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Emergent Spacetimes from Entanglement Renormalization

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Mainly Based on ArXiv:1208.3469
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① Introduction

Superstring theory as quantum gravity

⇒ ∃ Many evidences e.g.microscopic derivation of BH entropy.

However, we still do not have any good controls of quantum gravity in non-trivial spacetimes such as time-dependent and highly curved ones, which are very important in e.g. cosmology.

⇒ We need to understand the non-perturbative dynamics of string theory.

Many of string theorists have noticed that the holographic principle can be a very important clue for this purpose.

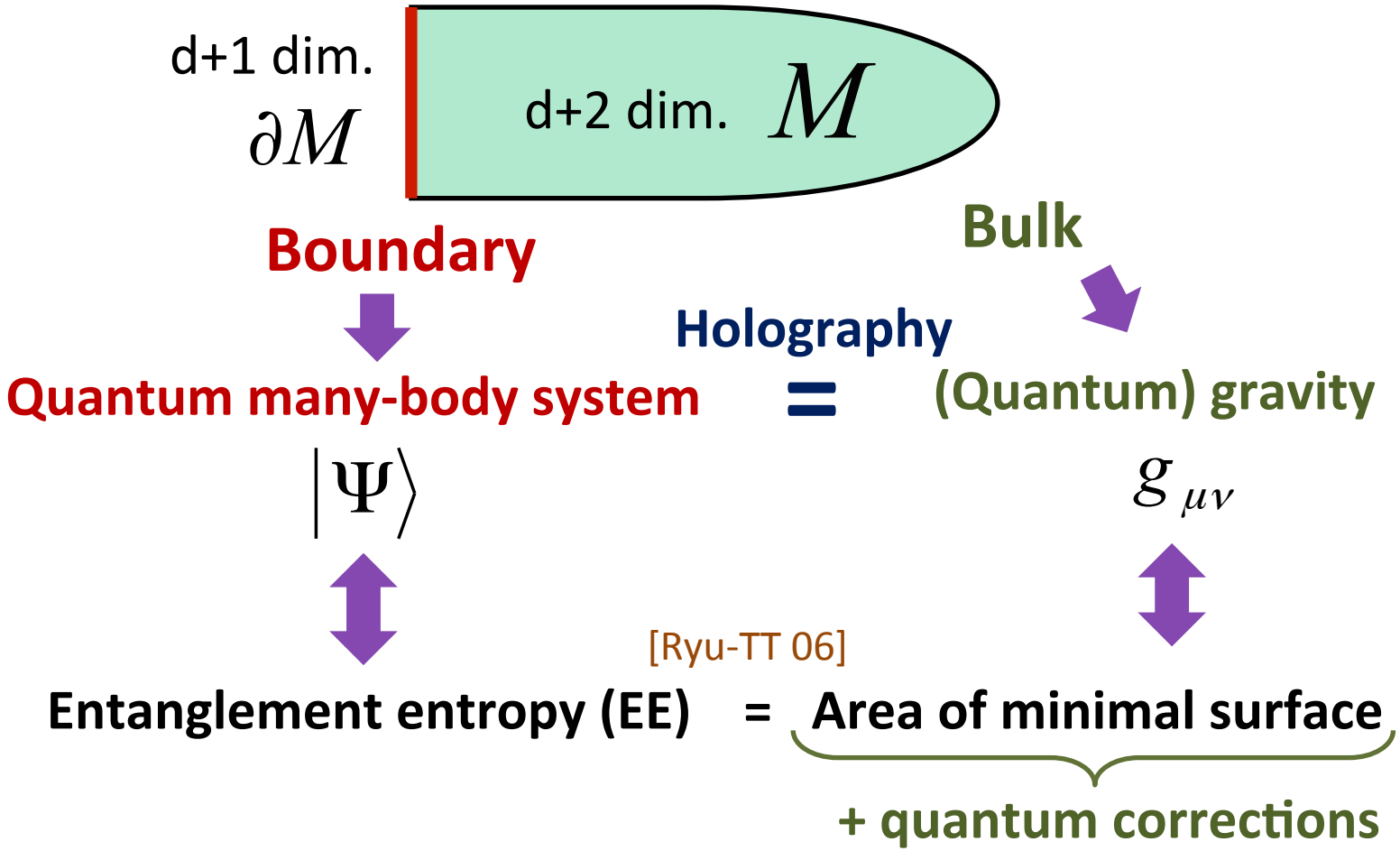
Ex. AdS/CFT [Maldacene 97] $\Rightarrow$ a non-perturbative formulation of quantum gravity on AdS in terms of QFTs.

However, we do not understand the mechanism of holographic principle so deeply as to apply the holography to other different backgrounds such as the flat space or de-Sitter space.

$\Rightarrow$ **We need to understand how the holography works.**

(This is the main motivation of this talk.)

