Blackfolds: effective worlvolume theory for black branes (part II)

BH workshop

U. Michigan, Ann Arbor, October 15, 2013

Niels Obers, NBI

0912.2352 (JHEP), 0910.1601 (JHEP), 0902.0427 (PRL) + 1106.4428 (JHEP0 (with R. Emparan, T. Harmark, V. Niarchos)

1012.5081 (PRD) (with J. Armas)

1012.1494 (JHEP), 1101,1297 (NPB), 1112.

with G. Grignani, T. Harmark, A. Marini, M. Orselli

0708.2181 (JHEP) (with R. Emparan, T. Harmark, V. Niarchos, M.J. Rodriguez)

1307.0504 & 1209.2127 (PRL) (with J. Armas, J. Gath)

1210.5197 (PRD) (with J. Armas)

1110.4835 (JHEP) (with J. Armas, J. Camps, T. Harmark)

Intro +overview

- long wave length perturbations of black branes
  - construction of new BH solutions in higher dimenions (ST)
  - properties of QFTs via holography

  in long-wave length regime: →
  black branes behave like any other type of continuous media with dynamics governed by some (specific) effective theory

- new insights into GR/geometry
- find BHs in higher dimensions and discover their properties
- effective theory that integrates out gravitational degrees of freedom
- AdS/CFT (fluid/gravity) inspired new way to look at gravity
- find universal features of black branes in long wave length regime
  described by "every day" physics
- reduce complicated gravitational physics to simple response coefficients
- cross-fertilization between classical elasticity/fluid theory and gravity
  (cf. rigorous development of fluid and superfluid dynamics using fluid/gravity correspondence)

Blackfold approach: a unified framework

two types of deformations:

- intrinsic:

time (in)dependent fluctuations along worldvolume/boundary directions

effective theory of viscuous fluid flows

Bhattacharyya,Hubeny,Minwalla,Rangamani
Erdmenger,Haack,Kaminski,Yarom/Nanerjee et al
(fluid/gravity ….)
Camps,Emparan,Haddad/Gath,Pedersen
Emparan,Hubeny,Rangamani

- extrinsic:

stationary perturbations along directions transverse to woldvolume

effective theory of thin elastic branes

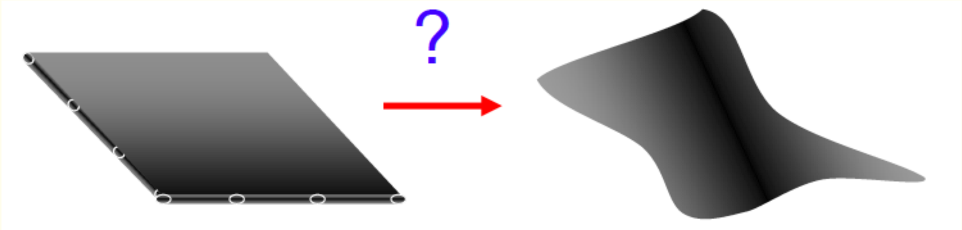
Emparan,Harmark,Niarchos,NO
Armas,Camps,Harmark,NO
Camps,Emparan
Armas,Gath,NO/Armas,NO/Armas

fluids living on dynamical surfaces ("fluid branes") = **blackfold aproach**
(unified general framework of the two descriptions)

Reviews: Emparan,Harmark,Niarchos,NO
Emparan/
Harmark,NO (to appear)

Blackfolds: framework for dynamics of black branes

- based on bending/vibrating of (flat) black branes



blackfold = black brane wrapped on a compact submanifold of spacetime

very much like other extended solitonic objects:

- Nielsen-Olesen vortices and NG strings
- open strings and DBI action

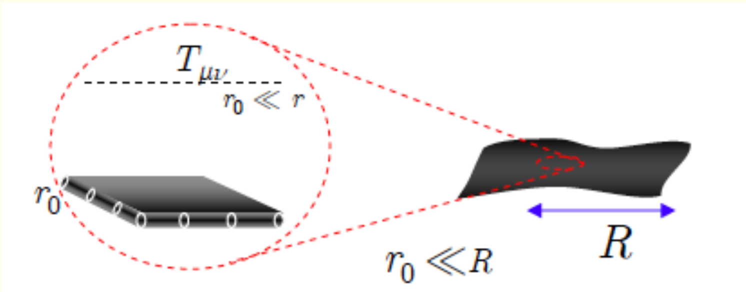
difference: - short-distance d.o.f. = gravitational short-wavelength modes

- extended objects posses black hole horizon

-> worldvolume thermodynamics

Effective worldvolume theory – leading order

widely separated scales: perturbed black brane looks locally like a flat black brane



- effective stress tensor of black branes correspond to specific type of fluid
  to leading order: perfect fluid

$$T^{ab} = (\varepsilon + P)u^a u^b + P\gamma^{ab}$$

for charged black branes of sugra:
novel type of (an)isotropic charged fluids

notation: spacetime $X^\mu,\ \mu,\nu \ldots = 0,\ldots,D-1$

worldvolume $\sigma^a,\ a,b \ldots = 0,\ldots,p$

$$n = D - p - 3$$

BF equations

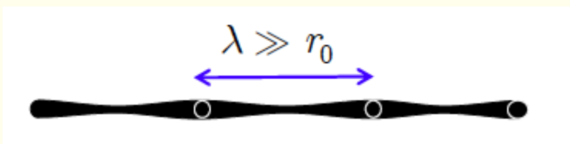
blackfold equations

Emparan,Harmark,Niarchos,NO

(liquid)

intrinsic (Euler equations of fluid + charge conservation)

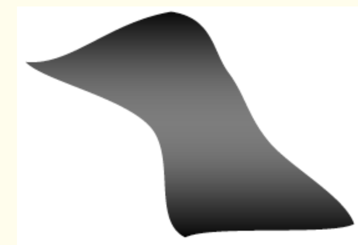
fluid excitations (+ charge waves)



(solid)

extrinsic (generalized geodesic eqn. for brane embedding)

elastic deformations



- gives novel stationary black holes (metric/thermo) + allows study of time evolution
- generalizes (for charged branes) DBI/NG to non-extremal solns. (thermal)
- possible in principle to incorporate higher-derivative corrections (self-gravitation + internal structure/multipole)
- BF equations have been derived from Einstein equations Camps,Emparan

general emerging picture (from hydro of non-extremal D3-branes)

Membrane paradigm $\subset$ Fluid/gravity correspondence $\subset$ Blackfolds.

