix} = \hat{\theta}_c$, $\hat{\theta}_{cn} \in \mathcal{R}^n$ and η is the $(n+1)$-th entry of $\hat{\theta}_c, \bar{r} = \{\hat{Z}_Q \frac{1}{\Lambda}\}r, \hat{\theta}_{Mf} \in \mathcal{R}^n, \hat{\theta}_{Qf} \in \mathcal{R}^{n+m}$, and $\hat{\theta}_{\lambda f} \in \mathcal{R}^{n+m}$ are the coefficient vectors of $\hat{Z}_M\Lambda_f$, $\hat{R}_Q\Lambda_f$, and $\Lambda\Lambda_f$, respectively. The derivations of (28) are given in Appendix C.

Step 2. Establish exponential stability of the homogeneous part of (28): A(t) has a block upper triangular structure, whose eigenvalues are the same as the eigenvalues of its diagonal matrices, i.e., for each fixed time t, $A(t)$ has the same eigenvalues as $A_1 = \begin{bmatrix} 0_{(n-1)\times 1} | I_{n-1} \\ (\hat{\theta}_a - \theta_\lambda)^T \end{bmatrix}$, $A_2 = \begin{bmatrix} 0_{(n+m-1)\times 1} | I_{n+m-1} \\ -\hat{\theta}_{Qf}^T \end{bmatrix}$, and $A_3 = \begin{bmatrix} 0_{(n-1)\times 1} | I_{n-1} \\ -\theta_\lambda^T \end{bmatrix}$.

Since $\hat{\theta}_a$ is the coefficient vector of $\Lambda - \hat{R}_M$, the eigenvalues of A_1 are the solutions of $\hat{R}_M = 0$, which have negative real parts $\forall t \geq 0$ because of the stability constraint enforced in the parameter identification, as discussed in Section II. $\hat{\theta}_{Qf}$ is the coefficient vector of $\hat{R}_Q\Lambda_f$, so the eigenvalues of A_2 are the solutions of $\hat{R}_Q\Lambda_f = 0$, which also have negative real parts $\forall t \geq 0$ using the same argument. The eigenvalues A_3 are the solutions of $\Lambda = 0$ which also have negative real parts.

By Theorem 3, the constrained identification guarantees that $\hat{\theta}_a, \hat{\theta}_b, \hat{\theta}_c, \hat{\theta}_d \in l_\infty, \Delta\hat{\theta}_a, \Delta\hat{\theta}_b, \Delta\hat{\theta}_c, \Delta\hat{\theta}_d \in l_\infty \cap l_2$. Their zero-order hold (ZOH) signals are used in $A(t)$. Thus, $A(t)$ is piecewise differentiable with respect to t. $\|A(t)\| \in \mathcal{L}_\infty$. Applying Lemma 9 of Appendix B, let $k_0 = T_s$, the system is exponentially stable, and the state transition matrix $\Phi(t, \tau)$ associated with $A(t)$ satisfies $\|\Phi(t, \tau)\| \leq k_1 e^{-k_2(t-\tau)}, \forall t \geq \tau \geq 0$ for some constants $k_1, k_2 > 0$.

Step 3. Establish signal boundedness: The $\mathcal{L}_{2\delta}$ norm $\|(\bullet)_t\|_{2\delta}$ for some $\delta > 0$ is the exponentially weighted $\mathcal{L}_2$ norm defined as $\|x_t\|_{2\delta} := (\int_0^t e^{-\delta(t-\tau)} x^T(\tau)x(\tau)d\tau)^{\frac{1}{2}}$. Applying Lemma 10 of Appendix B to the state-space equation (28), we can obtain

$$\begin{aligned} \|x_t\|_{2\delta} &\leq c\|(\hat{\epsilon}_M m_M^2)_t\|_{2\delta} + c \\ |x(t)| &\leq c\|(\hat{\epsilon}_M m_M^2)_t\|_{2\delta} + c \end{aligned} \tag{29}$$

where $|\bullet|$ is a vector norm, for any $\delta \in [0, \delta_1)$ where $\delta_1 > 0$ is any constant less than $2k_2$ and some finite constant $c \geq 0$.

For simplicity of the representation in this paper, c is used to represent a generic constant.

