Complex Systems 511: Homework 1

For full credit, show all your working.

1. **Fixed points:** For each of the following cases, give an equation of the form $\dot{x} = f(x)$ with the stated properties or, if there is no such equation, explain why not:

   (a) Every integer is a stable fixed point.

   (b) Every real number is a fixed point.

   (c) There are precisely three fixed points, and all of them are stable.

   (d) There are no fixed points.

   (e) There are precisely 100 fixed points.

2. **Linear stability analysis:** Use linear stability analysis to classify the fixed points of the following equations (or show that such an analysis breaks down):

   (a) $\dot{x} = x(1-x)(2-x)$

   (b) $\dot{x} = \tan x$

   (c) $\dot{x} = x^2(4-x)$

   (d) $\dot{x} = \mathrm{e}^{-1/x^2}$

   (e) $\dot{x} = \ln x$

3. **A model of the spread of disease:** An incurable disease spreads through a population according to the following process. Let x and y be the fractions of individuals infected and uninfected respectively. The rate at which the uninfected catch the disease from the infected is proportional to the number of uninfected and the number of infected. Write down an equation for the rate of change of x. Explain how y can be eliminated from this equation, and hence show that $\dot{x} = rx(1-x)$, where r is a parameter describing the virulence of the disease. Find and classify the fixed points and describe what each of them corresponds to in terms of the disease.

4. **Saddle-node bifurcations:** For each of the following equations show that there is a saddle-node bifurcation in the dynamics of x for some value of the parameter r, and determine that value. Sketch the bifurcation diagram of fixed points x^* as a function of r in each case:

   (a) $\dot{x} = r - \cosh x$

   (b) $\dot{x} = x^2 + rx + 1$

   (c) $\dot{x} = r + x - \ln(x+1)$

   (d) $\dot{x} = r^2 + \frac{1}{4}x - x/(1+x)$