Emparan,Hubeny,Rangamani

Main ingredients

- identify collective coordinates of the brane

$$X^\mu(\sigma), \varepsilon(\sigma), u^\mu(\sigma), q(\sigma), \ldots$$

positions transverse to worldvolume

energy density (horizon thickness)

local boost velocity

charge density

- blackfold equations of motion follow from conservation laws (stress tensor, currents,..)

$$\bar{\nabla}_\mu T^{\mu\nu} = 0, \bar{\nabla}_\mu J^\mu = 0, \ldots$$

⇨ effective (charged) fluid living on a dynamical worldvolume:

$$T^{\mu\nu} K_{\mu\nu}{}^\rho = 0$$

extrinsic equations (D-p-1)

$$D_a T^{ab} = 0$$

intrinsic equations (p+1)

$$D_a J^{a_1 \cdots a_{p+1}} = 0$$

leading order BF equations

Stationary solutions

◆ equilibrium configurations stationary in time = stationary black holes

$$u = \frac{k}{|k|}\,, \qquad \nabla_{(\mu}k_{\nu)} = 0\,, \qquad \mathcal{T}(\sigma^a) = \frac{T}{|k|} \qquad k = \xi + \Omega\chi$$

can solve intrinsic blackfold equations explicitly (e.g. for thickness and velocity)

→ only need to solve extrinsic equations for the embedding

$$r_0 = \frac{n\sqrt{1-V^2}}{2\kappa} \qquad V^2 = \sum_i \Omega_i^2 R_i^2(\sigma)$$

velocity field

• blackfolds with boundaries: fluid approaches speed of light at bdry. (horizon closes off !)

extrinsic equations: $K^\rho = \perp^{\rho\mu} \partial_\mu \ln(-P)$

⇨ derivable from action $\tilde{I} = \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma \sqrt{-\gamma}\, P$

- thermodynamics: all global quantities: mass, charge, entropy, chemical potentials by integrating suitable densities over the worldvolume

Action principle for stationary blackfolds and 1st law

- for any embedding (not nec. solution) the "mechanical" action is proportional to Gibbs free energy:

$$\beta^{-1} I = G = M - \sum_i \Omega_i J_i - TS$$

varying G -> 1st law of thermodynamics

$$dM = TdS + \Omega dJ \qquad (\text{fixed } Q_p)$$

⇨ 1st law of thermo = blackfold equations for stationary configurations

- can also use Smarr relation to show that:
  total tension vanishes for stationary blackfolds

$$(D-3)M - (D-2)(TS + \Omega J) - n\Phi_H^{(p)} Q_p = \mathcal{T}_{\text{tot}}$$

Applications of BF

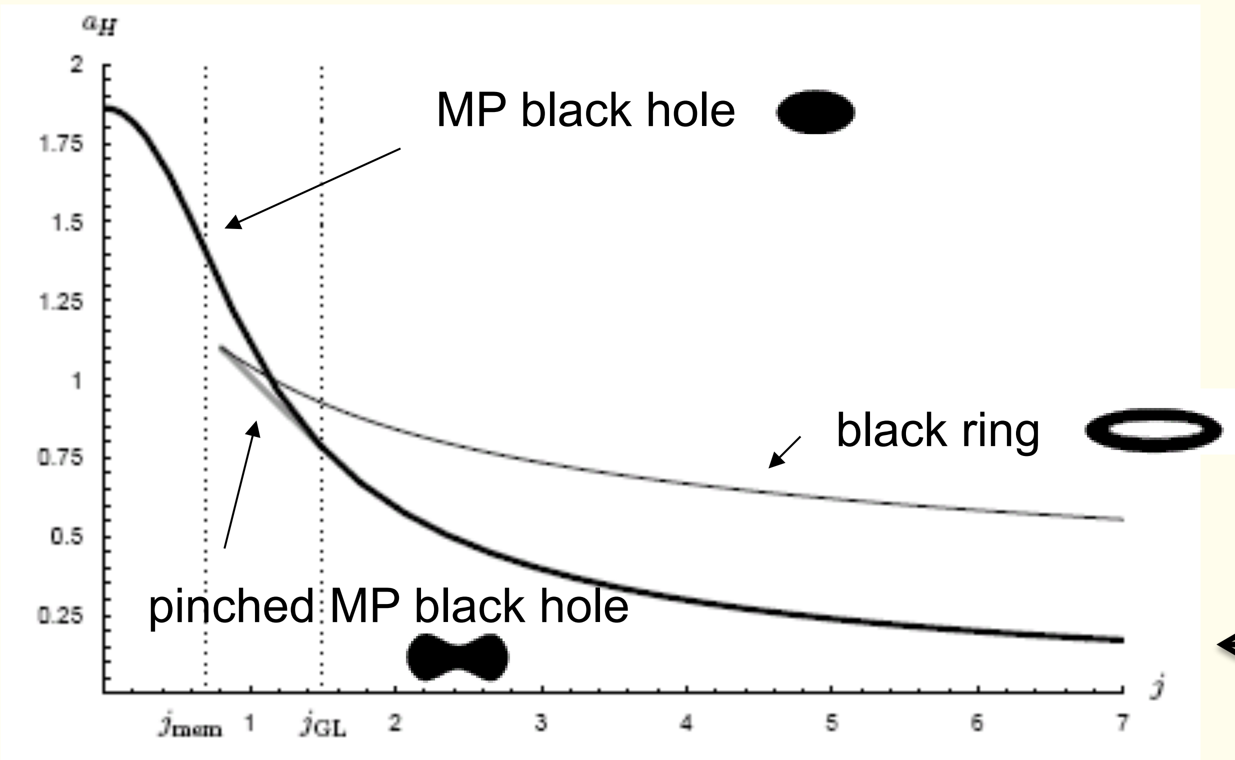
- new (approximate) stationary black hole solutions

- thermal probe brane method and connection to DB

- perturbations:
  - elasticity response coefficients
  - hydrodynamcis and GL instabilitity

I. New stationary BHs

- Solve the BF equations for stationary solutions:
  typically: zero tension condition
  - e.g. black ring: gravitational attraction balanced by centrifugal repulsion



black string curved along a circle

approaches black membrane (pancake)

hierarchy of scales:

- Kerr regime
- regime of mergers and connections
  i) new solution branches (via 0-modes_
  ii) topology changing transition
- BF regime (ultraspinning)

Example: Black ring

► wrap black string on a compact 1D space (topologically S^1)

specify embedding: S^1 in $\mathbb{R}^2$ (times point in $\mathbb{R}^{D-3}$)

$$\mathbb{R}^2 : \quad (r, \phi) \qquad r = R(\sigma) \ , \quad \phi = \sigma$$

action $\quad I_{WV} \propto \int \sqrt{-\gamma}(1-\Xi^2)^{\frac{n}{2}} = \int d\sigma \sqrt{(R')^2 + R^2}(1-\Omega^2 R^2)^{\frac{n}{2}}$

⟶ full EOM is:

$$(1-\Omega^2 R^2)RR'' + ((n+2)\Omega^2 R^2 - 2)R'^2 + ((n+1)\Omega^2 R^2 - 1)R^2 = ($$