We define the fictitious normalizing signal $m_f^2 := 1 + \|u_t\|_{2\delta}^2 + \|y_t\|_{2\delta}^2$. From the state-space equation, we have $\|u_t\|_{2\delta} + \|y_t\|_{2\delta} \le c\|x_t\|_{2\delta} + c\|(\hat{\epsilon}_M m_M^2)_t\|_{2\delta} + c$. With (29), we have $\|u_t\|_{2\delta} + \|y_t\|_{2\delta} \le c\|(\hat{\epsilon}_M m_M^2)_t\|_{2\delta} + c$, implying

$$m_f^2 \le c\|(\hat{\epsilon}_M m_M^2)_t\|_{2\delta}^2 + c.$$

From (18), applying Lemma 13 of Appendix B,

$$m_M = \sqrt{1 + \phi_M^T \phi_M} \le c m_f$$
$$m_f^2 \le c\|(\tilde{g} m_f)_t\|_{2\delta}^2 + c \tag{30}$$

where $\tilde{g} = \hat{\epsilon}_M m_M \in \mathcal{L}_{2e}$, or

$$m_f^2 \le c \int_0^t e^{-\delta(t-\tau)} \tilde{g}^2(\tau) m_f^2(\tau) d\tau + c$$

where $0 < \delta \le \delta^*$ and $\delta^* = \min[2\lambda, \delta_1]$, $\delta_1 \in (0, 2k_2)$.

Applying Bellman–Gronwall Lemma [15], we can establish that $m_f \in \mathcal{L}_\infty$. Then with (30), we have $m_M \in \mathcal{L}_\infty$ and therefore $\phi_M, x, \dot{x}, u, \{G_f\}y \in \mathcal{L}_\infty$, $\epsilon_Q m_Q^2 \in \mathcal{L}_\infty$, $m_Q = \sqrt{1 + \phi_Q^T \phi_Q} \in \mathcal{L}_\infty$. All the signals in the closed-loop system are uniformly bounded.

Step 4. Establish that the tracking error e converges to 0:

Since the tracking error e can be expressed as the sum of $e_M = \{G_f\}(y - y_M)$ and $e_Q = \{G_f\}l - y$ as in (11), we can establish convergence of e by demonstrating convergence of e_M and e_Q, respectively.

First, we consider the plant model estimation error $\hat{\epsilon}_M m_M^2 = y - y_M$ in (20). With the assumption that ϕ_M is PE, $\theta_M \to \theta_M^*$ exponentially according to Lemma 1. Therefore, $\epsilon_M m_M^2 = z_M - \theta_M^T \phi_M = (\theta_M^* - \theta_M)^T \phi_M \to 0$ as $t \to \infty$, and θ_M satisfies the stability constraints in (8) $\forall t \ge t_c$ for some finite t_c. Then according to Theorem 3, $\hat{\epsilon}_M m_M^2 \to \epsilon_M m_M^2 \to 0$ as $\gamma \to 0$. $e_M = \{G_f\}\hat{\epsilon}_M m_M^2$ according to (12). G_f is stable, therefore, $e_M \to 0$ as $\gamma \to 0$ and $t \to \infty$.

Following the same procedure, it can be shown that $\hat{\epsilon}_Q m_Q^2 \to 0$ as $t \to \infty$ and $\gamma \to 0$. With persistent excitation, from Lemma 1, we have $\Delta\theta_Q \to 0$. With the Lipschitz condition in Lemma 2, $\Delta\hat{\theta}_Q \to \Delta\theta_Q \to 0$. From (26), we have $e_Q = \hat{\epsilon}_Q m_Q^2 - \epsilon_6$, and $\epsilon_6 \to 0$ as $\Delta\hat{\theta}_Q \to 0$. Therefore, $e_Q \to 0$ as $\gamma \to 0$ and $t \to \infty$.

$e = e_M + e_Q$. Therefore, $e \to 0$ as $t \to \infty$ and $\gamma \to 0$, when the regressors ϕ_M and ϕ_Q are PE. ∎

Remark 2: Note that $\hat{\epsilon}_Q$ does not appear in the closed-loop representation (28); therefore, the property of $\hat{\epsilon}_Q$ is not required for establishing stability of CAIMC. It is only needed for establishing convergence of the tracking error.

Remark 3: The regularization term in the modified stability-constrained identification is critical for the closed-loop stability analysis of CAIMC. As shown in step 2 of the proof, boundedness and continuity of the modified stability-constrained identification is crucial for establishing exponential stability of the homogenous part of the closed-loop state-space equation using Lemma 9. The modified stability-constrained identification is general in the sense that it can be applied to any adaptive scheme where boundedness and continuity of the signals are required for establishing closed-loop stability.

Remark 4: Theorem 4 shows that the tracking error $e \to 0$ as $\gamma \to 0$ and $t \to \infty$ with the persistent excitation condition. γ, however, has to be nonzero to assure that the optimization problem (8) has an unique optimal solution. Therefore, the implication when γ is a small nonzero number is discussed.

