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Brief paper

Parameter governors for discrete-time nonlinear systems with pointwise-in-time state and control constraints[☆]

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Abstract

Parameter governors are add-on control schemes that adjust parameters (such as gains or offsets) in the nominal control laws to avoid violation of pointwise-in-time state and control constraints and to improve the overall system transient performance via the receding horizon minimization of a cost functional. As compared to more general model predictive controllers, parameter governors tend to be more conservative but the computational effort needed to implement them on-line can be relatively modest because the few parameters to be optimized remain constant over the prediction horizon. In this paper, we discuss the properties of several classes of parameter governors which have a common property in that the governed parameters do not shift the steady-state equilibrium of the states on which the incremental cost function explicitly depends on. This property facilitates the application of meaningful cost functionals. An example, together with simulation results, is reported to provide additional insights into the operation of the proposed parameter governor schemes.

Keywords: State and input constraints; Model-predictive control; Receding horizon optimal control; Reference governors

1. Introduction

The *parameter governors* are add-on control schemes that modify parameters of the nominal control systems to enforce pointwise-in-time state and input constraints, and to improve the system transient performance. The governed parameters are selected via a receding horizon minimization of a cost functional.

Fig. 1 illustrates the application of a parameter governor to a discrete-time nonlinear system which is controlled by a parameter-dependent feedback law:

$$x(t+1) = f(x(t), u(t), \theta(t), r(t)),$$
$$u(t) = u_c(x(t), \theta(t), r(t)). \tag{1}$$

Here $x(t)$ is the state, $u(t)$ is the control input, $\theta(t)$ is a vector of adjustable parameters in a specified control function u_c, $r(t)$ is a reference command, and $t \in \mathbf{Z}^+$, where $\mathbf{Z}^+$ is the set of non-negative integers. The state vector $x(t)$ may include both plant states and controller states, and depending on the form of the feedback law and parameter governor, f may explicitly depend on $\theta(t)$ and $r(t)$. The pointwise-in-time constraints, imposed on $x(t)$, $\theta(t)$, have the following form:

$$(\theta(t), x(t)) \in C(r(t)) \quad \forall t \in \mathbf{Z}^+, \tag{2}$$

where $C(r(t))$ is a specified set which may depend on $r(t)$. Note that the pointwise-in-time constraints on the control input $u(t)$ in (1) can always be recast as equivalent constraints of the form (2).

In a typical scenario, a nominal closed-loop system (corresponding to $\theta(t) = 0$) is first designed for closed-loop stability and good "small signal" behavior but without the consideration of the constraints. Well-developed control design methodologies exist for this purpose. The parameter governor is then added on to enforce the constraints and to improve the system transient performance via the on-line adjustment of $\theta(t)$. This

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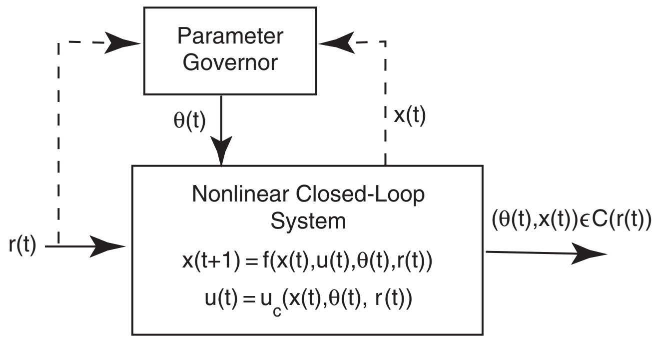

Fig. 1. The parameter governor.

adjustment is based on the following receding horizon principle. At the time instant t, $\theta(t)$ is selected so that with $\theta(t+k) \equiv \theta(t)$, $k=0,\ldots,T$, a cost functional of the form

$$J(x(t),\theta(t),r(t),T) \triangleq \|\theta(t)\|^2_{\Psi_\theta} + \sum_{k=0}^{T} \Omega(x^+(k|x(t),\theta(t),r(t)),\theta(t),r(t)) \tag{3}$$

is minimized with respect to the choice of $\theta(t)$ subject to the constraints being satisfied, i.e.,

$$(\theta(t), x^+(k|x(t),\theta(t),r(t))) \in C(r(t)), \quad k=0,1,\ldots,T. \tag{4}$$

Here T is the prediction horizon; $x^+(k|x(t),\theta(t),r(t))$, denotes the k steps ahead prediction of the system state, given the state of the system at time t, $x(t)$, and assuming that $\theta(t+k)=\theta(t)$, $r(t+k)=r(t)$ for $k=0,\ldots,T$. In the cost functional (3) a penalty on $\theta(t)$, of the form $\|\theta(t)\|^2_{\Psi_\theta} = \theta(t)^{\mathrm{T}}\Psi_\theta\theta(t)$, is used, where $\Psi_\theta = \Psi_\theta^{\mathrm{T}} \geqslant 0$. The incremental cost function is assumed to be of the form

$$\Omega(x,\theta,r) \triangleq Q(x - x_e(r), u_c(x,\theta,r) - u_e(r)), \tag{5}$$

where Q is non-negative and $u_e(r) = u_c(x_e(r), 0, r)$ is the steady-state value of the control input corresponding to $x_e(r)$ and $\theta(t) \equiv 0$. Here $x_e(r(t))$ denotes the desired equilibrium state of system (1) corresponding to $r(t)$ and $\theta(t) \equiv 0$.