• highly non-linear DE; simple solution with constant R. $\quad R = \frac{1}{\sqrt{n+1}}\frac{1}{\Omega}$

or directly from Carter equation: $\quad \frac{\tau_{11}}{R} = 0 \quad$ (total tension vanishes)

◄ zero tension condition is equivalent to balancing forces on ring

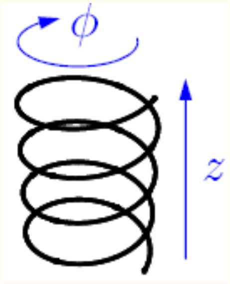
- centrifugal repulsion balances gravitational tension
- solution with horizon topology $S^1 \times S^{D-3}$

Neutral blackfolds

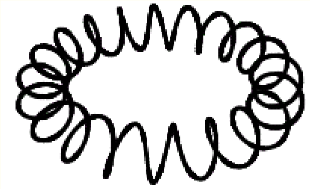
Emparan,Harmark,Niarchos,NO

First applied to neutral black branes of higher dim gravity:

Quick overview of results:

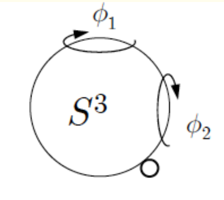


-new helical black strings and rings

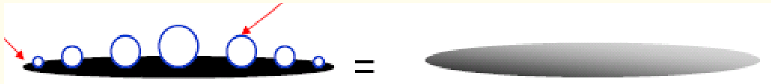


- odd-branes wrapped on odd-spheres
  (generalizes 5D black ring)

$$(\Pi_{p_a=\mathrm{odd}} S^{p_a}) \times s^{n+1}$$

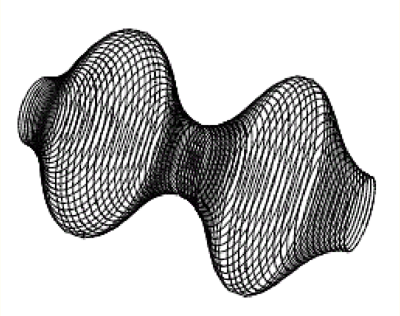


- even-branes wrapped on even-balls
  correctly reproduce MP BHs in
  ultraspinning (pancaked) limit



- non-uniform black cylinders

$$(\mathbb{R} \times S^1) \times s^{n+1}$$



- static minimal blackfolds
  (non-compact)

Blackfolds in supergravity and string theory

Emparan,Harmark,Niarchos,NO
Caldarelli,Emparan,v. Pol
Grignani,Harmark,Marini,NO,Orselli

- BF method originally developed for neutral BHs, but even richer dynamics when considering charged branes

- extra equations: charge conservation
  consider dilatonic black branes that solve action (includes ST black branes)

$$S = \frac{1}{16\pi G}\int d^D x\sqrt{-g}\left[R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2(q+2)!}e^{a\phi}H^2_{[q+2]}\right]$$

p-branes with q-charge: q=0, particle charge, q=1: string charge, etc.)

anistropic (charged) fluids

$$T_{ab} = \varepsilon u_a u_b + P_\perp\left(\gamma_{ab} + u_a u_b - v_a v_b\right) + P_{||} v_a v_b$$

q=1

$$J^{(0)}_{ab} = Q u_{[b} v_{a]}$$

spacelike vector v along the directions of the 1-charge (string)

Effective fluid

for charged dilatonic p-branes:
use exact sugra solution to read off the properties of the fluid:

$$\varepsilon = \frac{\Omega_{(n+1)}}{16\pi G} r_0^n \left(n + 1 + nN \sinh^2\alpha\right) , \qquad P = -\frac{\Omega_{(n+1)}}{16\pi G} r_0^n \left(1 + nN \sinh^2\alpha\right) ,$$

$$\mathcal{T} = \frac{n}{4\pi r_0 \left(\cosh\alpha\right)^N} , \qquad s = \frac{\Omega_{(n+1)}}{4G} r_0^{n+1} \left(\cosh\alpha\right)^N ,$$

$$Q_p = \frac{\Omega_{(n+1)}}{16\pi G} n\sqrt{N} r_0^n \sinh\alpha \cosh\alpha , \qquad \Phi_p = \sqrt{N} \tanh\alpha .$$

functions of r_0 and α, N = 1,2,3 (depending on case)

stress tensor takes form:

$$T_{ab} = \mathcal{T} s \left(u_a u_b - \frac{1}{n}\gamma_{ab}\right) - \Phi_p Q_p \gamma_{ab}$$

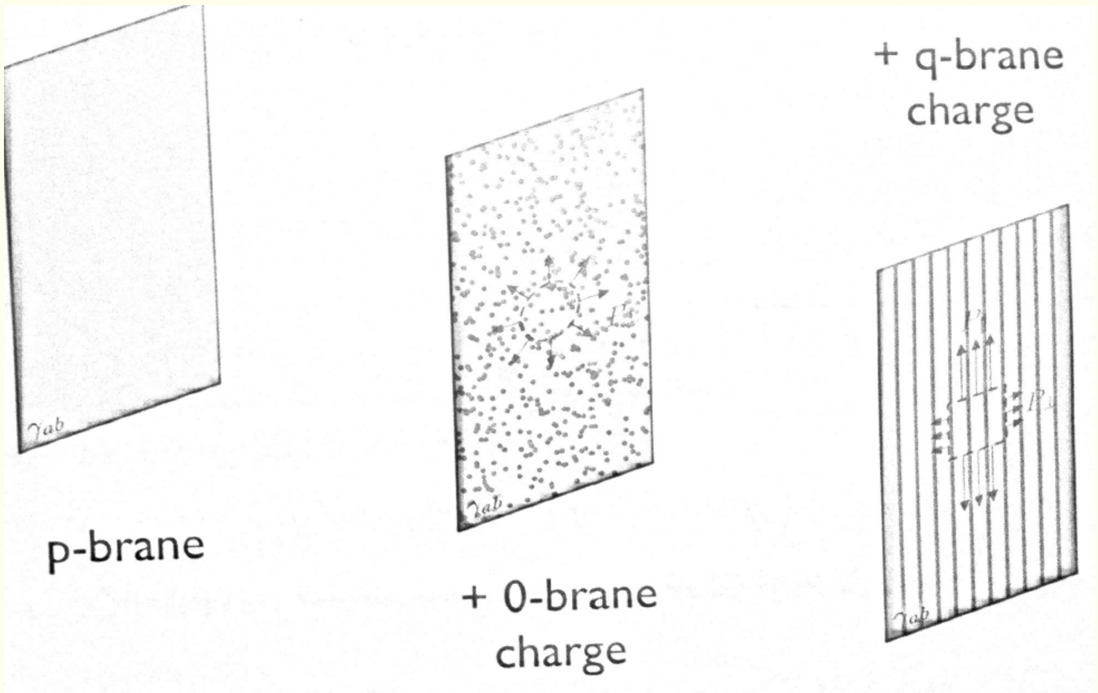
near extremality:

$$T_{ab}^{(\mathrm{exc})} \simeq \mathcal{T} s \left(u_a u_b + \left(\frac{N}{2} - \frac{1}{n}\right)\gamma_{ab}\right)$$

