The Science of Music

Exercises for Chapter 14

14.1 The pickup of a certain electric guitar is located $\frac{1}{5}$ of the way along the string from the bridge. The guitarist plucks the string $\frac{1}{4}$ of the way along.

a) If a single note on the guitar is played through a *clean* amp, what are the first five harmonics present in the note?
b) If the guitar is played through a *distorting* amp, what are the first five harmonics?

14.2 From the results in Sections 10.1.2 and 10.2.1 we can show that the vibration of the nth harmonic of a string plucked a fraction β of the way along its length is

$$y(x,t) = \frac{A}{n^2}\sin(\beta\pi n)\sin\left(\frac{\pi n x}{L}\right)\cos(2\pi n f t),$$

where A is a constant that controls the amplitude and depends on how hard the string is plucked, L is the length of the string, and f is the frequency of the note the string is tuned to. Suppose the pickup of an electric guitar is placed a fraction γ of the way along the string.

a) As discussed in Section 14.1.3, the signal from the pickup depends on the motion of the part of the string closest to it and specifically on how fast the string is vibrating, making it proportional to the velocity of the string at that point. With this in mind, what is the amplitude of the nth harmonic in terms of the parameters in the equation above?
b) Show that this amplitude is zero if either β or γ is $1/n$.

14.3 Most distorting guitar amplifiers have an asymmetric transfer function like the one shown in Fig. 14.4f, where the top and bottom halves of the curve are a different shape. This results in a waveform like that in Fig. 14.4g, whose top and bottom halves are also asymmetric. This is important for harmonic distortion because it adds both even and odd harmonics to the sound. Explain what could happen to the harmonics if the transfer function were symmetric. (Hint: you may want to read through the discussion of half-wave symmetry at the end of Section 4.3.5.)

14.4 The most common power chords on electric guitar are the fifth and fourth as discussed in Section 14.2.3, but there are a number of others that produce interesting sounds by creating subharmonics at musically pleasing intervals below the notes played. What note is generated as a subharmonic when you play the following note pairs through a distorting amplifier:

a) C4 and E4 (frequency ratio $\frac{5}{4}$)
b) C4 and A4 (ratio $\frac{5}{3}$)
c) C4 and A♭4 (ratio $\frac{8}{5}$)

14.5 The lowest string on a standard four-string electric bass plays the note E1 and the length of the vibrating part of the string is 86.4 cm. If the string is 2.7 mm in diameter and made of steel with density $7900\,\mathrm{kg/m^3}$, what is the tension on the string? How does this compare with the tension on a typical guitar string?

14.6 The tine for the note C4 on a Rhodes piano is 1.5 mm in diameter and made of steel with Young's modulus $200 \times 10^9\,\mathrm{N/m^2}$ and density $7900\,\mathrm{kg/m^2}$.

a) How long is the tine?
b) How long would the reed of a Wurlitzer piano be that plays the same note and is made of the same steel, 0.8 mm thick.

14.7 The tonewheel on a Hammond organ that makes the note D5 has 32 teeth around its edge. How fast in revolutions per second does it spin?

14.8 If one wanted to imitate the sound of a sawtooth wave on a Hammond organ, what registration would one use?

14.9 A child's toy consists of a whistle on the end of a string 1 meter long. When you whirl the whistle around in a circle on the end of the string, the movement of the air makes the whistle sound, producing the note C6.

a) A child playing with the toy whirls it around their head twice per second in a horizontal circle. What is the velocity of the whistle?
b) You are standing to one side listening to the sound. Instead of a steady note you hear a variation in pitch, the note going up and down as the whistle makes its circle. Briefly explain what is happening. Why does the pitch vary?
c) What are the highest and lowest frequencies the note attains?
d) Hence by how many musical cents does the pitch vary?

14.10 The African musical instrument known as an mbira or kalimba consists of a set of metal tines a few centimeters long, clamped onto a wooden frame at one end and free to vibrate at the other. The player holds the instrument in their hands and plucks the tines with their thumbs (or sometimes their forefingers) to produce a note. Suppose a particular instrument has tines made of steel with thickness 0.6 mm, density $7900\,\mathrm{kg/m^3}$, and Young's modulus $200 \times 10^9\,\mathrm{N/m^2}$, and ranges in pitch from C4 to C6. What are the lengths of the longest and shortest tines?