As an add-on control mechanism, the parameter governor has some common features with the well-studied reference governor (see e.g., Angeli & Mosca, 1999; Bemporad, 1998; Bemporad, Casavola, & Mosca, 1996, 1997; Gilbert, Kolmanovsky, & Tan, 1995; Gilbert & Kolmanovsky, 2002; Kapasouris, Athans, & Stein, 1990). In fact, the reference governor can be viewed as a special parameter governor that provides only a reference filtering mechanism without modifying the closed-loop dynamics. The virtual reference $v(t)$ in the reference governor case plays essentially the same role as $\theta(t)$ in our parameter governor formulation. In terms of what we are after, the reference governor has a disadvantage in that over the prediction horizon with $\theta(t+k) \equiv \theta(t)$, the state trajectories usually do not converge to a neighborhood of the desired equilibrium, $x_e(r(t))$, but to a neighborhood of $x_e(v(t))$; this complicates the application of the cost functionals of the form (3), (5) which penalize the deviation of the state from the *true* set-point corresponding to $r(t)$. See, for example, the work of Angeli and Mosca (1999) for the type of cost functionals which can be applied in the reference governor context.

The paper will discuss several parameter governor schemes which are designed in such a way that changes in $\theta(t)$ do not shift the equilibrium values of those states on which Q in (5) depends explicitly. The properties of one such scheme, the so-called *gain governor*, will be described first, in Section 2. The gain governor, which adjusts the controller gains at discrete time instants, is a generalization of the multi-mode controller studied, for example, in Kolmanovsky and Gilbert (1997). A generalization of the gain governor will be presented in Section 3 along with another parameter governor scheme, the so-called *feedforward governor*. Section 4 will describe an example of a gain governor applied to an engine control problem. The benefits of the parameter governor approach and other concluding remarks will be summarized in Section 5.

In the subsequent analysis of asymptotic properties of the parameter governors, we will assume that $r(t)$ remains constant for all t. The parameter governors can cope with large changes in $r(t)$ (as will be further commented on in the paper), but if arbitrary changes in $r(t)$ are permitted a reference governor needs to be included to ensure sufficient flexibility for the overall parameter governing scheme to rigorously enforce the constraints and improve transient performance.

2. The gain governor

In the gain governor case, (1) has the form,

$$x(t+1) = f(x(t), u(t)), \tag{6}$$

where $x(t) \in \mathbf{R}^p$ is the state, $u(t) \in \mathbf{R}^m$ is the control input and in the analysis we assume that $r(t) \equiv r$ for all $t \in \mathbf{Z}^+$. The function f is assumed to be continuous in its arguments. The equilibrium values of the state, $x_e(r)$, and control input, $u_e(r)$, satisfy $f(x_e(r), u_e(r)) = x_e(r)$; they are assumed to be unique for the given r.

The control input $u(t)$ is generated as a sum of the nominal feedforward term, $u_e(r)$, and a feedback term u_{fb}, which is assumed to depend continuously on $x(t)$ and the parameters in the control law $\theta(t) \in \mathbf{R}^s$:

$$u(t) = u_e(r) + u_{fb}(x(t), \theta(t), r). \tag{7}$$

We further assume that $u_{fb}(x_e(r), \theta, r) = 0$ for all $\theta \in \Theta \subset \mathbf{R}^s$. This property is characteristic of the gains in a feedback control law, as they usually multiply tracking errors. Hence, we refer to the parameter governor scheme which adjusts $\theta(t)$ in (7) as the *gain governor*.

The rationale for the gain governor can be easily understood in the case of systems with input constraints. Specifically, the gain governor can lower the gains when it becomes necessary to avoid violating the input constraints; the gain governor can increase the gains when there is no danger of constraint violation and doing so improves the performance. It is clear that large changes in $r(t)$ can also be accommodated in this situation.

The on-line selection of $\theta(t)$ for each $t \in \mathbf{Z}^+$ is based on the minimization of the cost functional (3) subject to the constraints (4). The horizon $T > 0$ needs to be selected in agreement with our subsequent assumptions. Note that (3) and (4) can be evaluated on-line by computing state predictions $x^+(k|x(t), \theta(t), r)$ based on simulating models (6)–(7). If $C(r)$ admits an inequality characterization

$$C(r) = \{(\theta, x) \in \mathbf{R}^{s+p} : g_j(\theta, x, r) \leqslant 0, \quad j = 1, \ldots, q\},$$

then constraint (4) reduces to

$$g_j(\theta(t), x^+(k|x(t), \theta(t), r), r) \leqslant 0, \quad j = 1, \ldots, q; \quad k = 0, \ldots, T. \tag{8}$$

The constraint (4) can be restated equivalently as

$$(\theta(t), x(t)) \in O_T(r), \quad O_T(r) \triangleq \{(\theta, x) \in \mathbf{R}^{s+p}: (\theta, x^+(k|x, \theta, r)) \in C(r), k = 0, 1, \ldots, T\}. \tag{9}$$

The rigorous theoretical results are based on the following assumptions. These assumptions are somewhat stronger than really needed but they simplify the exposition of the main ideas.

(A1) The set $C(r) \subset \mathbf{R}^{s+p}$ in (2) is compact and $\theta(t) \in \Theta$, where $\Theta \subset \mathbf{R}^s$ is a compact set.
(A2) There exists $\delta > 0$ such that for all $\theta \in \Theta$, $(\theta, x_e(r)) + \delta\mathscr{B}_{s+p} \subset C(r)$, where $\mathscr{B}_{s+p}$ is the unit ball in $\mathbf{R}^{s+p}$.
(A3) $x^+(k|\bar{x}, \theta, r) \to x_e(r)$ as $k \to \infty$ for all $\theta \in \Theta$ and $(\theta, \bar{x}) \in C(r)$.
(A4) There exists $k_1^* \in \mathbf{Z}^+$ and $0 \leqslant q < 1$ such that for all $(\theta, \bar{x}) \in C(r)$, $\theta \in \Theta$ and $k \geqslant k_1^*$,

$$\Omega(x^+(k|\bar{x}, \theta, r), \theta, r) \leqslant q \cdot \Omega(\bar{x}, \theta, r).$$

(A5) The function Q in (5) is continuous and is such that for all $\theta \in \Theta$, $\Omega(x_e(r), \theta, r) = 0$, and if $x \neq x_e(r)$ then $\Omega(x, \theta, r) > 0$.

