Complex Systems 511: Homework 2

For full credit, show all your working.

1. **Transcritical and pitchfork bifurcations:** Each of the following systems has either a transcritical or pitchfork bifurcation. Identify the type of bifurcation in each case and find the value of r at which they occur. For pitchfork bifurcations classify each as supercritical or subcritical.

   (a) $\dot{x} = rx - \ln(1 + x)$

   (b) $\dot{x} = x(r - e^x)$

   (c) $\dot{x} = rx - \sinh x$

   (d) $\dot{x} = x + rx/(x^2 + 1)$

2. **A two-dimensional linear system:** Consider the system $\dot{x} = 4x - y$, $\dot{y} = 2x + y$.

   (a) Show that the characteristic polynomial for the system is $\lambda^2 - 5\lambda + 6$ and so find the eigenvalues and eigenvectors.

   (b) Derive the general solution for the system.

   (c) Classify the fixed point at the origin.

   (d) Solve the system with the initial condition $x = 3$, $y = 4$.

3. **Phase portraits:** Plot the phase portrait and classify the fixed point at the origin for the following systems of equations:

   (a) $\dot{x} = y$, $\dot{y} = -2x - 3y$

   (b) $\dot{x} = 4x - 3y$, $\dot{y} = 8x - 6y$

4. **Steve Strogatz' messed-up couple:** In class we considered the case of Romeo and Juliet, whose tragic love affair is described by a pair of unromantic differential equations in which $R(t)$ is Romeo's love (or hate) for Juliet at time t and $J(t)$ is Juliet's for Romeo. Find and describe what happens in each of the following cases, if $a, b > 0$:

   (a) Do opposites attract? $\dot{R} = aR + bJ$, $\dot{J} = -bR - aJ$.

   (b) What if they have everything in common? $\dot{R} = aR + bJ$, $\dot{J} = bR + aJ$. Should they expect boredom or bliss?

   (c) Nothing can ever change the way Romeo feels: $\dot{R} = 0$, $\dot{J} = aR + bJ$. Does Juliet end up loving him or hating him?