Claim: The *quantum entanglement* can be a key concept to understand the holography.

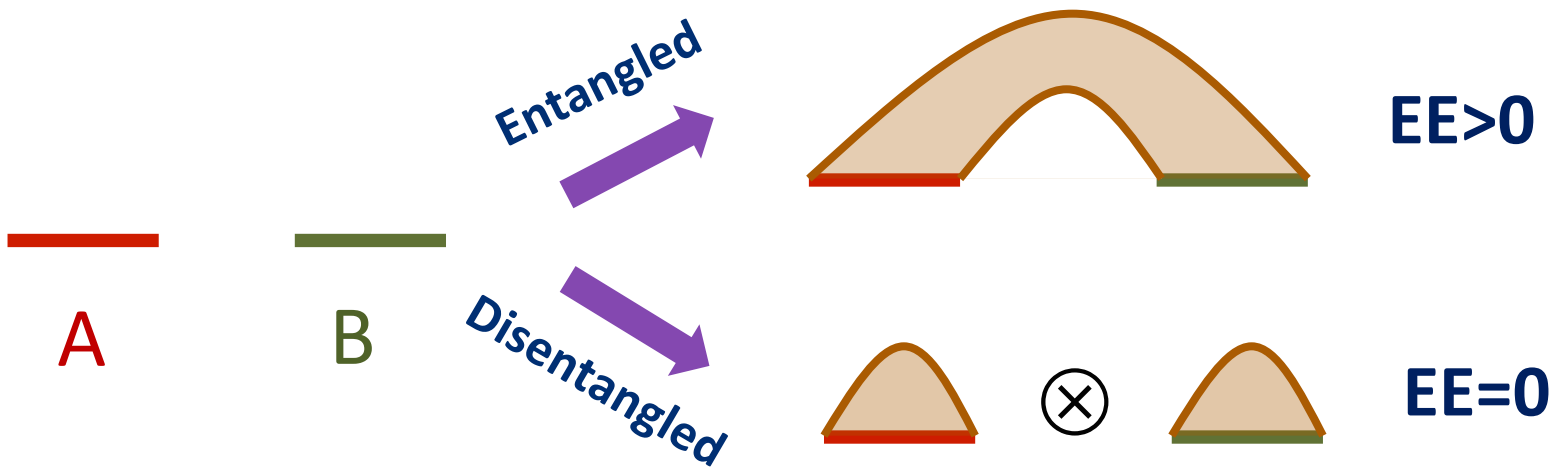


Advantages of EE

- EE is defined for *any quantum many-body systems*. ⇒Universal
  (In cond-mat, EE = a quantum order parameter)

- In the presence of quantum corrections, the metric may not be a good description of the spacetime. But, the EE is robust.

- EE can *capture spacetime topologies*. For example,



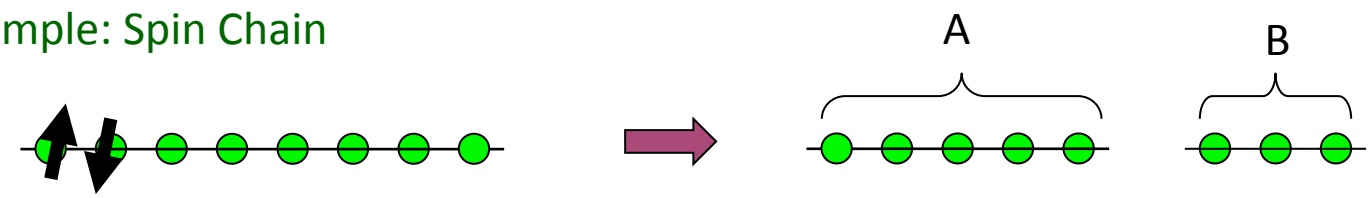
② Holographic Entanglement Entropy

(2-1) Definition of Entanglement Entropy

Divide a quantum system into two subsystems A and B.

$$H_{tot} = H_A \otimes H_B \quad .$$

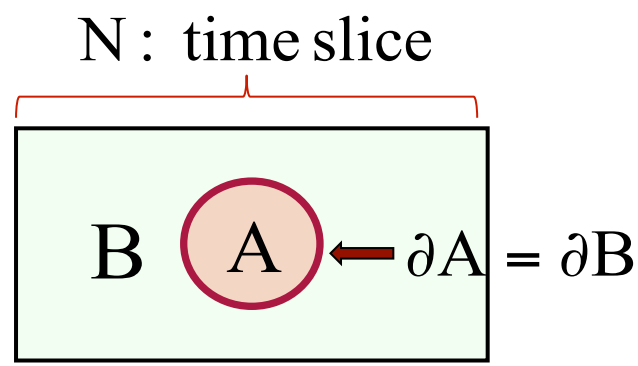
Example: Spin Chain



Define the reduced density matrix ρ_A for A by $\rho_A = \mathrm{Tr}_B \rho_{tot}$, taking trace over the Hilbert space of B .