Assumption (A1) may require that artificial constraints be added for the state variables that are unconstrained by the virtue of the problem formulation. Compactness of Θ and $C(r)$ may be relaxed to their boundness if (A2) and (A3) hold for closures of Θ and $C(r)$. Assumption (A2) can be interpreted as a strict steady-state feasibility condition. Assumption (A3) characterizes the needed stability properties of the system when $\theta(t)$ is maintained at a constant value. Given that (6), (7) are intended to represent a stable closed-loop system, both (A3) and (A4) are reasonable and not very limiting. Assumption (A5) is imposed on the incremental cost function and not on the original system itself. It holds, for example, if Q is continuous, $Q(0, 0) = 0$, and Q is positive-definite in the state variable, i.e., $Q(a, b) > 0$ if $a \neq 0$.

Assumptions (A2) and (A3) and the compactness of $C(r)$ and Θ imply the following:

Proposition 1. *There exists $k_2^* \in \mathbf{Z}^+$ such that for all $\theta \in \Theta$ and $(\theta, \bar{x}) \in C(r)$, if $(\theta, x^+(k|\bar{x}, \theta, r)) \in C(r)$ for $k = 0, \ldots, k_2^*$, then $(\theta, x^+(k|\bar{x}, \theta, r)) \in C(r)$ for all $k \in \mathbf{Z}^+$.*

The result in Proposition 1 enables to relax the conditions $(\theta, x^+(k|x(t), \theta, r)) \in C(r)$ *for all* $k \in \mathbf{Z}^+$ to $(\theta, x^+(k|x(t), \theta, r)) \in C(r)$ *for* $k = 0, 1, \ldots, T$, provided that T is sufficiently large. A similar property has been exploited in the reference governor case (Bemporad, 1998) and it is related to finite determination of maximum constraint admissible sets (Gilbert et al., 1995).

The main result characterizing the response properties of the gain governor is given by the following theorem.

Theorem 2. *Suppose assumptions* (A1)–(A5) *hold*, $T > max\{k_1^*, k_2^*\}$, *where k_1^* is defined in* (A4) *and k_2^* is defined in Proposition* 1, *and the initial state $x(0)$ is feasible in the sense that there exists $\theta(0) \in \Theta$ such that $(\theta(0), x(0)) \in O_T(r)$ for all $k \geqslant 0$. Suppose further that $\theta(t) = \theta^*(t)$, $t \geqslant 0$, has been selected and let $x^*(t)$, $u^*(t)$ denote, respectively, the resulting state and control trajectories. If for each $t \in \mathbf{Z}^+$,*

$$J(x^*(t), \theta^*(t), r, T) \leqslant J(x^*(t), \theta^*(t-1), r, T), \tag{10}$$

and $(\theta^*(t), x^*(t)) \in O_T(r)$, then $x^*(t)$ remains feasible for all $t \geqslant 0$ (in particular, constraints $(\theta^*(t), x^*(t)) \in C(r)$ are satisfied for all $t \geqslant 0$) and $x^*(t) \to x_e(r)$, $u^*(t) \to u_e(r)$ as $t \to \infty$. Furthermore, $\|\theta^*(t)\|^2_{\Psi_\theta}$ converges to a limit.

Proof. The proof of Theorem 2 is similar to the stability proofs for receding horizon optimal controllers (see, for example, Mayne, Rawlings, Rao, & Scokaert, 2000). Using (A4), (3), (10) and that $\theta^*(t)$ is a feasible choice (guarantees constraint satisfaction) at time $t + 1$, we obtain,

$$\begin{aligned} &J(x^*(t+1), \theta^*(t+1), r, T) \leqslant J(x^*(t+1), \theta^*(t), r, T) \\ &\leqslant J(x^*(t), \theta^*(t), r, T) - (1-q) \cdot \Omega(x^*(t), \theta^*(t), r). \end{aligned} \tag{11}$$

Note that the first inequality in (11) is based on (10) while the second inequality in (11) is based on (A4). Since $0 \leqslant q < 1$, and Ω takes only non-negative values, the sequence $\{J(x^*(t), \theta^*(t), r, T)\}$ is bounded and non-increasing with t. Therefore, it has a limit as $t \to \infty$ and

$$\Omega(x^*(t), \theta^*(t), r) \to 0 \quad \text{as } t \to \infty. \tag{12}$$

By (A5) and continuity of u_{fb}, $x^*(t) \to x_e(r)$ and $u^*(t) \to u_e(r)$. Finally, note that $J(x^*(t), \theta^*(t), r, T)$ converging to a limit, (12) and (3) imply that $\|\theta^*(t)\|^2_{\Psi_\theta}$ converges too. The proof is complete. □

Remark 1. The cost non-increase condition (10) allows significant flexibility in applying numerical optimization to (3) and (4). In particular, the exact minimizer is not required and (10), (4) for $t > 0$ can be trivially satisfied with $\theta^*(t) = \theta^*(t-1)$ if the numerical optimizer fails to provide a better value. Clearly, $\theta^*(t-1)$ can be used as an initial guess by the numerical optimizer when computing $\theta^*(t)$ at the time instant t.

Remark 2. Theorem 2 applies when Θ consists only of a *finite* number of elements. In this case the minimization of (3) subject to (4) can be accomplished by performing a finite number of on-line model simulations for each value of $\theta(t) \in \Theta$

and comparing the cost values for those trajectories which satisfy (4). Each of the simulations can be run up to the horizon T or until the first time instant when constraints become violated.

