# The Science of Music

## Exercises for Chapter 15

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**15.1** Sketch a picture of an ADSR envelope with:

a) Attack 10 milliseconds, decay 20 milliseconds, sustain 50%, and release 50 milliseconds;
b) Attack 100 milliseconds, decay 1000 milliseconds, sustain 0%, and release 50 milliseconds;
c) Attack 300 milliseconds, decay 100 milliseconds, sustain 90%, and release 0 milliseconds.

**15.2** What will be the five lowest harmonics in the tone produced by an FM synthesizer when the carrier is a sine wave with frequency $f$ and the modulation is a sine wave with frequency $5f$?

**15.3** Sketch a picture of the output waveform produced by frequency modulation when the carrier is a sine wave of frequency $f$ and the modulation is a sine wave of frequency $\frac{1}{2}f$.

**15.4** We perform frequency modulation on a sine-wave carrier with frequency 220 Hz using a sine-wave modulation with frequency 550 Hz. What note will the ear hear and what harmonics will be present in the sound?

**15.5** The low-pass filter of Fig. 7.8 cuts the amplitude of an input sine wave with frequency $f$ by a factor of $\sqrt{1 + 4\pi^2 f^2 R^2 C^2}$, where $R$ is the resistance in ohms and $C$ is the capacitance in farads.

a) The filter on a particular synthesizer uses this circuit with a capacitance of $C = 10^{-6}$ farads and a variable resistor that allows us to change the value of $R$ by turning a knob. We set $R = 500\,\Omega$ and feed a sine wave with frequency 440 Hz into the filter. How much will the amplitude of the signal be cut down by?
b) In a sawtooth wave the amplitude of the second harmonic is a half the amplitude of the fundamental. If you pass a sawtooth wave with frequency 440 Hz through the filter how big will the amplitude of the second harmonic be relative to the fundamental after it passes through the filter?
c) To what value would you have to set the resistance $R$ to make the amplitude of the second harmonic a third the amplitude of the fundamental?

**15.6** How much memory would it take, in bytes, to

a) Record a 5-second trumpet note on A4 at the industry standard 44 100 samples per second and 16-bit resolution?
b) The same trumpet note if it is divided into a 50 millisecond transient and a loop consisting of ten cycles of the sustained part of the note.

**15.7** Consider a looped sample of the kind discussed in Section 15.4.1. The note has frequency $f$ and the sample consists of $n$ cycles of the waveform of the loop.

a) Show that the only frequencies that can be present in the sound are whole-number multiples of $f/n$.
b) Because of the slight inharmonicity on a piano the second harmonic of a note is typically a few cents sharp of the ideal frequency $2f$. How long a sample, in terms of cycles $n$ of the wave, would you need to capture inharmonicity of this magnitude?