# Physics 406: Homework 1

1. **Work done on a ideal gas:** The pressure $p$ and volume $V$ of $n$ (gram) moles of a ideal gas obey the equation of state $pV = nRT$, where $R = 8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$.

   (a) How much work must be done to compress a mole of this gas at constant temperature $T$ from volume $V_1$ to volume $V_2$? Verify that your answer is indeed positive when the volume of the gas gets *smaller*.

   (b) How much work must be done to compress a mole of this gas at constant temperature $T$ from pressure $p_1$ to pressure $p_2$?

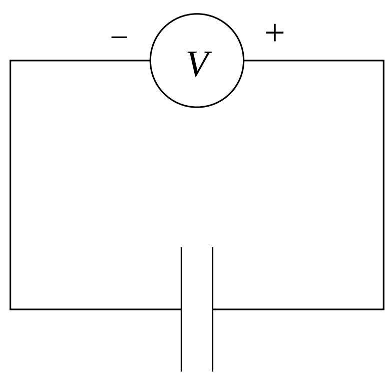
   (c) How much work must be done to compress *half* a mole of this gas at constant temperature $T = 21°\mathrm{C}$ from one atmosphere pressure to two atmospheres? (Recall that $0°\mathrm{C} \equiv 273.15\,\mathrm{K}$.)

   (d) The **isothermal compressibility** $\kappa_T$ of a gas measures the fractional change in volume as we change the pressure on the gas at constant temperature:

   $$\kappa_T = -\frac{1}{V}\frac{\partial V}{\partial p}\bigg|_T.$$

   Derive an expression for $\kappa_T$ as a function of volume for one mole of a ideal gas. How would the expression change if we had 2 moles?

2. **Electrical work:** Suppose we have a circuit in which a voltage source is connected to a capacitor with capacitance $C$ (which may depend on temperature):




$C$

   Starting with an uncharged capacitor and zero voltage, we isothermally increase the voltage, so as to charge the capacitor.

   (a) If the charge across the capacitor is $q$ at some point, what is the voltage across it? (Note: we use lower-case $q$ to avoid confusion with heat $Q$, but charge is the extensive variable here and voltage is the intensive one.)

   (b) The definition of electrical work is the product of charge moved times the potential difference it moves across. So the amount of work $đW$ that needs to be done by the voltage source in order to move an extra small amount of charge $\mathrm{d}q$ from one side of the capacitor to the other is $đW = V\mathrm{d}q$. This is the equivalent of $đW = -p\mathrm{d}V$ in our fluid system. If we charge the capacitor isothermally from empty up to a charge $q$, what is the total work done as a function of $q$ and the capacitance $C$?

(c) And as a function of the voltage $V$?

(d) If we do not fix the temperature of the capacitor as we charge it, but instead allow it to change freely, the capacitor will heat up as it is charged. What is the differential form of the First Law for a change in internal energy of the capacitor (i.e., what is the expression for an infinitessimal change $\mathrm{d}U$ in the internal energy of the capacitor)? And what is the amount of heat absorbed $\bar{\mathrm{d}}Q$ by the capacitor for a given change $\mathrm{d}U$ in internal energy and $\mathrm{d}q$ in charge?

(e) Hence write down an expression for the heat capacity $c_q$ at constant charge of the capacitor, and another (more complicated) one for the heat capacity $c_V$ at constant voltage. If the capacitance is independent of temperature, what is the expression for $c_V$?

(f) An infinitessimal change in the enthalpy of a pressure/volume system is, as we have seen, $\mathrm{d}H = \bar{\mathrm{d}}Q + V\mathrm{d}p$. What is the corresponding expression for the enthalpy of a capacitor?

3. **Heat capacity of copper:** A 0.1 kg piece of copper metal is heated to 100°C and dropped into a thermally insulated vessel containing 0.2 liters of water at 15°C. Pressure is held constant. After the system has come to equilibrium, the temperature is measured again and both copper and water are found to be at 18.6°C. Assuming the heat capacities of both water and copper to be constant over the range of temperatures involved, calculate the ratio of the specific heat capacities of copper and water.