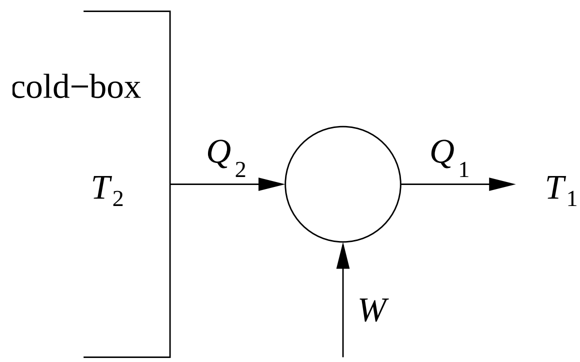
Physics 406: Homework 3

1. **Refrigerator:** An (improbable) domestic refrigerator is maximally efficient, i.e., it is as efficient as any refrigerator can be. A refrigerator's efficiency is the ratio between the amount of heat Q_2 extracted from the cold-box and the amount of work W done: $\eta = Q_2/W$.



   (a) What is the efficiency of the refrigerator in terms of the temperatures T_1 and T_2 of the kitchen and the cold-box?

   (b) If the cold-box is at $-4°$C and the kitchen is at $23°$C, what is the efficiency?

   (c) If the refrigerator consumes 100 Watts when running, at what rate does heat come out the back of the refrigerator?

   (d) A real refrigerator is of course not perfectly efficient. If a refrigerator with only 20% the efficiency is used to cool the same selection of objects to the same temperature, how much power will the fridge consume when running? And how much heat will come out the back?

2. **Charging a capacitor:** Returning to the problem of a charging capacitor that we saw earlier in the semester, we can now write a full expression for a change in the internal energy as

$$\mathrm{d}U = T\,\mathrm{d}S + V\,\mathrm{d}q.$$

   (a) Derive a Maxwell relation from this expression for $(\partial T/\partial q)_S$.

   (b) Write down the expressions for small changes in the other three thermodynamic potential functions and the corresponding Maxwell relations. (No need to write out the derivations in full, although you can if it's useful for working them out.)

   (c) Suppose we charge the capacitor in thermal isolation. By using the reciprocity rule and one of your Maxwell relations show that

$$\left.\frac{\partial T}{\partial V}\right|_S = -\frac{T}{C_V}\left.\frac{\partial q}{\partial T}\right|_V,$$

   where C_V is the heat capacity at constant voltage. If the capacitance is independent of temperature, what is the change in temperature when we charge the (initially uncharged) capacitor up to one volt?

   (d) A different capacitor has a capacitance that does depend on temperature: it has $C = a/T$, where a is a constant. Suppose that this capacitor is initially is at temperature T_1 and that its heat capacity is constant in the temperature range of interest. What then is the final temperature T_2 of the capacitor if we charge it adiabatically up to one volt?

3. **A weight on a spring:** A light spring is hung from the ceiling and a weight of mass m attached to its free end, under normal gravity g and at room temperature T_0. The weight stretches the spring by a small extension x_1 (small in the sense that the spring is linear and obeys Hooke's law). This stretching happens fast and can be considered adiabatic. As a result the spring heats up to temperature T_1. Then, over a longer period of time, the spring cools back down—still with the weight attached—to T_0 and in the process contracts to a new length with extension x_2.

   (a) Show that the ratio of the isothermal and adiabatic spring constants, k_T and k_S, satisfies

   $$\frac{k_T}{k_S} = \frac{C_L}{C_f},$$

   where C_L and C_f are the heat capacities at constant length and force respectively.

   (b) Hence find an expression for the final extension x_2 of the spring in terms of the initial extension x_1 and the two heat capacities. You can assume small extensions and heat capacities that are independent of temperature over the temperature range considered.

   (c) The spring is made of iron and has mass 100 grams. The temperature of the room is 70°F and the spring heats up to 75°F when we stretch it. The two extensions are measured to be 20cm and 10cm. Approximately how much heat leaves the spring as it cools? (You can assume small extensions again, and you'll probably have to look up some property of iron somewhere.)