Remark 3. If T satisfies the assumptions of Theorem 2, then the terminal set and terminal penalty conditions (Mayne et al., 2000) are not needed to guarantee the state convergence. In the gain governor case, these conditions would require that a terminal penalty function term, $F(x^+(T|x(t), \theta(t), r))$, where $F(x_e(r)) = 0$, be added to (3) and a terminal set condition $x^+(T|x(t), \theta(t), r) \in \Gamma$ be imposed. The terminal set Γ must be positively invariant *for all* $\theta \in \Theta$ (i.e., $x^+(k|\bar{x}, \theta, r) \in \Gamma$ if $\bar{x} \in \Gamma$) and constraint-admissible (i.e., $(\theta, x^+(k|\bar{x}, \theta, r) \in C$ for all $\bar{x} \in \Gamma$, $\theta \in \Theta$ and $k \in \mathbf{Z}^+$). The terminal penalty function F must satisfy $F(x^+(1|\bar{x}, \theta, r)) - F(\bar{x}) \leqslant -\Omega(x^+(1|\bar{x}, \theta, r), \theta, r)$ for all $\bar{x} \in \Gamma, \theta \in \Theta$. As these terminal set and terminal penalty conditions complicate the optimization, not requiring them is an advantage.

Remark 4. The dimensionality of the optimization problem for determining $\theta(t)$ does not grow with T (it remains equal to s).

Remark 5. The number of constraints in (4) or (8) grows with T. This can complicate on-line optimization if T is large. As in the reference governor case (Gilbert & Kolmanovsky, 2002), the use of a simple off-line functional characterization of a subset, $M(r) = \{(\theta, x) : V(x, \theta, r) \leqslant 0\} \subset O_T(r)$, in place of $O_T(r)$ provides an alternative. In this case, multiple inequalities in (4) or (8) can be replaced by a single inequality, $V(x(t), \theta(t), r) \leqslant 0$. With $M(r)$ used in place of $O_T(r)$ no feasible $\theta(t) \in \Theta$ may exist for some t, i.e., it can happen that $V(x(t), \theta) > 0$ for all $\theta \in \Theta$. In this case, setting $\theta(t) = \theta(t-1)$ preserves the response properties in Theorem 2. Another scheme to minimize the on-line computational burden is to implement an *explicit* gain governor by calculating the optimal values of $\theta = \theta^*(x, r)$ off-line for different x and r and then developing a functional approximation, $\bar{\theta}^*(x, r)$ of $\theta^*(x, r)$ for on-line implementation. Suppose that such an explicit solution is available for $x \in \Pi \subset \mathbf{R}^p$ where Π is a set such that $x_e(r) \in \text{int}\Pi$. As long as $x(t) \in \Pi$, $\bar{\theta}^*(x, r)$ is defined. If the trajectory of x starts in Π but exits Π at a time instant t, then $\bar{\theta}^*(x(t), r)$ is not defined but $\theta(t)$ can be set to the value of $\theta(\tilde{t}) = \theta^*(x(\tilde{t}), r)$, where $\tilde{t} < t$ is the last time instant for which $x(\tilde{t}) \in \Pi$. Even if $x(t)$ exits Π at a time instant t, the condition $x_e(r) \in \text{int}\Pi$ and (A3) guarantee that $x(t)$ must re-enter Π in finite-time where $\bar{\theta}^*(x, r)$ can again be applied. The reduction in the size of the set Π over which the functional approximation to the explicit solution is developed and deployed provides a mechanism for decreasing the complexity of this functional approximation and for improving its accuracy.

Remark 6. A practical numerical procedure to approximately determine an adequate horizon T is available. This procedure is based on computing two quantities, $L_1(k)$ and $L_2(k)$, $k \in \mathbf{Z}^+$:

$$L_1(k) = \max_{j=1,\ldots,q,\theta\in\Theta,(\theta,x)\in C(r)} g_j(\theta, x^+(k|x, \theta, r), r),$$

$$L_2(k) = \max_{\theta\in\Theta,(\theta,x)\in C(r)} \frac{\Omega(x^+(k|x, \theta, r), r, \theta)}{\Omega(x, r, \theta)},$$

where g_j is defined in (8). The $L_1(k)$, $L_2(k)$ are, respectively, the maximum constraint violation and the minimum decay ratio of the incremental cost due to $x^+(k|x, \theta, r)$ as x, θ and r vary ($\theta \in \Theta$, $(\theta, x) \in C(r)$ and r varies within the intended operating range). An acceptable T must satisfy the conditions $L_1(k) \leqslant 0$ and $L_2(k) \leqslant q$ for all $k \geqslant T$ and some $0 \leqslant q < 1$. Either off-line numerical optimization or multiple off-line simulations of the model for different x, θ and r can be used to estimate $L_1(k)$ and $L_2(k)$. An acceptable T can be easily picked from the graphical plots of $L_1(k)$ and $L_2(k)$ versus k. We note that the resulting T is a numerical approximation to the required horizon and not a guaranteed upper bound.

Remark 7. In a common situation when (6) and (7) represent a discrete-time approximation of a continuous-time system and Δ is the physical time period between two subsequent parameter updates, it is usually the underlying continuous-time dynamics that dictate an acceptable value for $T \cdot \Delta$ to yield properties required in (A4) and Proposition 1. In particular, selecting larger Δ (i.e., using less frequent parameter updates) can lead to smaller T while the effort to simulate the continuous-time model to a desired level of accuracy does not increase with Δ. The drawback of larger Δ is in cruder enforcement of constraints for the original continuous-time system. This drawback can be addressed by using a finer time grid for constraint enforcement, with (4) replaced by

$$(\theta(t), \bar{x}^+(n\delta|x(t), \theta(t), r)) \in C(r(t)), \quad n = 0, \ldots, N.$$

Here $\bar{x}^+(n\delta|x(t), \theta(t), r))$ is the predicted state of the continuous-time system at time $n\delta$, where $\delta < \Delta$ and $N\delta > T\Delta$. If the approach of Remark 5 is used, the number of constraints in the resulting optimization problem may not be large.