According to Lemma 2, when θ satisfies the stability constraints in (8), $\hat{\theta}(\theta, \gamma)$ is Lipschitz continuous with respect to $\gamma \ge 0$. Therefore, $\hat{\epsilon}_M m_M^2 = z_M - \hat{\theta}_M^T \Phi_M$ and $\hat{\epsilon}_Q m_Q^2 = z_Q - \hat{\theta}_Q^T \Phi_Q$ are Lipschitz continuous with respect to $\gamma \ge 0$, which implies that e is Lipschitz continuous with respect to $\gamma \ge 0$. Therefore, when θ satisfies the stability constraints in (8) and $\gamma << 1$, e is small.

With the persistent excitation condition, θ will eventually converge to θ^*, and the estimated plant model and plant inverse model will be stable. One may remove the constrained optimization problem (8) when θ converges to a stable parameter, which leads to $\hat{\theta} = \theta$. In that case, $e = e_M + e_Q = \{G_f\}(\hat{\epsilon}_M m_M^2) + \hat{\epsilon}_Q m_Q^2 - \epsilon_6$ according to (11), (20), and (26). With persistent excitation, $\hat{\epsilon}_M m_M^2$, $\hat{\epsilon}_Q m_Q^2$, and ϵ_6 all converge to zero exponentially according to Lemma 1. Exponential convergence of the tracking error to zero is guaranteed.

Remark 5: For simplicity and clarity, the stability analysis is performed under the assumption that there are no unmodeled dynamics. In general, however, there are unmodeled dynamics in the presentation of the physical plant. To handle the unmodeled dynamics, a deadzone is typically added to the estimation error for robustness [15]. The robust CAIMC stability proof follows a very similar procedure by expanding the proof here as shown in [15] for robust adaptive pole placement control, and its tracking error e is bounded.

V. CAIMC Simulation Results

In this section, the CAIMC scheme is applied to the third-order LTI plant (9). Algorithm 1 is used to identify the unknown parameters of the plant and its inverse simultaneously. With this example, we demonstrate that the unconstrained adaptive law yields unstable estimates due to transients even when the initial estimates and the true parameters are stable, and Algorithm 1 guarantees stability of the estimated parameters. The tracking performance improves as the identified parameters converge.

Equation (9) is stable and minimum-phase, with relative degree 1. Assume that the desired bandwidth is approximately 0.8 Hz, then a first-order filter $G_f = \frac{1}{0.2s+1}$ is adopted.

The plant inverse modeled from the inverse of (9) appended with the filter G_f is

$$u = \left\{ \frac{\theta_{c3}^* s^3 + \theta_{c2}^* s^2 + \theta_{c1}^* s + \theta_{c0}^*}{s^3 + \theta_{d2}^* s^2 + \theta_{d1}^* s + \theta_{d0}^*} \right\} y \tag{31}$$

where $\theta_c^* = [2.5, 5, 5, 7.5]^T$, $\theta_d^* = [6, 5.5, 2.5]^T$ are assumed to be unknown. The inverse is stable. There are no unmodeled dynamics.

The CAIMC scheme, as summarized in Section III, is applied to the plant. The plant (9) and its inverse (31) are identi-

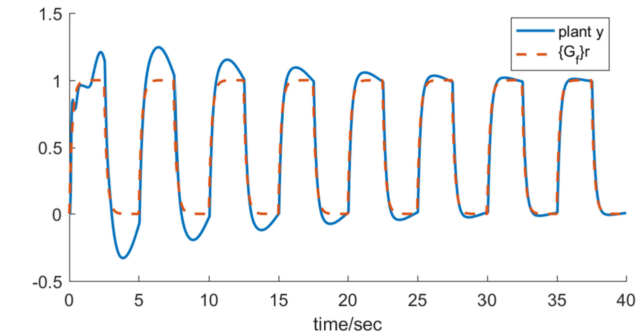

Fig. 4. CAIMC simulation result.