Blackfolds with brane currents

- can perform general analysis of blackfolds with brane currents on them (q=0 and q=1 simplest): anistropic charged perfect fluids (entirely new type of fluid dynamics)
- able to capture thermal excitations of e.g. D-branes with lower D-brane or F-string currents

brane currents induce differences in pressures in directions parallel and transverse to them (due to effective tension $\Phi_q Q_q$ along the current)



- charge density conserved along the q-brane but can redistribute itself in transverse directions

- stability of charge waves governed by isothermal permittivity

$$\epsilon_q \equiv \left(\frac{\partial \Phi_q}{\partial Q_q}\right)_{Q_p,T}$$

ST/M: BF with dipole p-brane charge & brane currents

- -> find new odd-sphere (+products) stationary black hole solutions with dipole-like (local) charge (checks with exact 5D dipole rings (Emparan))

- new stationary black holes in string/M-theory, w. novel horizon topology
- new type of charge (generalizing dipole charge of ring) entering 1st law of thermo (cf. Copsey, Horowitz)
- (presumably) stable for sufficiently high charge (positive specific heat)
- standard extremal limit gives Dirac: $T_{ab} = P\gamma_{ab}$
- interesting new extremal limits with null waves (beyond DBI):

$$T_{ab} = \mathcal{K}\, l_a l_b - \sqrt{N} Q_p\, \gamma_{ab} \qquad l_a l^a = 0$$

■ can perform general analysis of blackfolds with brane currents on them (q=0 and q=1 simplest): anistropic charged perfect fluids (entirely new type of fluid dynamics)

- able to capture thermal excitations of e.g. D-branes with lower D-brane or F-string currents

- 3-charge example: D1-D5-P (e.g. in D=6: horizon S^1 x s^3) with finite entropy in extremal limit could be first example of stable, asympotically flat, extremal, non-supersymmetric brane in ST with non-spherical horizon topology in D > 5

Odd-sphere blackfolds in string theory

Emparan,Harmark,Niarchos,NO

Brane (IIA)	Worldvolume	$\perp$ Sphere
F1	S^1	s^7
D2	$\mathbb{T}^2$	s^6
D4	$S^3 \times S^1$, $\mathbb{T}^4$	s^4
NS5	S^5 , $S^3 \times \mathbb{T}^2$	s^3
D6	$S^3 \times S^3$, $S^5 \times S^1$	s^2

Brane (IIB)	Worldvolume	$\perp$ Sphere
D1	S^1	s^7
F1	S^1	s^7
D3	S^3, $\mathbb{T}^3$	s^5
D5	S^5 , $S^3 \times \mathbb{T}^2$	s^3
NS5	S^5 , $S^3 \times \mathbb{T}^2$	s^3

Table 1: *A list of horizon topologies for stationary non-extremal black holes in type IIA/IIB string theory based on the singly-charged blackfolds of the theory with worldvolumes curved into products of odd-spheres. The s^{n+1} denotes the 'small' sphere in horizon directions orthogonal to the worldvolume. The number ℓ of 'large' odd-spheres spanned by the worldvolume is limited by* (3.11).

Brane	Worldvolume	$\perp$ Sphere
M2	$\mathbb{T}^2$	s^7
M5	S^5, $S^3 \times \mathbb{T}^2$, $\mathbb{T}^5$	s^4

Table 2: *The analogue of Table 1 in M-theory for M2 and M5 black branes.*

Examples (2 charges)

blackfolds based on 2-charge brane systems: D0-Dp (p=2,4,6) F1-Dp (p >0)

		Worldvolume	$\perp$ Sphere
	F1-D1	(helical) S^1	s^7
D0-D2	F1-D2	$\mathbb{T}^2$	s^6
	F1-D3	S^3 , $\mathbb{T}^3$	s^5
D0-D4	F1-D4	$S^3 \times S^1$, $\mathbb{T}^4$	s^4
	F1-D5	S^5 , $S^3 \times \mathbb{T}^2$	s^3
D0-D6	F1-D6	$S^3 \times S^3$, $S^5 \times S^1$	s^2

Examples (3 charges)

3-charge example: D1-D5-P (e.g. in D=6: horizon S^1 x s^3

with finite entropy in extremal limit

could be first example of stable, asympotically flat, extremal,
non-supersymmetric brane in ST with non-spherical horizon topology in D > 5

s)

Dimension (non-compact)	Worldvolume	$\perp$ Sphere
$D = 10$	S^5 , $S^3 \times \mathbb{T}^2$	s^3
$D = 9$	$S^3 \times S^1$, $\mathbb{T}^4$	s^3
$D = 8$	S^3 , $\mathbb{T}^3$	s^3
$D = 7$	$\mathbb{T}^2$	s^3
$D = 6$	S^1	s^3

Table 4: *A list of horizon topologies for stationary extremal rotating black holes with D1-D5 dipoles in a spacetime with D non-compact dimensions and $10 - D$ compact KK circles. We do not distinguish whether the D1-P current wraps some of the compact directions, which gives different kinds of black holes.*

other new extremal black holes in ST ?

Thermal probe branes and BFs

- Brane/string probes widely used in ST, including AdS/CFT

- uncover features of backgrounds, phase transitions, stringy observables, non-perturbative aspects of FT, dual operators in CFT, ads/CMT
- learn new things about fundamentals of ST/M-theory by studying low energy theories on D/M-branes

conventionally used (at weak coupling):

F-stings: NG (Wilson loops, q-qbar potential, energy loss of quarks)

D-branes: DBI (Wilson loops in large sym/antisym reps, flavors, meson spectroscopy, giant gravitons)

M-branes: PST (giant gravitons, self-dual string)

what happens when we heat this up ?