From Theorem 2, $\|\theta^*(t)\|^2_{\Psi_\theta}$ converges to a limit. Suppose that

$$\lim_{t\to\infty} \|\theta^*(t)\|_{\Psi_\theta} = \lim_{t\to\infty} \sqrt{(\theta^*(t))^{\mathrm{T}}\Psi_\theta\theta^*(t)} = v_{\lim} \geqslant 0. \tag{13}$$

It turns out that under appropriate, additional assumptions, $v_{\lim} = 0$. In other words, *asymptotically* the gain governor becomes inactive and the closed loop system functions under the nominal control law (corresponding to $\theta = 0$). The additional assumptions are:

(A6) The function $x^+(k|x, \theta, r)$ is locally Lipshitz as a function of θ and the function $u_{fb}(x, \theta, r)$ is locally Lipshitz as a function of x and θ for all $(\theta, x) \in C(r)$, $\theta \in \Theta$ and $k = 0, \ldots, T$.

(A7) $0 \in \text{int}\Theta$, Θ is convex.

(A8) Q is twice continuously differentiable.
(A9) The matrix Ψ_θ in (3) is positive definite, $\Psi_\theta > 0$.

Theorem 3. *Suppose assumptions* (A6)–(A9) *hold in addition to the assumptions of Theorem* 2 *and* $\theta^*(t)$ *is the minimizer of* (3) *subject to* (4). *Then, all conclusions of Theorem* 2 *remain valid, and* $\|\theta^*(t)\|_{\Psi_\theta} \to 0$.

Proof. See Appendix A. □

Remark 8. The result in Theorem 3 holds if an exact minimizer is computed at each time instant t. The result in Theorem 2 does not depend on this condition.

3. More general parameter governors and a feedforward governor

We now consider a class of parameter governors which are more general than the gain governors. Suppose that the state of the system, x, in (6) can be partitioned as

$$x = \begin{bmatrix} x_p \\ x_i \end{bmatrix}, \tag{14}$$

so that u_{fb} in the control law (7) has the form,

$$u_{fb}(x, \theta, r) = \bar{u}_{fb}(x_p, x_i, \theta, r). \tag{15}$$

Unlike in the gain governor case, here we no longer assume that system (6), (7), (14), (15) has the steady-state equilibrium which does not depend on θ; this property is assumed only for x_p. Thus the unique equilibrium corresponding to $\theta(t) \equiv \theta$ and $r(t) \equiv r$ is assumed to have the form,

$$x_e(\theta, r) = \begin{bmatrix} x_{pe}(r) \\ x_{ie}(\theta, r) \end{bmatrix} \tag{16}$$

and

$$\bar{u}_{fb}(x_{pe}(r), x_{ie}(\theta, r), \theta, r) = 0 \quad \text{for all } \theta \in \Theta. \tag{17}$$

The cost function penalizes the deviation of x_p from $x_{pe}(r)$ and u from $u_e(r)$ so that in (3)

$$\Omega(x, \theta, r) = \bar{Q}(x_p - x_{pe}(r), \bar{u}_{fb}(x_p, x_i, \theta, r)). \tag{18}$$

The results similar to Theorems 2 and 3 for this more general parameter governor follow immediately under essentially the same assumptions as in the gain governor case. See the conference version of this paper (Kolmanovsky & Sun, 2004) for details. Specifically, in (A1)–(A4) and (A8), $x_e(r)$ is replaced by $x_e(\theta, r)$ and Q is replaced by $\bar{Q}$. Assumption (A5) needs to be replaced by $\bar{Q}$ being continuous and $\bar{Q}(a, b) > 0$ if $(a, b) \neq 0$. The results require an additional technical assumption that $\bar{u}_{fb}$ in (15) is invertible with respect to x_i and the inverse is a continuous function of x_p and θ for all $\theta \in \Theta$ and x_p sufficiently close to $x_{pe}(r)$. This assumption guarantees that $x_p(t) \to x_{pe}(r)$, $u(t) \to u_e(r)$ imply $x_{ie}(t) \to x_{ie}(\theta, r)$.

Clearly the gain governor represents a special case of this more general parameter governor, with $x = x_p$. Another special case is the feedforward governor, for which the complete system of equations has the following form:

$$x_p(t+1) = f_p(x_p(t), u(t)), \tag{19}$$

$$x_i(t+1) = x_i(t) + y(t) - r, \tag{20}$$

$$y(t) = h(x_p(t)), \tag{21}$$

$$u(t) = u_e(r) + \tilde{u}_{fb}(x_p(t), x_i(t), r) + \theta(t). \tag{22}$$

Thus the adjustable parameter vector $\theta(t)$ appears as a feedforward offset in the control law (22).

The integrator (20) is essential to the feedforward governor operation to eliminate the influence of the constant offset term, $\theta(t)$, on the steady-state values of x_p and u, and thus guarantee the equilibrium properties in (16), (17). Note that to achieve this, the dimensionality of y must be equal or exceed the dimensionality of u. If the original plant does not contain such an integrator, it may be artificially added in the process of the feedforward governor design.

As shown in Kolmanovsky and Sun (2004), system (19)–(22), under appropriate assumptions and owing to the presence of an integrator, exhibits slow and fast dynamics decomposition with the slow manifold satisfying the constraints; then the fast portion of the trajectory can be made to satisfy the constraints through the adjustment of $\theta(t)$. This mechanism permits to handle large changes in $r(t)$.