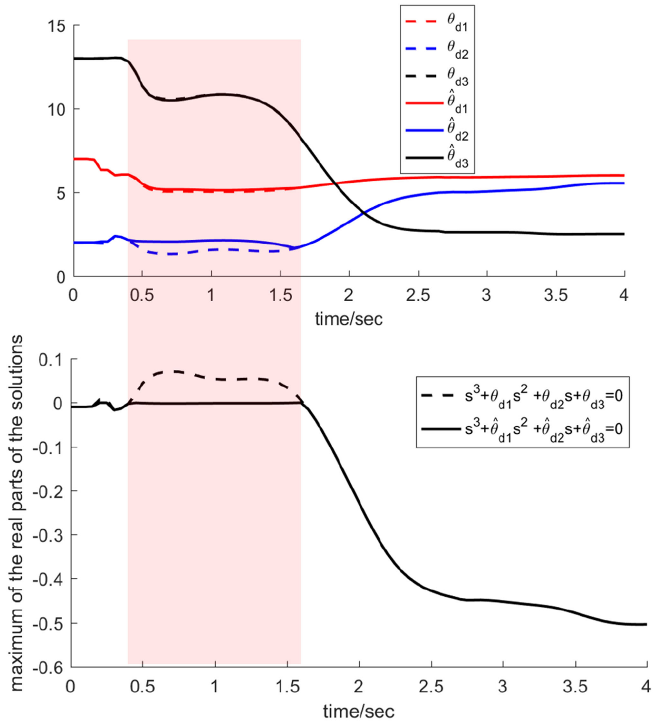

Fig. 5. CAIMC simulation parameters.

fied simultaneously using Algorithm 1. The constraints are imposed by stability of $\hat{\theta}_a$, $\hat{\theta}_c$, and $\hat{\theta}_d$. The initial conditions of the parameters are $\theta_a(0) = [10, 5, 4]^T, \theta_b(0) = [4, 2, 2]^T, \theta_c(0) = [2, 5, 2, 10]^T, \theta_d(0) = [7, 2, 13]^T$. They represent the stable plant and inverse models to start with.

The closed-loop response is shown in Fig. 4. The reference r is a square wave with period 5 s and amplitude 1. The plant response y tracks $\{G_f\}r$, and the performance is improved with the online identification of the parameters. Due to the space limitation, not all identified parameters are shown here. The identified parameters for θ_d^* at 0–4 s are included in Fig. 5. The initial condition $\theta_d(0)$ is stable, and the true θ_d^* is stable; however, θ_d from the unconstrained adaptive law is unstable in the shaded area around 0.4–1.6 s, as the maximum of the real parts of the poles is positive. $\hat{\theta}_d$ from the constrained optimization problem is always stable, as the maximum of the real parts of the poles is always negative. When θ is stable, $\hat{\theta}_d \approx \theta_d$. All the parameters converge to their desired values. The result is consistent with the simulations in Section II.

VI. Conclusion

CAIMC is generalized to SISO n-th-order LTI plants. Simultaneous identification of the model and inverse reduces modeling errors, thereby reducing the tracking error and improving control performance. The stability constraint for an n-th-order plant is re-parameterized as an LMI constraint, and the constrained identification is cast into a convex optimization problem. A regularization term is added for boundedness and continuity of the identified parameters to assure closed-loop stability of the adaptive scheme. The closed-loop stability proof and asymptotic performance are established for CAIMC in the ideal case. The effectiveness of the proposed identification algorithm and CAIMC is demonstrated on a third-order example.

Appendix A
Convex Programming Analysis Preliminaries

Consider the optimization problem

$$\underset{\chi}{\text{minimize}}\ f(\chi), \quad \text{subject to } \chi \in \Phi \tag{32}$$

where $\chi \in \mathcal{R}^n$ is the optimization variable, and the function $f : \mathcal{R}^n \to \mathcal{R}$ is the objective or cost function. The optimal value of the cost function is defined as $v^* = \inf\{f(\chi)|\chi \in \Phi\}$. χ^* is an optimal solution if $\chi^* \in \Phi$ and $f(\chi^*) = v^*$.

Lemma 5: [29] For (32), let Φ be a nonempty closed convex set and f be a strictly convex function over Φ, then the optimal solution χ^* is unique.

Consider the parameterized optimization problem

$$\underset{\chi}{\text{minimize}}\ f(\chi, p), \quad \text{subject to } \chi \in \Phi \tag{33}$$

where the cost function $f(\chi, p)$ depends on the parameter vector $p \in \mathcal{R}^m$, and the feasible region Φ is independent of p. Let $v^*(p) : \mathcal{R}^m \to \mathcal{R}$ represent the optimal cost value function.

Lemma 6: [30] For (33), the optimal cost value function $v^*(p)$ is upper semi-continuous, i.e., $\lim_{p\to p_0} v^*(p) \le v^*(p_0)$.

Assume that the optimal solution is unique and let $\chi^*(p) : \mathcal{R}^m \to \mathcal{R}^n$ represent the optimal solution function.