Open/closed perspectives

worldvolume (DBI,NG)	⟷	spacetime (SUGRA)
microscopic, open		macroscopic, closed
weak coupling		strong coupling

- for SUSY configs can interpolate between the two (exactly)
- for non-SUSY (finite T): qualitative matching (more control for near-extremal)

"shapes of branes/strings" are determined dynamically

most work on curved D-branes performed using open string picture:

probe (N=1)

⇨ $$K_{ab}{}^{\rho} T^{ab} = J \cdot F^{\rho}$$

can use symmetries, ansatze consistency to construct the exact backgrounds in SUGRA (N >> 1)

external force

extrinsic curvature (2nd fundamental form)

(DBI) EM tensor of the brane

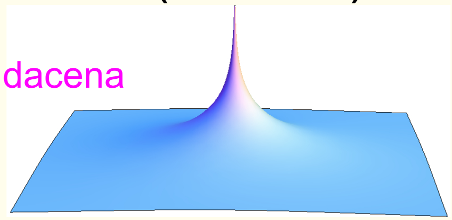
Note: any probe brane will have EOM of this form   Carter

Example: BIon

DBI

D3-brane DBI with constant electric flux (D3->F1)

Callan,Maldacena



SUGRA

derive appropriate PDEs and prove existence of SUSY sol

Lunin

branes follow harmonic profiles (unique, given BCs)

match: open/closed duality beyond decoupling limit

⇨ Q: Can one extend this open/closed picture to finite T ? (non-SUSY)

interesting since:

- develop horizon, learn about BH physics
- branes are used to probe spaces at finite T (hot flat or AdS space, AdS BH)
- thermal states in gauge theories (AdS/CFT)

Intermezzo: conventional method for probe branes in thermal background

conventionally used method: 'Euclidean DBI probe' method:

- Wick rotate background and classical DBI action
- find solns. of EOM
- identify the radii of thermal circle in background and DBI soln.

(see also: Kiritsis/Kiritsis,Taylor/Kiritsis,Kehagias)

boils down to: solving same (local) EOMs but different BCs

$$(T_E)^{ab}_{\rm DBI}(K_E)_{ab}{}^{\rho} = (\perp_E)^{\rho\lambda}\frac{1}{4!}(J_E)^{abcd}(F_E)_{\lambda abcd}$$

this global condition is not enough to ensure that probe is in thermal equilibrium with the background

reason: to ensure thermal equilibrium we need to also modify the EOMs (via the stress tensor) since the brane DOFs get thermally excited

Example: single D3-brane near extremality (at weak coupling)

- gas of photons (+ superpartners)

$$T_{ab} = -T_{\rm D3}\eta_{ab} + T^{\rm (NE)}_{ab}\ , \quad T^{\rm (NE)}_{00} = \rho\ , \quad T^{\rm (NE)}_{ii} = p\ , \ i = 1,2,3 \qquad \rho = 3p = \pi^2 T^4/2.$$

Open/closed at finite T

open (weak coupling) , N=1

thermal DBI
(thermal SUSY gauge theory
+ string corrections)

Grignani,Harmark,Marini,Orselli (to appear)

thermal NG: quantize string in finite T background

de Boer,Hubeny,Rangamani,Shigemori

closed (strong coupling), N >>1

black branes (solitons)
- curved black brane solutions in SUGRA

exact solutions already hard at T=0

↓

go to regime where brane is approximately **locally** flat:
-> can use probe approximation
= 0^{th} order blackfold construction

gives the geometry to leading order in perturbative expansion governed r0/R

like DBI/NG this is (to leading order) probe computation:
dynamics in both cases described by Carter equation:

⇨ - difference is EM tensor that you put
+ different regime ! (match when T->0,N=1)

Applications

-> Heating up DBI/NG solutions using:
blackfolds as thermal probe branes/strings in string theory

shows new qualitative & quantitative effects

- Thermal Bion solutions (wormhole & spike)
  Grignani,Harmark,Marini,NO,Orsell

  M5-M2 system and self-dual string
  Niarchos,Siampos

- Thermal string probes in AdS &
  finite T Wilson loops
  Grignani,Harmark,Marini,NO,Orsell

- Thermal (spinning) giant gravitons
  (also: new null-wave giant gravitons)
  Armas,Harmark,NO,Orselli,Vigand Pedersen
  Armas,NO,Vigand Pedersen

black probe	background
D3-F1	hot flat 10D
M5-M2	11D
F1	AdS BH
	hot..
D3	AdS5 x S5
M2	AdS7 x S4
M5	AdS4 x S7

Black branes as fluids and elastic materials

Goal: show that asymptotically flat (charged) black branes have both elastic and fluid properties

Method: perturb -> consider derivative corrections

two ways:

- intrinsic perturbations parallel to the worldvolume (wiggle)

viscosities (shear, bulk)
charge diffusion
gives connection to GL instability, fluid/gravity, …

Camps,Emparan,Haddad
Gath,Pedersen
Emparan,Hubeny,Rangamani,

can be used in AdS/Ricci flat map

Caldarelli,Camps,Gouteraux,Skenderis

- extrinsic perturbations transverse to the worldvolume (bend)

response coefficients are inputs to effective theory

* generalizes Polyakov QCD string + actions considered in theoretical biology

Elasticity: Fine structure corrections to blackfolds

Armas, Camps, Harmark, NO

- can explore corrections in BF approach that probe the fine structure: go beyond approximation where they are approximately thin

$$\hat{T}^{\mu\nu}(x^\alpha) = \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma\sqrt{-\gamma}\left[T^{\mu\nu}_{(0)}(\sigma^a)\frac{\delta^D(x^\alpha - X^\alpha(\sigma^a))}{\sqrt{-g}} - \nabla_\rho\left(T^{\mu\nu\rho}_{(1)}(\sigma^a)\frac{\delta^D(x^\alpha - X^\alpha(\sigma^a))}{\sqrt{-g}}\right) + \ldots\right]$$

Vasilic, Vojinovic

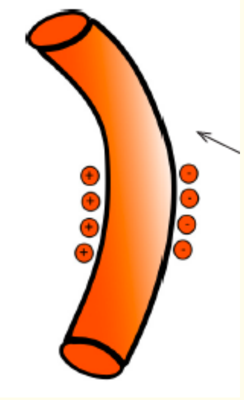
$$T^{\mu\nu\rho}_{(1)} = u_b{}^{(\mu}j^{\nu)\rho b} + u^\mu_a u^\nu_b d^{ab\rho} + u^\rho_a T^{\mu\nu a}_{(1)}$$

$$u^\mu_a \equiv \partial_a X^\mu$$

accounts for:

- dipole moment of wv stress energy
  = bending moment (density)

$$D^{ab\rho} = \int_\Sigma d^{D-1}x\sqrt{-g}\,\hat{T}^{ab}x^\rho = \int_{\mathcal{B}_p} d^p\sigma\sqrt{-\gamma}\,d^{ab\rho}$$



- internal spin degrees of freedom
  (conserved angular momentum density)

$$J^{\mu\nu}_\perp = \int_\Sigma d^{D-1}x\sqrt{-g}\left(\hat{T}^{\mu 0}x^\nu - \hat{T}^{\nu 0}x^\mu\right) = \int_{\mathcal{B}_p} d^p\sigma\sqrt{-\gamma}\,j^{0\mu\nu}$$

Fine structure: Charged branes

Armas,Gath,,NO

- branes charged under Maxwell fields: multipole expansion of current

$$\hat{J}^\mu(x^\lambda) = \int_{\mathcal{W}_{p+1}} dV \left[\frac{J^\mu_{(0)}(\sigma^a)}{\sqrt{-g}} \delta^{(D)}(x^\lambda - X^\lambda(\sigma^a)) - \nabla_\rho \left(\frac{J^{\mu\rho}_{(1)}(\sigma^a)}{\sqrt{-g}} \delta^{(D)}(x^\lambda - X^\lambda(\sigma^a)) \right) + \ldots \right]$$

$$J^{\mu\nu}_{(1)} = m^{\mu\nu} + u^\mu_a p^{a\nu} + J^{\mu a}_{(1)} u^\nu_a$$

dipoles of charge

electric dipole moment:

$$P^{a\rho} = \int_\Sigma d^{D-1}x \sqrt{-g}\, \hat{J}^\mu u^a_\mu x^\rho = \int_{\mathcal{B}_p} d^p\sigma \sqrt{-\gamma}\, p^{a\rho}$$

$$p^{a\rho} = u^a_\mu \perp^\rho{}_\nu J^{\mu\nu}_{(1)}$$

can also write generalization for p-branes carrying q-charge (omit details)

corrected pole/dipole BF equations generalize those of general relativistic (charged) spinning point particle (p=0, q=0) to extended charged objects

Relativistic Young modulus

bending moment a priori unconstrained -> assume classical Hookean elasticity theory:

$$d^{ab\rho} = \tilde{Y}^{abcd} K_{cd}{}^{\rho}$$

bending moment (not present for point particlel)

relativistic Young modulus

extrinsic curvature like Lagrangian strain (measures variation of induced metric transverse to wv.)

general structure of Y can be classified using effective action approach (done for neutral isotropic fluids):

generalization to (isotropic) case with wv. charge:

$$\tilde{Y}^{abcd} = -2\Big(\lambda_1(\mathrm{k};T,\Phi_H)\gamma^{ab}\gamma^{cd} + \lambda_2(\mathrm{k};T,\Phi_H)\gamma^{a(c}\gamma^{d)b} + \lambda_3(\mathrm{k};T,\Phi_H)\mathrm{k}^{(a}\gamma^{b)(c}\mathrm{k}^{d)}$$
$$+\lambda_4(\mathrm{k};T,\Phi_H)\frac{1}{2}(\mathrm{k}^a\mathrm{k}^b\gamma^{cd} + \gamma^{ab}\mathrm{k}^c\mathrm{k}^d) + \lambda_5(\mathrm{k};T,\Phi_H)\mathrm{k}^a\mathrm{k}^b\mathrm{k}^c\mathrm{k}^d\Big) ,$$

k = Killing vector, T = global temperature, Phi = chemical potential

upshot: (charged) black branes are described by this effective theory + characterized by particular values of the response coefficients lambda

piezo electric moduli

- for piezo electric materials: dipole moment proportional to strain (q=0)

$$p^{a\rho} = \tilde{\kappa}^{abc} K_{cd}{}^{\rho}$$

relativistic generalization of piezo-electric modulus found in electro-elasticity

structure of kappa not yet classified from effective action, but from symmetries/covariance

$$\tilde{\kappa}^{abc} = -2\left(\kappa_1(\mathbf{k};T,\Phi_H)\gamma^{a(b}\mathbf{k}^{c)} + \kappa_2(\mathbf{k};T,\Phi_H)\mathbf{k}^a\mathbf{k}^b\mathbf{k}^c + \kappa_3(\mathbf{k};T,\Phi_H)\mathbf{k}^a\gamma^{bc}\right)$$

similarly: for p-branes with q-charge:
- possible anomalous terms in Young-modulus
- piezo electric effect with new types of piezo electric moduli

(note: piezo electric effect also encountered in context of superfluids)

Erdmenger,Fernandez,Zeller

upshot: (charged) black branes are described by this effective theory
+ characterized by particular values of the response coefficients kappa

Measuring Young/piezo electric moduli for charged BB

- can be measured in gravity by computing the first order correction to bent charged black branes

simplest example: charged black branes of EMD theory
obtained by uplift-boost-reduce from neutral bent branes

more involved: charged black p-branes with q-charge of E[(q+1)-form]D theory
can use again same procedure to charge up branes
+ use in string theory setting U-dualities to generate higher form charge

- bending of black string (or brane) induces dipole moments of stress
  can be measured from approximate analytic solution (obtained using MAE)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}^{(M)} + h_{\mu\nu}^{(D)} + \mathcal{O}\left(r^{-n-2}\right)$$

$$\nabla_\perp^2 \bar{h}_{\mu\nu}^{(D)} = 16\pi G d_{\mu\nu}{}^{r_\perp} \partial_{r_\perp} \delta^{(n+2)}(r)$$

- bending of charged black string (or brane) induces dipole moments of charge
  can be measured from approximate analytic solution (obtained using MAE)

$$A_\mu = A_\mu^{(M)} + A_\mu^{(D)} + \mathcal{O}\left(r^{-n-2}\right)$$

$$\nabla_\perp^2 A_\nu^{(D)} = 16\pi G p_\nu{}^{r_\perp} \partial_{r_\perp} \delta^{(n+2)}(r)$$

Examples of results for new response coeffs

p-branes with 0-form (Maxwell) charge: 3+1 response coefficients

Young modulus

$$\lambda_1(\mathbf{k};T,\Phi_H) = \frac{\Omega_{(n+1)}}{16\pi G}\xi_2(n)\left(\frac{n}{4\pi T}\right)^{n+2}|\mathbf{k}|^{n+2}\left(1-\frac{\Phi_H^2}{|\mathbf{k}|^2}\right)^{\frac{n}{2}}\left(\frac{3n+4}{2n^2(n+2)} - \bar{k}\left(1-\frac{\Phi_H^2}{|\mathbf{k}|^2}\right)\right)$$

$$\lambda_2(\mathbf{k};T,\Phi_H) = \frac{\Omega_{(n+1)}}{16\pi G}\xi_2(n)\left(\frac{n}{4\pi T}\right)^{n+2}|\mathbf{k}|^{n+2}\left(1-\frac{\Phi_H^2}{|\mathbf{k}|^2}\right)^{\frac{n}{2}+1}\frac{1}{2(n+2)} \ ,$$

$$\lambda_3(\mathbf{k};T,\Phi_H) = \frac{\Omega_{(n+1)}}{16\pi G}\xi_2(n)\left(\frac{n}{4\pi T}\right)^{n+2}|\mathbf{k}|^{n}\left(1-\frac{\Phi_H^2}{|\mathbf{k}|^2}\right)^{\frac{n}{2}} \ ,$$

piezo electric

$$\kappa_1(\mathbf{k};T,\Phi_H) = \frac{\Omega_{(n+1)}}{16\pi G}\frac{\xi_2(n)}{2}\left(\frac{n}{4\pi T}\right)^{n+2}\Phi_H|\mathbf{k}|^{n}\left(1-\frac{\Phi_H^2}{|\mathbf{k}|^2}\right)^{\frac{n}{2}}$$

similar expressions for p-branes with q-form charge: 3+1 response coefficients

Effective action for elastic expansion of branes

old physical problem: fluids living on surfaces: response to bending
(e.g. biconcave shape of red blood cells: cannot be described by standard soap bubble action, with minimal surface)