4. Example

In this section, we illustrate the application of the gain governor to an example of an engine with an electronic throttle and variable cam phasing. The objective is to use the gain governor to coordinate these two actuators to provide fast and monotonic air flow response into the engine cylinders. The background for this problem is described in Stefanopoulou and Kolmanovsky (1999). Several other examples, for both the gain governor and the feedforward governor cases, were reported in the conference version of this paper (Kolmanovsky & Sun, 2004).

The engine breathing dynamics, in a simplified form as compared to Stefanopoulou and Kolmanovsky (1999), have the following form in continuous time:

$$\dot{p} = c_m(k_1 \cdot u_{\text{th}} \cdot \sqrt{p - p^2} - W),$$

$$W = k_2 \cdot p \cdot \left(1 - \frac{\phi}{90}\right),$$

$$\dot{\phi} = -\tau(\phi - \phi_e),$$

$$\ddot{u}_{\text{th}} = -2\zeta\omega_n \dot{u}_{\text{th}} - \omega_n^2(u_{\text{th}} - u_{\text{th},e}),$$

where p is the intake manifold pressure, u_{th} is the throttle angle, W is the cylinder flow, ϕ is the cam phasing angle, and the subscript e signifies the equilibrium value of a variable. The constants are $c_m = 0.0414$, $k_1 = 4.0$, $k_2 = 30.0$, $\omega_n = 24.5$. The governed parameters are τ and ζ so that $\tau = 8 + \theta_1$, $\zeta = 1 + \theta_2$, where $\theta \in \Theta = [-6, 8] \times [-0.8, 1.0]$. They determine, respectively, the speed of cam phasing adjustment and the damping

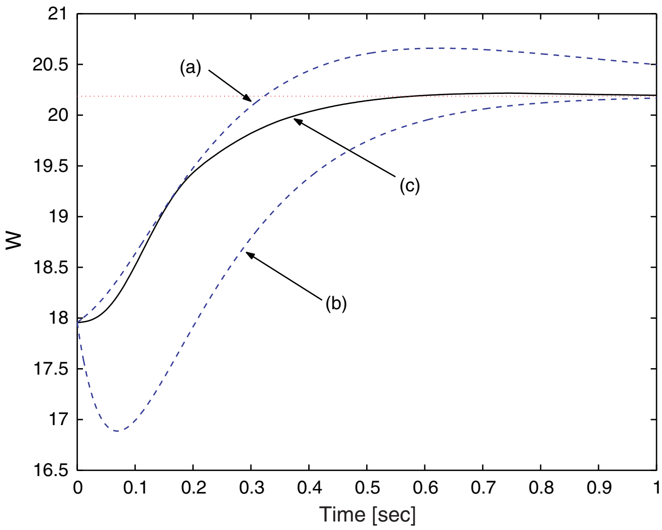

Fig. 2. Time histories of cylinder flow with (a) $\tau(t) \equiv 2$, $\zeta(t) \equiv 1$; (b) $\tau(t) \equiv 16$, $\zeta(t) \equiv 1$; (c) $\tau(t)$, $\zeta(t)$ prescribed by the gain governor.

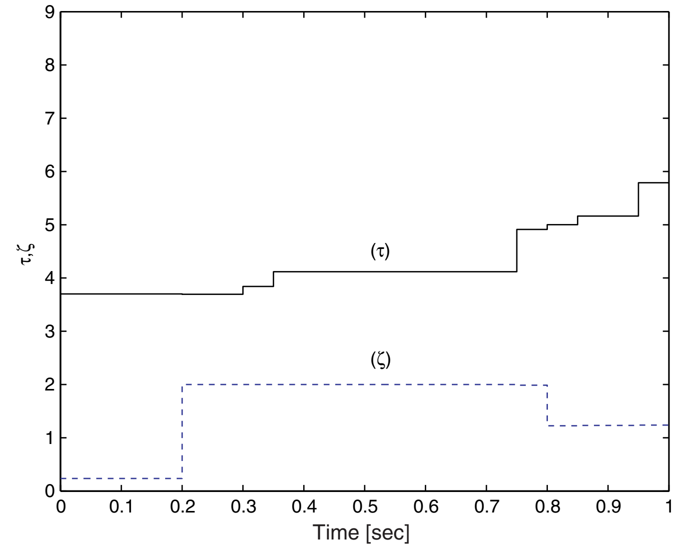

Fig. 3. Time histories of $\tau(t)$ and $\zeta(t)$ prescribed by the gain governor.

ratio in the throttle position response. These parameters are updated every $T_s = 50$ ms by the gain governor. The incremental cost function Q in (3) is

$$Q = q_1 \cdot (W - W_e)^2 + q_2 \cdot (\phi - \phi_e)^2 + q_3 \cdot (u_{\text{th}} - u_{\text{th},e})^2 + q_4 \cdot \dot{u}_{\text{th}}^2,$$

where $u_{\text{th},e} = r = 20$, $\phi_e = 25$, $q_1 = 100$, $q_2 = 0.01$, $q_3 = 10^{-4}$, $q_4 = 10^{-4}$, while $\Psi_\theta = \text{diag}(0.001, 0.001)$. Due to a large value of q_1, the cost emphasizes fast cylinder flow response to provide better engine responsiveness and drivability.