Now the entanglement entropy S_A is defined by the von-Neumann entropy:

$$S_A = -\mathrm{Tr}_A \, \rho_A \log \rho_A \, .$$



(2-2) Area law [Bombelli-Koul-Lee-Sorkin 86, Srednicki 93]

EE in QFTs includes UV divergences.

Area Law

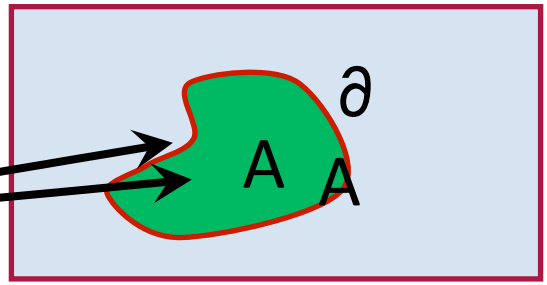
In a d+1 dim. QFT (d>1) with a UV fixed point, the leading term of EE at its ground state behaves like

$$S_A \sim \frac{\mathrm{Area}(\partial \mathrm{A})}{a^{d-1}} + (\text{subleading terms}),$$

where a is a UV cutoff (i.e. lattice spacing). [d=1: log div.]

Intuitively, this property is understood like:

Most strongly entangled



(2-3) Holographic Entanglement Entropy (HEE)

[Ryu-TT 06]

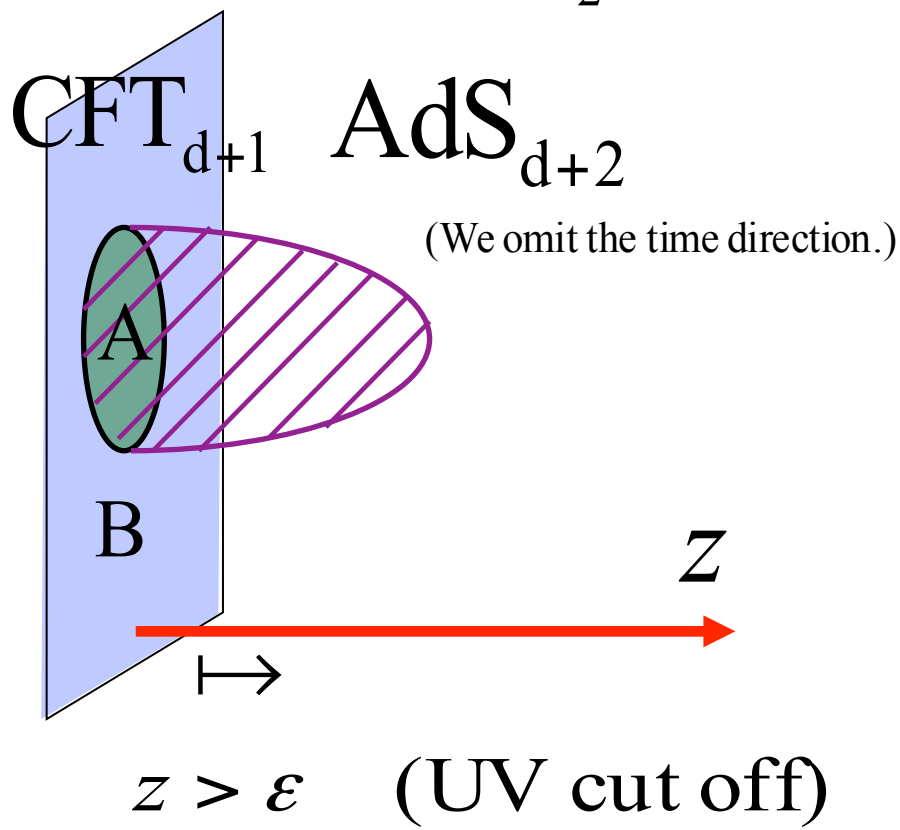
$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N}$$