Helfrich-Canham bending energy: add
(K = mean curvature vector)

$$\mathcal{F}[X^\mu] = \alpha \int dA\, K^2$$

Carter/Capovilla,Gueven

in physics: improved effective action for QCD string (Polyakov & Kleinert)

general framework for higher order corrections (stationary brane fluids) Armas

$$I[X^\mu] = \int_{\mathcal{W}_{p+1}} \mathcal{L}(\sqrt{-\gamma}, \gamma_{ab}, \mathbf{k}^a, \nabla_a, K_{ab}{}^i, \omega_a{}^{ij})$$

$$K_{ab}{}^\mu = \nabla_a u_b^\mu$$

$$\omega_a{}^{ij} = -n^j{}_\mu \nabla_a n^{i\mu}$$

EM tensor

$$T^{ab} = \frac{2}{\sqrt{-\gamma}} \frac{\delta \mathcal{L}}{\delta \gamma_{ab}}$$

dipole moment spin current

$$\mathcal{D}^{ab}{}_i = \frac{1}{\sqrt{-\gamma}} \frac{\delta \mathcal{L}}{\delta K_{ab}{}^i} \quad , \quad \mathcal{S}^a{}_{ij} = \frac{1}{\sqrt{-\gamma}} \frac{\delta \mathcal{L}}{\delta \omega_a{}^{ij}}$$

Leading order effective action

$$I[X^\mu] = \int_{\mathcal{W}_{p+1}} \mathcal{L}\left(\sqrt{-\gamma}, \mathbf{k}\right) = \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma \sqrt{-\gamma}\ \lambda_0(\mathbf{k})$$

gives perfect fluid

$$T^{ab} = T^{ab}_{(0)} = \lambda_0(\mathbf{k})\gamma^{ab} - \lambda_0'(\mathbf{k})\mathbf{k}u^a u^b$$

$$P = \lambda_0(\mathbf{k})\ ,\ \epsilon + P = -\lambda_0'(\mathbf{k})\mathbf{k}\ .$$

strain tensor

$$U_{ab} = -\frac{1}{2}\left(\gamma_{ab} - \bar{\gamma}_{ab}\right)$$

$$dU_{ab} = N_\rho K_{ab}{}^\rho$$

Armas,NO

can define elasticity tensor (measuring compression/stretching)

$$E^{abcd} = 2\left(\lambda_0(\mathbf{k})\gamma^{a(c}\gamma^{d)b} - \left(\frac{\partial\lambda_0(\mathbf{k})}{\partial\gamma_{ab}}\right)\gamma^{cd} - 2\left(\frac{\partial^2\lambda_0(\mathbf{k})}{\partial\gamma_{ab}\partial\gamma_{cd}}\right)\right)$$

$$dT^{ab} = E^{abcd}dU_{cd}$$

extrinsic dynamics in transverse directions to the surface correspond to that of elastic brane

Second order corrections

Armas

first order: $\mathbf{k}^a \nabla_a \mathbf{k}\ ,\ \nabla_a \mathbf{k}^a$. are zero

in agreement with analysis of stationary & non-dissipative fluids (expansion and shear vanish)

2nd order elastic

$$\lambda_1(\mathbf{k}) K^i K_i\ ,\ \ \lambda_2(\mathbf{k}) K^{abi} K_{abi}\ ,\ \ \lambda_3(\mathbf{k}) \mathbf{k}^a \mathbf{k}^b K_{ac}{}^i K^c{}_{bi}\ ,$$

$$\lambda_4(\mathbf{k}) \mathbf{k}^a \mathbf{k}^b K_{ab}{}^i K_i\ ,\ \ \lambda_5(\mathbf{k}) \mathbf{k}^a \mathbf{k}^b \mathbf{k}^c \mathbf{k}^d K_{ab}{}^i K_{cdi}\ .$$

2nd order spin

$$\varpi_1(\mathbf{k}) \omega^a{}_{ij} \omega_a{}^{ij}\ ,\ \ \varpi_2(\mathbf{k}) \mathbf{k}^a \mathbf{k}^b \omega_{aij} \omega_b{}^{ij}$$

2nd order hydrodynamic

$$\upsilon_1(\mathbf{k}) \nabla_a \nabla^a \mathbf{k}\ ,\ \ \upsilon_2(\mathbf{k}) \mathcal{R}\ ,\ \ \upsilon_3(\mathbf{k}) \mathbf{k}^a \mathbf{k}^b \mathcal{R}_{ab}\ ,$$

$$\upsilon_4(\mathbf{k}) \nabla_{[a} \mathbf{k}_{b]} \nabla^{[a} \mathbf{k}^{b]}\ ,\ \ \upsilon_5(\mathbf{k}) \nabla_a \mathbf{k} \nabla^a \mathbf{k}\ ,\ \ \upsilon_6(\mathbf{k}) R^a{}_{ba}{}^b\ ,\ \ \upsilon_7(\mathbf{k}) \mathbf{k}^a \mathbf{k}^b R^c{}_{acb}$$

coupling between elastic and hydrodynamic modes

using field redefinitions, ibp and other props:
one finds for codimension higher than 1 branes

- 3 elastic response coefficents
- 5 hydrodynamic response coefficients
- 1 spin response coefficient

but: coupling between elastic and hydro due to geometric constraints

Gauss-Codazzi $$R_{abcd} = \mathcal{R}_{abcd} - K_{ac}{}^{i} K_{bdi} + K_{ad}{}^{i} K_{bci}$$

new terms compared to stationary and non-dissipative space-filling fluids

cf. Banerjee,Bhattacharya,Bhattarcharyya,Jain,Minwalla et al
Jensen,Kaminski,Kovtun,Meyer,Ritz et al
Bhattacharya,Bhattacharyya,Rangamani

Young modulus from effective action

using 2nd order elastic corrections one finds from the action

$$\mathcal{D}^{abi} = \mathcal{Y}^{abcd} K_{cd}{}^{i}$$

with

$$\mathcal{Y}^{abcd} = 2\left(\lambda_1 \gamma^{ab}\gamma^{cd} + \lambda_2 \gamma^{a(c}\gamma^{d)b} + \lambda_3 \mathbf{k}^{(a}\gamma^{b)(c}\mathbf{k}^{d)} + \frac{\lambda_4}{2}\left(\gamma^{ab}\mathbf{k}^c\mathbf{k}^d + \gamma^{cd}\mathbf{k}^a\mathbf{k}^b\right) + \lambda_5 \mathbf{k}^a\mathbf{k}^b\mathbf{k}^c\mathbf{k}^d\right)$$

black branes in gravity are a particular case of this !