Lemma 7: [30] For (33), suppose that

1) The second-order growth condition holds for $f(\chi, p)$ at $\chi^*(p_0)$, i.e., there exists a neighborhood N of $\chi^*(p_0)$ and a constant $c > 0$ such that $f(\chi) \ge f(\chi^*) + c(\chi - \chi^*)^2, \forall \chi \in \Phi \cap N$.
2) The difference function $f(\chi, p) - f(\chi, p_0)$ is Lipschitz continuous with respect to χ modulus κ on $\Phi \cap N$, i.e., $\exists \kappa < \infty, \|(f(\chi_1, p) - f(\chi_1, p_0)) - (f(\chi_2, p) - f(\chi_2, p_0))\| \le \kappa \|\chi_1 - \chi_2\|, \forall \chi_1, \chi_2 \in \Phi \cap N$.

Then, $\|\chi^*(p) - \chi^*(p_0)\| \le c^{-1}\kappa$.

Appendix B
Mathematical Preliminaries for Stability Proof

The stability proof of CAIMC is done by representing the closed-loop system as a linear time-varying (LTV) system. Relevant results are introduced here to establish exponential stability and signal boundedness of linear systems.

Lemma 8. Swapping Lemma: [15] Let $\tilde{\theta}$ be differentiable, and $\omega : \mathcal{R}^+ \to \mathcal{R}^n$. Let W be a proper stable rational transfer function with a minimal realization (A_W, B_W, C_W, d_W). Then,

$$\{W\}\tilde{\theta}^T\omega = \tilde{\theta}^T\{W\}\omega + \{W_c\}((\{W_b\}\omega^T)\dot{\tilde{\theta}})$$

where $W_c{=}{-}C_W^T(sI{-}A_W)^{-1}$, and $W_b{=}(sI{-}A_W)^{-1}B_W$.

Lemma 9: Consider an LTV system $\dot{x} = A(t)x$, where $x \in \mathcal{R}^n$, and the elements of $A(t)$ are piecewise differentiable and bounded. Assume that $Re\{\lambda_i(A(t))\} < -\delta_s\ \forall t \geq 0$ and for $i = 1, 2, \ldots, n$, where $\delta_s > 0$ is some constant. Also, assume that $\|A(t)\| \leq c$, for some constant $c > 0, \forall t \geq 0$, where $\|A(t)\|$ is the induced norm.

If $\exists k_0 > 0, \delta_0 > 0$, $\sup_{0\leq\tau\leq k_0} \|A(t+\tau) - A(t)\| \leq \delta_0$, then the equilibrium state $x_e = 0$ is exponentially stable, i.e., the state transition matrix

$$\|\Phi(t,\tau)\| \leq \lambda_0 e^{-\alpha_0(t-\tau)}, \forall t \geq \tau \geq 0$$

for some $\lambda_0, \alpha_0 > 0$ [31].

Lemma 10: Consider an LTV system given by $\dot{x} = A(t)x + B(t)u$, where $x \in \mathcal{R}^n, y \in \mathcal{R}^r, u \in \mathcal{R}^m$, and the elements of the matrices $A(t), B(t)$ are bounded piecewise continuous functions of time. If the state transition matrix $\|\Phi(t,\tau)\| \leq \lambda_0 e^{-\alpha_0(t-\tau)}$ for some $\lambda_0, \alpha_0 > 0$ and $u \in \mathcal{L}_{2e}$,[5] then for any $\delta \in [0, \delta_1)$ where $0 < \delta_1 < 2\alpha_0$ is arbitrary, we have

- $|x(t)| \leq \frac{c\lambda_0}{\sqrt{2\alpha_0-\delta}}\|u_t\|_{2\delta} + \epsilon_t$
- $\|x_t\|_{2\delta} \leq \frac{c\lambda_0}{\sqrt{(\delta_1-\delta)(2\alpha_0-\delta_1)}}\|u_t\|_{2\delta} + \epsilon_t$

  where $c = \sup_t \|B(t)\|$, and ϵ_t is an exponentially decaying to zero term due to the initial condition [15].

Lemma 11: Consider an LTI system $y = \{H\}u$. If $u \in \mathcal{L}_{2e}$ and $h \in \mathcal{L}_1$,[6] where h is the impulse response of H, then

$$\|y_t\|_2 \leq \sup_\omega |H(j\omega)|\|u_t\|_2.$$

Lemma 12: Let H be a strictly proper rational function of s. Then, H is analytic in Re$[s] \geq 0$ if and only if $h \in \mathcal{L}_1$.