$$ds^2_{AdS} = R^2_{AdS} \frac{-dt^2 + \sum_{i=1}^{d} dx_i^2 + dz^2}{z^2}$$

γ_A is the minimal area surface (codim.=2) such that

$$\partial A = \partial \gamma_A \text{ and } A \sim \gamma_A \ .$$

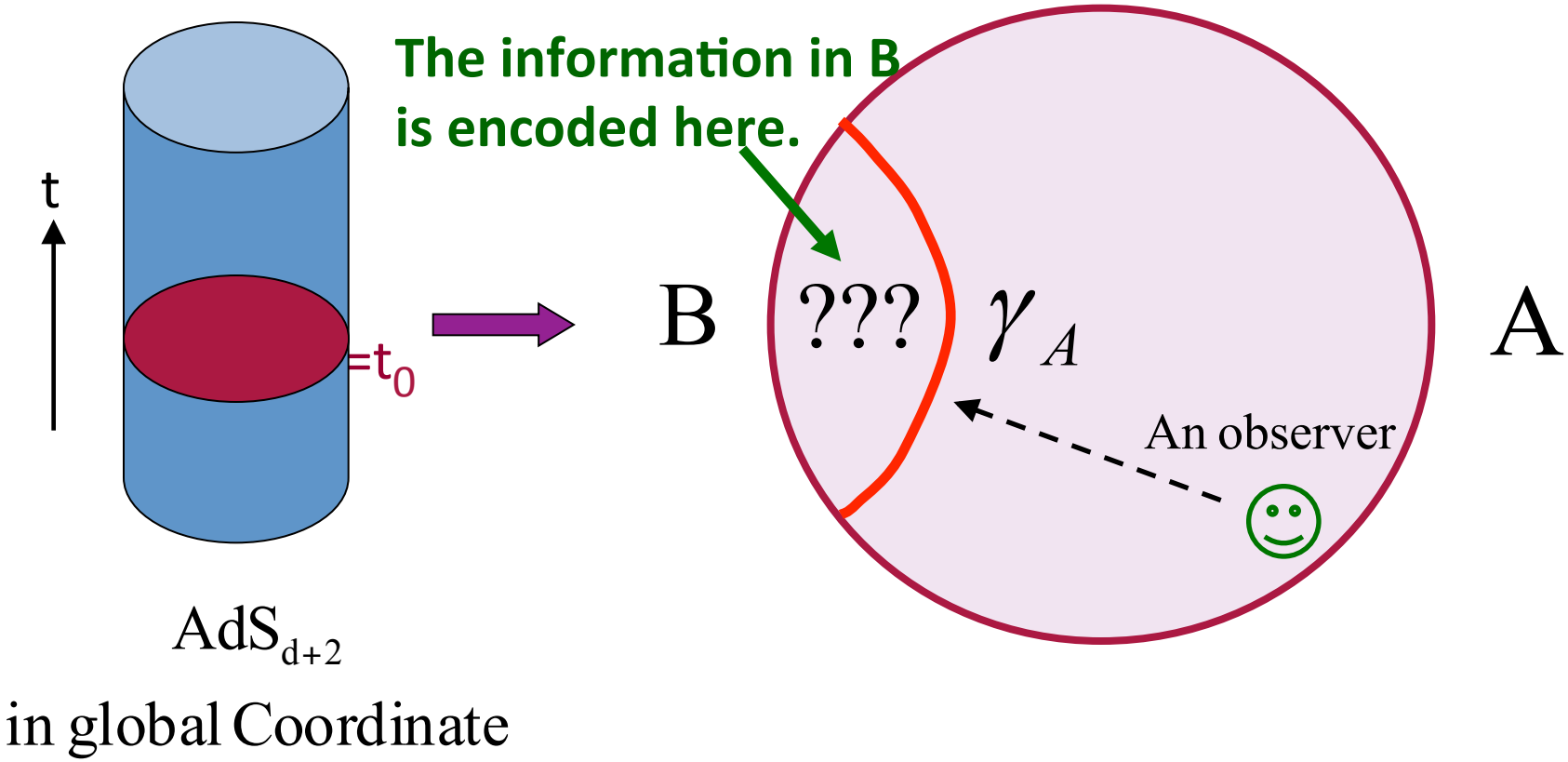
homologous



Note: In time-dependent b.g., we need to employ the covariant version [Hubeny-Rangamani-TT 07].

An Intuitive Interpretation of HEE

Here we employ the global coordinate of AdS space and take its time slice at $t=t_0$.



- In spite of a heuristic argument [Fursaev, 06] , there has been no complete proof. But, so many evidences and no counter examples.

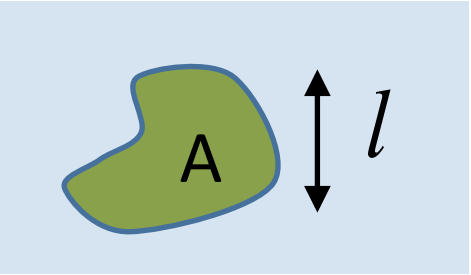
[A Partial List of Evidences]

- Area law [Ryu-TT 06]
- Analytical Agreements in AdS3/CFT2 [Ryu-TT 06]
- Strong subadditivity [Headrick-TT 07]
- Disconnected subsystems [Headrick 10, (CFT: Calabrese-Cardy-Tonni 09)]
- Agreements on the coefficients of log term in 4d CFT (~a+c)

  [Ryu-TT 06, Solodukhin 08,10, Lohmayer-Neuberger-Schwimmer-Theisen 09, Dowker 10, