

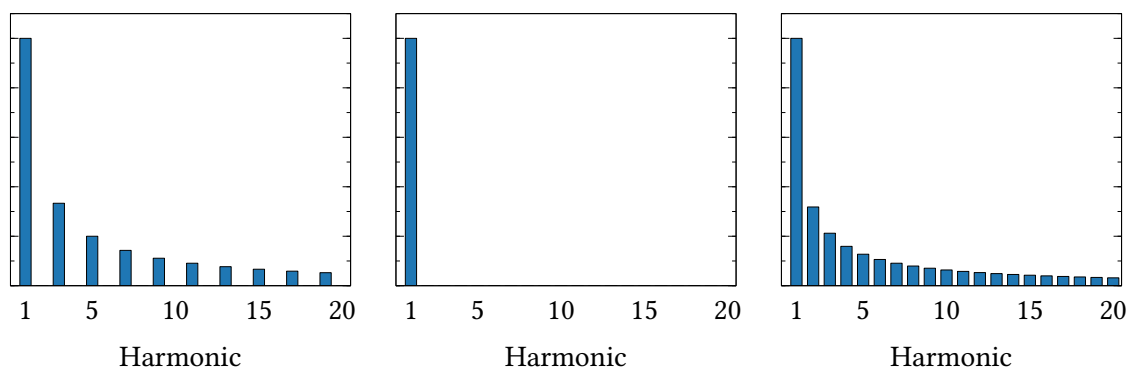
THE SCIENCE OF MUSIC

EXERCISES FOR CHAPTER 4

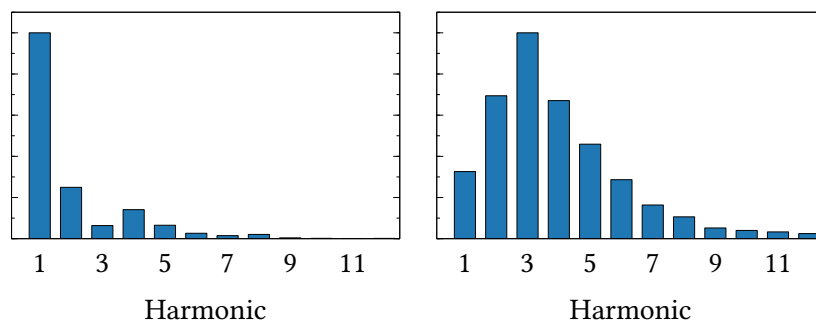
4.1 The note middle C is played on a piano.

- What is the musical interval (“octave”, “fifth,” etc.) between the fundamental frequency of the note and its third, fifth, and tenth harmonics?
- For a middle C what notes are the third, fifth, and tenth harmonics?

4.2 Here are bar charts showing the amount of each harmonic for a sine wave, a square wave, and a sawtooth wave. Which is which, and what are the giveaway features that let you know?



4.3 Here are the amounts of each harmonic for two different instruments playing the same note. Which one would have the brighter sound and how can you tell?



4.4 Using Eq. (4.10) calculate the amount A_n of the n th harmonic for the following waveforms:

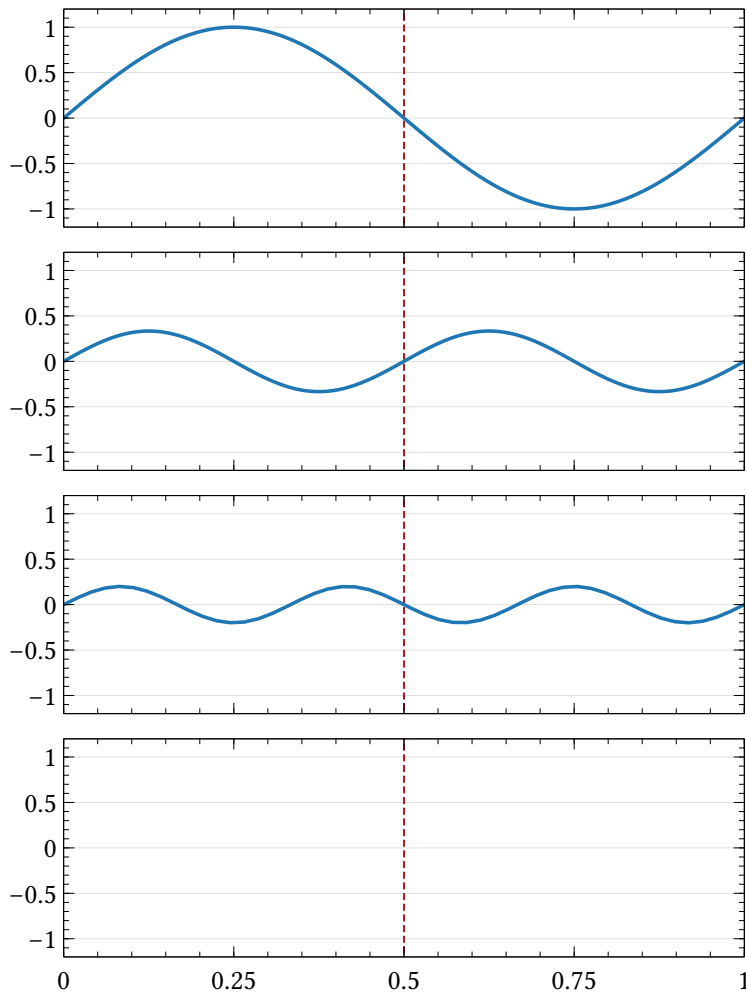
- A sawtooth wave with the form $p(t) = t/T$ in the interval from $t = 0$ to T .

b) A square wave with the form

$$p(t) = \begin{cases} +1 & \text{if } 0 \leq t < \frac{1}{2}T, \\ -1 & \text{if } \frac{1}{2}T \leq t < T. \end{cases}$$

You should find that the sawtooth wave contains all harmonics but the square wave contains only the odd harmonics.

4.5 By any means you like, create a rough picture of the waveform you would get if you added together the three sine waves shown here. If you like you can draw your picture in the empty box at the bottom (or print or copy the page then draw in the box). The horizontal grid lines and the vertical dashed lines are provided as a guide to the eye only.



4.6 A waveform with period T is *half-wave symmetric* if it satisfies $p(t) = -p(t - \frac{1}{2}T)$ for all t .

- a) Use Eq. (4.10) to show that for such a waveform $A_n = 0$ for all even values of n , and hence that half-wave symmetric waveforms contain only odd harmonics as discussed at the end of Section 4.3.5.

- b) Show that the reverse is also true: any waveform that contains odd harmonics only is half-wave symmetric.
- c) Name two examples of waveforms that are half-wave symmetric and one example of a waveform that is not.

4.7 The lowest three harmonics contained in a note played on a certain instrument have frequencies 440 Hz, 660 Hz, and 880 Hz. What musical note will the ear hear?

4.8 What note will the ear hear if the lowest harmonics of a sound are:

- a) 220 Hz, 440 Hz, and 660 Hz
- b) 220 Hz, 330 Hz, 440 Hz, and 660 Hz
- c) Explain why the addition of one harmonic changes the pitch one hears.