Lemma 13: Consider an LTI system $y = \{H\}u$, where H is strictly proper and analytic in $Re(s) \geq -\frac{\delta}{2}$ for some $\delta > 0$ and $u \in \mathcal{L}_{2e}$. Then, we have $|y(t)| \leq c\|u_t\|_{2\delta}$ for some c [15].

Lemma 14. Bellman–Gronwall (B-G) Lemma: [15] Let $\lambda(t), g(t), k(t)$ be nonnegative piecewise continuous functions of time t. If a function $f(t)$ satisfies the inequality $f(t) \leq g(t)\int_{t_0}^t k(s)f(s)ds + \lambda(t), \forall t \geq t_0 \geq 0$, then

$$f(t) \leq g(t)\int_{t_0}^t \lambda(s)k(s)\left[\exp\left(\int_s^t k(\tau)g(\tau)d\tau\right)\right]ds + \lambda(t)$$
$$\forall t \geq t_0 \geq 0.$$

In particular, if $\lambda(t) \equiv \lambda$ is a constant and $g(t) \equiv 1$, then

$$f(t) \leq \lambda \exp\left(\int_{t_0}^t k(s)ds\right), \quad \forall t \geq t_0 \geq 0.$$

[5]A signal $x \in \mathcal{L}_{2e}$, if the truncated signal $x_t(\tau) = \begin{cases} x(\tau), & \text{if } 0 \leq \tau \leq t \\ 0, & \text{if } \tau > t \end{cases}$ belongs to $\mathcal{L}_2$ for any finite t.

[6]A continuous signal $x \in \mathcal{L}_1$ when $\int_0^\infty |x(t)|dt$ is finite.

Appendix C
Derivation of the Closed-Loop State-Space Equation of CAIMC

Let S_n represent the vector $[1, s, \ldots, s^{n-1}]^T$. Combining (18), (19), and (27), let $\hat{\theta}_{Mf} \in \mathcal{R}^{n+m}$ be the coefficient vector of $\hat{Z}_M\Lambda_f$, we have $\hat{\epsilon}_M m_M^2 = z_M - \hat{\theta}_M^T\phi_M = \{s^n\}y_f + \theta_\lambda^T\{S_n\}y_f - \hat{\theta}_a^T\{S_n\}y_f - \hat{\theta}_{Mf}^T\{S_n\}u_f$. Therefore,

$$y_f^{(n)} = (\hat{\theta}_a - \theta_\lambda)^T\{S_n\}y_f + \hat{\theta}_{Mf}^T\{S_n\}u_f + \hat{\epsilon}_M m_M^2. \quad (34)$$

From (22) and (27), and $l = r - \hat{\epsilon}_M m_M^2$, let $\bar{r} = \{\hat{Z}_Q\frac{1}{\Lambda}\}r$ and $\hat{\theta}_c = \begin{bmatrix}\hat{\theta}_{cn} \\ \eta\end{bmatrix}$, where $\hat{\theta}_{cn} \in \mathcal{R}^n$ and η is the $(n+1)$-th entry of $\hat{\theta}_c$. Then,

$$\begin{aligned}\{\hat{R}_Q\Lambda_f\}u_f &= \bar{r} - \left\{\hat{Z}_Q\frac{1}{\Lambda}\right\}\hat{\epsilon}_M m_M^2 \\ &= \bar{r} - \hat{\theta}_{cn}^T\{S_n\}\epsilon_f - \eta s^n\epsilon_f \\ &= \bar{r} - \hat{\theta}_{cn}^T\{S_n\}\epsilon_f - \eta(\Lambda - \theta_\lambda\{S_n\})\epsilon_f \\ &= \bar{r} - (\hat{\theta}_{cn}^T - \eta\theta_\lambda)\{S_n\}\epsilon_f - \eta\hat{\epsilon}_M m_M^2.\end{aligned}$$

Let $\hat{\theta}_{Qf} \in \mathcal{R}^{n+m}$ represent the coefficient vector of $\hat{R}_Q\Lambda_f$. Then,

$$\begin{aligned}u_f^{(n+m)} = &-\hat{\theta}_{Qf}^T\{S_{n+m}\}u_f - (\hat{\theta}_{cn} - \eta\theta_\lambda)^T\{S_n\}\epsilon_f \\ &- \eta\hat{\epsilon}_M m_M^2 + \bar{r}.\end{aligned} \quad (35)$$

From (27),

$$\epsilon_f^{(n)} = -\theta_\lambda^T\{S_n\}\epsilon_f + \hat{\epsilon}_M m_M^2. \quad (36)$$

Combining (34), (35), and (36), we have the state-space equation as shown in (28).