Assuming the command is to increase the cylinder flow, the constraint which ensures the monotonic cylinder flow response is $\dot{W}(t) \geqslant 0$. Strictly speaking, our theory does not permit the treatment of constraints in this form because in steady-state $\dot{W}(t) = 0$ and (A2) is violated. We therefore relax the constraint to $\dot{W}(t) \geqslant -0.1$. In this example, artificial constraints, defined by $0.6 \leqslant p \leqslant 0.99$, $10 \leqslant \phi \leqslant 30$, $10 \leqslant u_{\text{th}} \leqslant 30$, $-80 \leqslant \dot{u}_{\text{th}} \leqslant 80$ were added to complete the definition of the set $C(r)$ in (4) and formally satisfy (A1). The horizon, $T = 44$, was estimated based on the off-line numerical procedure of Remark 6 applied to 10^5 simulated trajectories, each corresponding to values of $\theta(t)$ and $x(0)$ selected at random.

Fig. 2 demonstrates that the gain governor is able to coordinate throttle and cam phasing to produce a monotonic cylinder flow response. Figs. 3 and 4 indicate that the gain governor creates an initial overshoot in throttle response (by decreasing the damping ratio). This increases the air flow through the throttle and into the engine intake manifold, and it mitigates the increase in the residuals due to changing cam phasing. The gain governor initially adjusts the cam phasing position slowly and then speeds it up. For comparison, two cases where $\tau(t)$ and $\zeta(t)$ are maintained constant are also included in Figs. 3 and 4. The overall response of the cylinder flow in these two cases is slower than with the gain governor and the cylinder flow monotonicity constraint is violated.

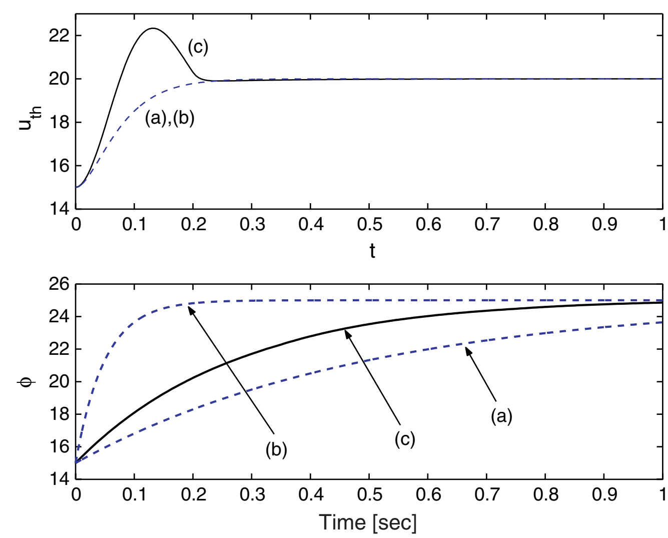

Fig. 4. Time histories of u_{th}, ϕ with (a) $\tau(t) \equiv 2$, $\zeta(t) \equiv 1$; (b) $\tau(t) \equiv 16$, $\zeta(t) \equiv 1$; (c) $\tau(t)$, $\zeta(t)$ prescribed by the gain governor.

5. Concluding remarks

A parameter governor uses receding horizon optimization for on-line adjustment of parameters in a nominal control law to avoid violation of pointwise-in-time state and control input constraints, and to improve transient performance. The adjustable parameters remain constant over the prediction horizon. Thus the dimensionality of the optimization problem being solved does not depend on the horizon and is equal to the number of parameters. As was demonstrated for the gain governor case, if the horizon satisfies appropriate assumptions, terminal set and terminal penalty conditions are not required to guarantee state and control input convergence. Furthermore, a large degree of flexibility exists in accommodating the on-line optimization. For example, the exact minimizer is not required or need not be computed at every sample time instant. Key results hold even if the parameter values are restricted to a finite set; in this case the optimization reduces to a finite number of on-line simulations. In addition, an explicit implementation (wherein the solution to the receding horizon optimization problem is pre-computed off-line and its functional approximation is applied on-line) can be generated for a subset of the state space and then extended in a simple way beyond this subset while preserving the state convergence.

The parameter adjustment approach provides only a limited mechanism for dealing with constraints and for on-line performance improvement. At the same time, the required on-line computational effort to implement it may be relatively modest as compared to more general model predictive control (MPC) schemes. This may make the parameter governor approach suitable for systems with fast dynamics and limited computational resources.

Two special parameter governors, the gain governor and the feedforward governor, were discussed in the paper in more detail. More research is needed to delineate situations when the gain governor or when the feedforward governor should be used. The considerations in the paper and the examples which we treated so far suggest that the feedforward governor may be preferable over the gain governor when large and frequent changes in $r(t)$ are permitted. The gain governor, on the other hand, may be a preferred choice when changes in $r(t)$ are less frequent (so the operation of the overall system resembles repeated stabilization) and in the case when there are only control input constraints.

Future research may help identify other effective classes of parameter governors and to understand their robustness properties as well as modifications needed to deal with uncertainties and disturbances. The related developments in the reference governor and multimode controller cases suggest that similar treatment should be possible in a general parameter governor case. Additional insights can be gained by applications of parameter governors to realistic practical problems and through the experimental validation.

Appendix A. Proof of Theorem 3

The idea of the proof is to compare the optimal decision at time t, $\theta^*(t)$, with an alternative decision,

$$\hat{\theta}(t) \triangleq \theta^*(t) \cdot \frac{v_{\lim}}{v_{\lim} + \varepsilon_2}, \quad \varepsilon_2 > 0, \tag{23}$$

and to demonstrate that if $v_{\lim} > 0$, t is sufficiently large and ε_2 is sufficiently small, then $\hat{\theta}(t)$ is a feasible choice at time t and actually results in a smaller value of the cost (3) than $\theta^*(t)$.