Intrinsic transport coefficients: viscosities

Camps,Emparan,Haddad

- can study in derivative expansion the intrinsic perturbations for flat branes

  - to first order in derivative expansion

$$T^{ab} = \left(\epsilon u^a u^b + P P^{ab} - 2\eta\sigma_{ab} - \zeta\vartheta P^{ab}\right)\delta^{n+2}(x^\rho - X^\rho)$$

higher derivative corrections to monopole part of stress tensor

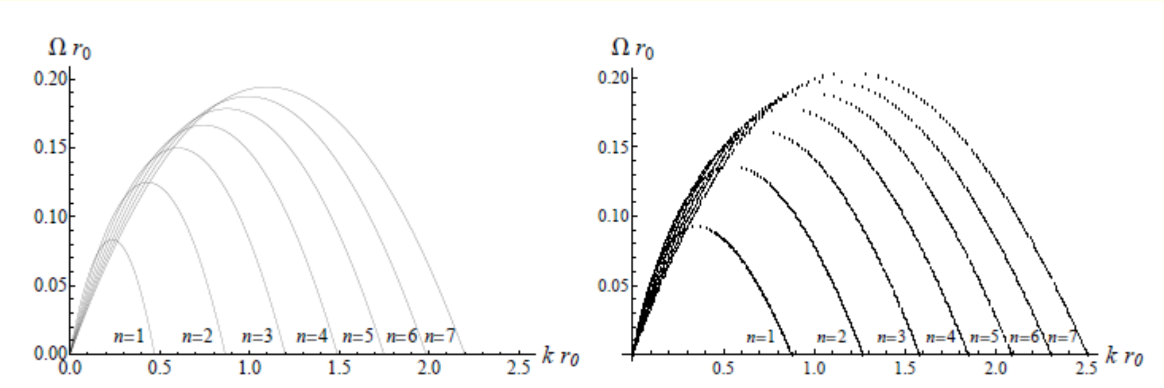
$$\eta = \frac{s}{4\pi}\ , \quad \zeta = 2\eta\left(\frac{1}{p} - c_s^2\right)$$

shear and bulk viscosity

can apply e.g. to GL instability: include viscous damping of the soundmode perturbations in dispersion relation

$$\Omega = \frac{k}{n+1}\left(1 - \frac{n+1}{n\sqrt{n+1}}k r_0\right)$$

fits very well with GL curve



recnetly: 2nd order transport via AdS/Ricci flat map

Caldarelli,Camps,Gouteraux,Skenderis

Viscious corrections to charged black branes

Gath,Pedersen

charged branes:

simplest case is Reisner-Nordstrom branes (solution of higher-D EM theory)

extra contribution to charge current: $$\Upsilon^a_{(1)} = -\mathfrak{D}\left(\frac{\mathcal{Q}\mathcal{T}}{w}\right)^2 \Delta^{ab}\partial_b\left(\frac{\Phi}{\mathcal{T}}\right)$$

results:

charge diffusion constant

$$\eta = \frac{\Omega_{(n+1)}}{16\pi G} r_0^{n+1}(1+\gamma_0)^{\frac{N}{2}}, \quad \frac{\zeta}{\eta} = \frac{2}{p}\frac{(n+p+1)(n+1)}{(n+1+pB\gamma_0)^2}$$

$$\frac{\mathfrak{D}}{\eta} = \frac{1}{N\gamma_0\mathcal{T}} = \frac{s}{\Phi\mathcal{Q}}$$

- shear viscosity/entropy satisfies usual bound, bulk viscosity provides counterexamples to various (proposed) bounds in certain regimes of charge

analyzed effect on first order dispersion relation for effective fluid
-> results are in agreement with thermo for smeared black D0-branes

Harmark,Niiachros,NO

interesting relations with AdS/fluid-gravity (using AdS/Ricci flat map)

Effective hydrodynamics of black D3-branes

Emparan,Hubeny,Rangamani,

relation between various fluid descriptions encountered:

Membrane paradigm $\subset$ Fluid/gravity correspondence $\subset$ Blackfolds.

study effective hydrodynamcis on non-extremal black D3-brane
(geometry includes three regions, Rindler, AdS throat, flat Minkowski)

enclose D3-brane in a "box"; Dirichlet boundary conditions + allow long wavelength fluctuations along D3-brane worldvolume

-> constitutive relations for the effective theory and transport coeffs can be read off
(charge is fixed, only energy density fluctuates)
gives neutral relativistic fluid capturing low wave length modes on surface outside horizon

two non-trivial sales (horizon & cutoff scale, in terms of charge radius scale)
-> can interpolate between the different descriptions

Relevance of BF method

EHONR/EHON/ Caldarelli,Emparan,Rodriguez
Armas,NO/Camps,Emparan,Giusto,Saxena/..

- new stationary BH solutions:

approximate analytic construction of BH metrics in higher D gravity/ supergravities (cf. String Theory)
- possible horizon topologies, thermodynamics, phase structure, …
- new non-extremal and extremal BH solutions
- useful for insights/checks on exact analytic/numeric solutions

Camps,Emparan,Haddad Armas,Camps,Harmark,NO
Camps,Emparan/Armas,Gath,NO/Armas,NO

- BH instabilities and response coefficients:

understand GL instabilities in long wavelength regime, dispersion relation, elastic (in) stabilities, new long wavelength response coefficients for BHs, Young modulus (hydro + material science)

- Thermal probe branes/strings:

new method to probe finite T backgrounds with probes that are in thermal equilibrium with the background (e.g. hot flat space, BHs)

Grignani,Harmark,Marini,NO,Orsell
Armas,Harmark,NO,Orselli,Vigand Pedersen
Armas,NO,Vigand Pedersen/Niacos,SIampos

- AdS/CFT:

many potential applications
(new black objects in AdS, connection with fluid/gravity, thermal probes thermal giant gravitons, BHs on branes, …)

+ interrelations between the four items above

Outlook

• systematic effective actions for elastic/hydrodynamic properties of charged fluid branes
 - obtained useful inputs/insights from gravity

Cf. development of fluids/superfluids inspired by gravity and holography

- elastic corrections for D3-branes and AdS/CFT !
- responses for spinning charged branes
- response coefficients in other backgrounds with non-zero fluxes (susceptibility, polarizability)
- Chern-Simons couplings
- multi-charge bound states
- entropy current
- effective hydrodynamics of spinning D3-branes (Dp,M)
- further explore AdS/Ricci flat connection

The end