The Science of Music

Exercises for Chapter 10

10.1 A string is plucked exactly a third of the way along its vibrating part. What are the lowest five harmonics that will be present in the sound it produces?

10.2 Show that Eq. (10.5) is indeed a solution of the wave equation, Eq. (10.2), and find the value of f.

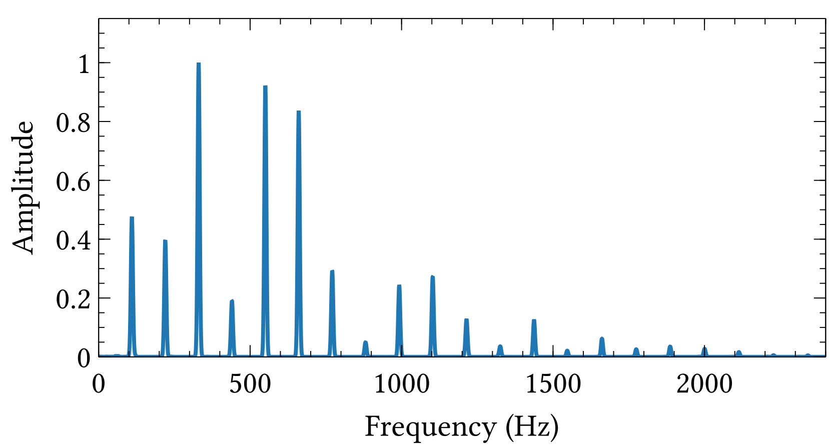
10.3 Using Eq. (10.23) show that the first harmonic of the vibration of a plucked string will always be stronger than any other harmonic, no matter where the string is plucked. (You can ignore the effect of the soundboard and instrument resonances—we're just talking about the vibration of the string itself.)

10.4 The lowest two strings on a guitar are tuned to E2 and A2.

a) If the lowest string is set vibrating in its fourth mode, what note will it sound?
b) If the second-lowest string is set vibrating in its third mode, what note will it sound?
c) Guitarists learn to play exactly these modes by plucking the strings in the right way. They typically do this when tuning the strings of the guitar. Why would this be a useful thing to do when tuning?

10.5 Calculate how far it is from the nut to the second fret on a standard sized guitar, for which the vibrating part of the strings is 64.8 cm long.

10.6 Here is the spectrum of a plucked guitar string. At approximately what point along its length was the string plucked?



10.7 As discussed in Section 10.4.1, the sound "board" of a banjo consists of a flexible membrane, usually made of mylar plastic, which has low acoustic impedance compared to the wooden soundboard of a guitar.

a) Suppose the acoustic impedance for a particular banjo is $20\,\mathrm{N\,s/m}$ and the strings have length 67 cm and tension 6 kg. What is the decay time of a note on this banjo due to bridge loss?
b) How does this compare with an acoustic guitar, and how will the instrument sound as a result?

10.8 The corner in the Helmholtz motion travels with velocity v given by Eq. (10.65). Calculate this velocity for a violin string 33 cm long that plays the note A4.

10.9 We have seen that a disturbance will travel along a stretched string at a velocity given by Eq. (10.65).

a) Show that in terms of the diameter and density of the string this velocity can also be written

$$v = \frac{2}{d}\sqrt{\frac{T}{\pi\rho}}.$$

b) A jump rope, being held by two people with a tension of 2 kg, is shaken at one end producing waves that travel down the rope. If the diameter of the rope is 8 mm and its density is $1000\,\mathrm{kg/m^3}$, calculate the speed with which the waves travel.
c) A long piece of string is tied between two tin cans to make a "tin can telephone." The string is held with tension 1 kg and the string has diameter 2 mm and density $800\,\mathrm{kg/m^3}$. How fast is the velocity of vibrations traveling down the string in miles or kilometers per hour? (The vibration traveling along the string in this case is what carries the sound from one can of the telephone to the other.)

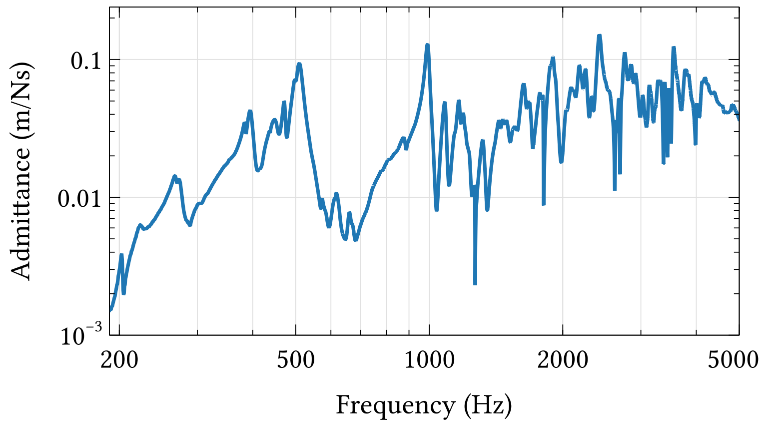
10.10 Though the speed of a violin bow does not affect the pitch of the note, it does affect its volume. Consider for instance the second string of a violin, which plays the note A4 when the player does not place any fingers on the fingerboard.

a) Given the frequency of the note, about how long does it take for the stick-slip motion of the bow to drag the string from one side to the other?
b) If the bow is moving at 0.5 meters per second, how far does the string move from side to side?
c) If the player moves the bow twice as fast, how far does the string now move?
d) Hence about how much louder, in terms of decibels, do you expect the sound to be when the player moves the bow twice as fast?

10.11 In Section 10.8.4 we derived the mathematical form of the first half of a cycle of the Helmholtz motion, Eqs. (10.78) and (10.79). Perform the equivalent derivation for the second half of the motion and show that for $L/v < t \leq 2L/v$ the string has the shape

$$y(x,t) = \begin{cases} 4A(vt - L)x & \text{if } x \leq 2L - vt, \\ 4A(2L - vt)(L - x) & \text{if } x > 2L - vt. \end{cases}$$

10.12 This graph shows the measured admittance of the soundboard of a violin:



In tests this violin was rated poorly by expert listeners. Suggest some possible issues with the instrument based on the admittance curve.

10.13 The lowest string on a violin plays the note G3, with frequency 196 Hz. Looking at the admittance curve in the previous exercise:

a) What is the *impedance* of the violin bridge and soundboard at this frequency?
b) The length and tension of the vibrating part of a steel violin string are about 33 cm and 5 kg respectively. What would the decay time be, due to energy loss through the bridge, if you plucked the note G3 on this violin?
c) The A string plays the note A4. What would the decay time be for this string?
d) Decay time plays less of a role on violin than on guitar for instance. Why is this?

10.14 A piano string playing the note C4 is 60 cm long and has a tension of 80 kg, and the note it produces has a decay time of $\tau = 3$ seconds.

a) Using Eq. (10.40), estimate the impedance Z of the bridge and soundboard of the piano.
b) Looking at the admittance curve in Exercise 10.12, estimate the impedance of a violin bridge and soundboard at the same frequency.
c) How do your figures for piano and violin compare? What is the reason for the difference?

10.15 The body of a violin is typically about 36 cm long and its air resonance falls at about 270 Hz.

a) Given that the body of a double bass is about 110 cm long, what frequency would you expect its air resonance to have if the instrument were simply a scaled-up version of the violin?
b) Assuming this value for the air resonance frequency, to what note should the lowest string on the double bass be tuned if we want it to have the same position relative to the air resonance as the lowest note of a violin?
c) How does your calculation compare with the actual lowest note of a double bass? How do you explain the difference?