References

[1] M. Morari and E. Zafiriou, *Robust Process Control*. Prentice-Hall, Englewood Cliffs, New Jersey, 1989.

[2] C. E. Garcia and M. Morari, “Internal model control. A unifying review and some new results,” *Ind. Eng. Chem. Process Des. Dev.*, vol. 21, no. 2, pp. 308–323, 1982.

[3] I. G. Horn, J. R. Arulandu, C. J. Gombas, J. G. VanAntwerp, and R. D. Braatz, “Improved filter design in internal model control,” *Ind. Eng. Chem. Res*, vol. 35, no. 10, pp. 3437–3441, 1996.

[4] N. L. Ricker, “Use of quadratic programming for constrained internal model control,” *Ind. Eng. Chem. Res*, vol. 24, no. 4, pp. 925–936, 1985.

[5] M. Campi, W. S. Lee, and B. D. O. Anderson, “New filters for internal model control design,” *Int. J. Robust Nonlinear Control*, vol. 4, pp. 757–775, 1994.

[6] A. Datta, *Adaptive Internal Model Control*. Springer-Verlag, New York, New York, 1998.

[7] A. Datta and J. Ochoa, “Adaptive internal model control: Design and stability analysis,” *Automatica*, vol. 32, no. 2, pp. 261–266, 1996.

[8] A. Datta and J. Ochoa, “Adaptive internal model control: H_2 optimization for stable plants,” *Automatica*, vol. 34, no. 1, pp. 75–82, 1998.

[9] A. Datta and L. Xing, “Adaptive internal model control: H_∞ optimization for stable plants,” *IEEE Trans. Autom. Control*, vol. 44, no. 11, pp. 2130–2134, 1999.

[10] D. Rupp and L. Guzzella, "Adaptive internal model control with application to fueling control," *Control Eng. Practice*, vol. 18, no. 8, pp. 873–881, 2010.

[11] N. R. Lakshmi Narayanan, P. R. Krishnaswamy, and G. P. Rangaiah, "An adaptive internal model control strategy for pH neutralization," *Chem. Eng. Sci.*, vol. 97, no. 18, pp. 3067–3074, 1997.

[12] X. Shao, J. Zhang, Z. Zhao, and X. Wen, "Adaptive internal model control of permanent magnet synchronous motor drive system," in *Proc. Eighth Int. Conf. Elect. Machines Syst*. ICEMS, 2005, vol. 3, pp. 1843–1846.

[13] Z. Qiu, M. Santillo, M. Jankovic, and J. Sun, "Composite adaptive internal model control and its application to boost pressure control of a turbocharged gasoline engine," *IEEE Trans. Control Syst. Technol.*, vol. 23, no. 6, pp. 2306–2315, 2015.

[14] Z. Qiu, M. Santillo, J. Sun, and M. Jankovic, "Enhanced composite adaptive IMC for boost pressure control of a turbocharged gasoline engine," *Amer. Control Conf.*, 2016, pp. 3286–3291.

[15] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Dover, Mineola, New York, 2012.

[16] G. van der Veen, J.-W. van Wingerden, M. Bergamasco, M. Lovera, and M. Verhaegen, "Closed-loop subspace identification methods: An overview," *IET Control Theory Appl.*, vol. 7, no. 10, pp. 1339–1358, 2013.

[17] N. L. C. Chui and J. M. Maciejowski, "Realization of stable models with subspace methods," *Automatica*, vol. 32, no. 11, pp. 1587–1595, 1996.

[18] T. Van Gestel, J. A. K. Suykens, P. Van Dooren, and B. De Moor, "Identification of stable models in subspace identification by using regularization," *IEEE Trans. Autom. Control*, vol. 46, no. 9, pp. 1416–1420, Sep. 2001.

[19] S. L. Lacy and D. S. Bernstein, "Subspace identification with guaranteed stability using constrained optimization," *IEEE Trans. Autom. Control*, vol. 48, no. 7, pp. 1259–1263, Jul. 2003.

[20] L. Vandenberghe and S. Boyd, "Semidefinite programming," *Soc. Ind. Appl. Math.*, vol. 38, no. 1, pp. 49–95, 1996.

[21] D. N. Miller and R. A. Callafon, "Subspace identification with eigenvalue constraints," *Automatica*, vol. 49, no. 8, pp. 2468–2473, 2013.

[22] Z. Qiu, J. Sun, M. Jankovic, and M. Santillo, "Generalized composite adaptive IMC: Design and analysis," in *Proc. IEEE Conf. Decis. Control*, 2016, pp. 334–340.