Indeed, define $\hat{x}(t+k) = x^+(k|x(t), \hat{\theta}(t), r)$, $x^*(t+k) = x^+(k|x(t), \theta^*(t), r)$, $\hat{u}(t+k) = u_e(r) + u_{fb}(\hat{x}(t+k), \hat{\theta}(t), r)$, $u^*(t+k) = u_e(r) + u_{fb}(x^*(t+k), \theta^*(t), r)$ and consider the difference of the cost values corresponding to $\hat{\theta}(t)$ and $\theta^*(t)$. Noting that

$$\|\hat{\theta}(t)\|^2_{\Psi_\theta} - \|\theta^*(t)\|^2_{\Psi_\theta} = (\hat{\theta}(t) - \theta^*(t))^{\mathrm{T}} \Psi_\theta (\hat{\theta}(t) + \theta^*(t)) = -\varepsilon_2 \frac{\|\theta^*(t)\|_{\Psi_\theta}}{(v_{\lim} + \varepsilon_2)^2} (2v_{\lim} + \varepsilon_2),$$

we obtain

$$\begin{aligned} J(x(t), \hat{\theta}(t), r, T) &- J(x(t), \theta^*(t), r, T) \\ &= -\varepsilon_2 \frac{\|\theta^*(t)\|_{\Psi_\theta}}{(v_{\lim} + \varepsilon_2)^2} (2v_{\lim} + \varepsilon_2) \\ &\quad + \sum_{k=0}^{T} (Q(\hat{x}(t+k) - x_e(r), \hat{u}(t+k) - u_e(r)) \\ &\quad - Q(x^*(t+k) - x_e(r), u^*(t+k) - u_e(r))). \end{aligned} \tag{24}$$

By (A8) and Taylor series expansion properties,

$$\begin{aligned} &Q(\hat{x}(t+k) - x_e(r), \hat{u}(t+k) - u_e(r)) \\ &\quad - Q(x^*(t+k) - x_e(r), u^*(t+k) - u_e(r)) \\ &= DQ(x^*(t+k) - x_e(r), u^*(t+k) - u_e(r)) \cdot \begin{bmatrix} \hat{x}(t+k) - x^*(t+k) \\ \hat{u}(t+k) - u^*(t+k) \end{bmatrix} \\ &\quad + \begin{bmatrix} \hat{x}(t+k) - x^*(t+k) \\ \hat{u}(t+k) - u^*(t+k) \end{bmatrix}^{\mathrm{T}} \frac{D^2 Q(z_\zeta(t+k))}{2} \begin{bmatrix} \hat{x}(t+k) - x^*(t+k) \\ \hat{u}(t+k) - u^*(t+k) \end{bmatrix}, \end{aligned}$$

where

$$z_\zeta(i) = \zeta(i) \cdot \begin{bmatrix} x^*(i) - x_e(r) \\ u^*(i) - u_e(r) \end{bmatrix} + (1 - \zeta(i)) \cdot \begin{bmatrix} \hat{x}(i) - x_e(r) \\ \hat{u}(i) - u_e(r) \end{bmatrix},$$

$0 \leqslant \zeta(i) \leqslant 1$. By (A6) and (A9), for all $\varepsilon_2 > 0$ sufficiently small we can find $L_Q > 0$ such that

$$\sup_{k=0,\ldots,T} \left\| \begin{matrix} \hat{x}(t+k) - x^*(t+k) \\ \hat{u}(t+k) - u^*(t+k) \end{matrix} \right\| \leqslant \frac{L_Q \varepsilon_2}{v_{\lim} + \varepsilon_2} \|\theta^*(t)\|_{\Psi_\theta}, \tag{25}$$

where $\|\cdot\|$ denotes the usual 2-vector norm. From (A8), $x^*(t+k) \to x_e(r)$, $u^*(t+k) \to u_e(r)$, and since $(0, 0)$ is a minimum of Q (so that $DQ(0, 0) = 0$) and since DQ is continuous, it follows that

$$\sup_{k=0,\ldots,T} \|DQ(x^*(t+k) - x_e(r), u^*(t+k) - u_e(r))\| \to 0 \quad \text{as } t \to \infty. \tag{26}$$

Consider now $\hat{\theta}(t)$ defined by (23). By (A7), $\hat{\theta}(t) \in \Theta$. In view of (A2) and (25), for all $\varepsilon_2 > 0$ sufficiently small, if t is sufficiently large, then $\hat{\theta}(t)$ is a feasible choice at time t. From (24)–(26),

$$\begin{aligned} J(x(t), \hat{\theta}(t), r, T) &- J(x(t), \theta^*(t), r, T) \\ &\leqslant -\varepsilon_2 \frac{\|\theta^*(t)\|_{\Psi_\theta}}{(v_{\lim} + \varepsilon_2)^2} (2v_{\lim} + \varepsilon_2) \\ &\quad + \frac{T L_Q \varepsilon_2 \|\theta^*(t)\|_{\Psi_\theta}}{v_{\lim} + \varepsilon_2} \sup_{k=0,\ldots,T} \|DQ(x^*(t+k) \\ &\quad - x_e(r), u^*(t+k) - u_e(r))\| \\ &\quad + O(\varepsilon_2^2). \end{aligned} \tag{27}$$

Note that $\|\theta^*(t)\|_{\Psi_\theta} \to v_{\lim}$ as $t \to \infty$. If $v_{\lim} > 0$, $\varepsilon_2 > 0$ is sufficiently small and $t \in \mathbf{Z}^+$ is sufficiently large, the first term in (27) can be made to strictly dominate in absolute value the third term and, in view of (26), the first term can also dominate the second term so that

$$J(x(t), \hat{\theta}(t), r, T) < J(x(t), \theta^*(t), r, T),$$

which contradicts the fact that $\theta^*(t)$ is a minimizer for J. The proof is complete.

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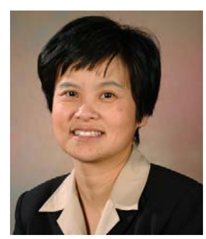

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