

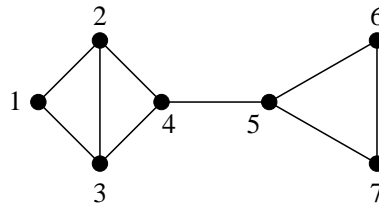
Complex Systems 535/Physics 508: Homework 6

1. **The number of communities in a network:** The following file contains the adjacency matrix of a small undirected network of 32 vertices:

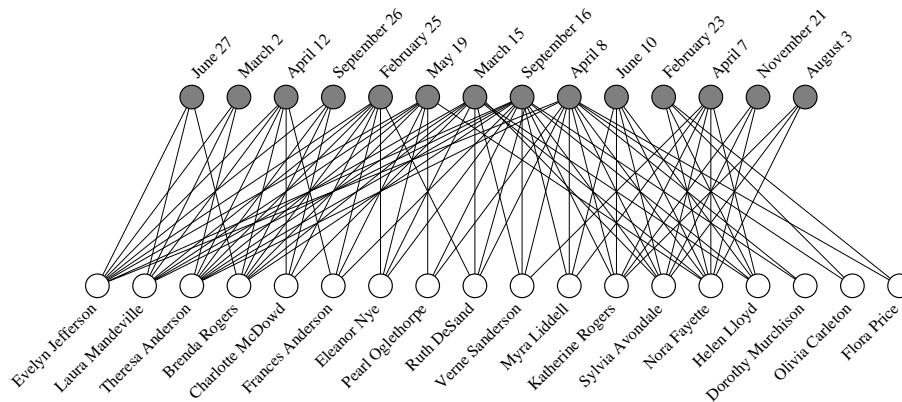
<http://www-personal.umich.edu/~mejn/courses/2005/cscs535/groups.adj>

Download the file and calculate the spectrum of the Laplacian for the graph. Make a plot or sketch showing the spectrum. Hence make a statement about how many communities you think the network contains.

2. **Hierarchical clustering for a small network:** Here is a small network which I just made up off the top of my head:



- (i) Write down the adjacency matrix and use it to calculate the cosine similarity for all $\frac{1}{2} \times 7 \times 6 = 21$ pairs of vertices. List the similarities in descending order.
 - (ii) Add edges to an initially empty graph of 7 vertices and so construct the dendrogram for the single-linkage hierarchical clustering of this network. Does the method find the two (seemingly) obvious communities of the four vertices on the left and the three on the right?
3. **Singular value decomposition of a social network:** Here is another small network, this one real. This is the famous “southern women” network of Davis, Davis, and Gardner (1941), which shows $n = 18$ women (by name) and $m = 14$ social events (by date) that they attended in 1936:



The 14×18 incidence matrix for the network is here:

<http://www-personal.umich.edu/~mejn/courses/2004/cscs535/southern.inc>

- (i) Perform a singular value decomposition of the incidence matrix. Give the complete list of singular values for the eigenvectors.

- (ii) What percentage of the sum of the squares of the elements of the incidence matrix is accounted for by the leading eigenvector? And by the leading two? If you were to divide the women into two groups on the basis of the SVD, what would the two groups be? Are any of the members of the groups ambiguous?