[23] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Volume 15 of Studies in Applied Mathematics. SIAM, Philadelphia, Pennsylvania, 1994.

[24] S. Boyd and L. Vandenberghe, *Convex Programming*. Cambridge University Press, New York, New York, 2004.

[25] H. K. Khalil, *Nonlinear Systems*. Prentice Hall, Upper Saddle River, New Jersey, 2002.

[26] D. Bernstein and W. Haddad, "Robust stability and performance analysis for linear dynamic systems," *IEEE Trans. Autom. Control*, vol. 34, no. 7, pp. 751–758, Jul. 1989.

[27] D. Arzelier, J. Bernussou, and G. Garcia, "Pole assignment of linear uncertain systems in a sector via a Lyapunov-type approach," *IEEE Trans. Autom. Control*, vol. 38, no. 7, pp. 1128–1131, Jul. 1993.

[28] K. S. Tsakalis and P. A. Ioannou, *Linear Time-Varying Systems*. Prentice Hall, Upper Saddle River, New Jersey, 1993.

[29] A. Nedic, *Network Mathematics Graduate Programme* lecture notes, Maynooth, Ireland: Hamilton Institute, 2008.

[30] J. F. Bonnans and A. Shapiro, *Perturbation Analysis of Optimization Problems*. Springer-Verlag, New York, New York, 2000.

[31] A. Ilchmann, D. H. Owens, and D. Pratzel-Wolters, "Sufficient conditions for stability of linear time-varying systems," *Syst. Control Lett.*, vol. 9, no. 2, pp. 157–163, 1987.

Zeng Qiu received the B.S. degree from the Beijing University of Posts and Telecommunications, Beijing, China, in 2011, and the M.S. and Ph.D. degrees from the University of Michigan, Ann Arbor, MI, USA, in 2013 and 2017, respectively, all in electrical engineering.

She is currently with the Ford Motor Company, Research and Advanced Engineering, Dearborn, MI, USA. Her research interests include internal model control, adaptive control, optimization, and their automotive applications.

Jing Sun (F'04) received the Ph.D. degree from the University of Southern California, Los Angeles, CA, USA, in 1989.

From 1989 to 1993, she was an Assistant Professor with the Electrical and Computer Engineering Department, Wayne State University, Detroit, MI, USA. She joined Ford Research Laboratory, Dearborn, MI, USA, in 1993, where she worked at the Powertrain Control Systems Department. After spending almost 10 years in industry, she came back to academia and joined the faculty of the College of Engineering at the University of Michigan, Ann Arbor, MI, USA, in 2003, where she is now Michael G. Parsons Professor and the Chair in the Department of Naval Architecture and Marine Engineering, with courtesy appointments as a Professor in the Department of Electrical Engineering and Computer Science and Department of Mechanical Engineering. Her research interests include system and control theory and its applications to marine and automotive propulsion systems. She holds 41 US patents and has co-authored a textbook on Robust Adaptive Control.

Dr. Sun is a Fellow of National Academy of Inventors, and a recipient of the 2003 IEEE Control System Technology Award.

Mrdjan Jankovic (F'16) received the Bachelor degree from the University of Belgrade, Beograd, Serbia (1986), and Master and Doctoral degrees from the Washington University, St. Louis, MO, USA (1989 and 1992).

He held postdoctoral positions with Washington University and UC Santa Barbara, CA, USA. He has joined Ford Research in 1995 where he is currently a Senior Technical Leader overseeing a team working on development of automotive control technologies. He has co-authored one book, four book chapters, and more than 100 technical papers. He is a co-inventor on more than 70 US patents, 18 of which are implemented in Ford products worldwide. Dr. Jankovic received IEEE Control Systems Technology Award, AACC Control Engineering Practice Award, two Ford Research Technical Achievement Awards, and three best paper awards from IEEE, SAE, and AVEC. He served as an Associate Editor of the *IEEE Transactions on Control Systems Technology* and as a Chair of several IEEE and SAE committees.

Mario Santillo received the B.S. degree in aeronautical and mechanical engineering from the Rensselaer Polytechnic Institute, Troy, NY, USA, in 2003, and the M.S.E. and Ph.D. degrees in aerospace engineering from the University of Michigan, Ann Arbor, MI, USA, in 2005 and 2009, respectively.

He is currently with Ford Motor Company, Research and Advanced Engineering, Dearborn, MI, USA. His research interests include adaptive control and advanced control methods for automotive and robotics applications.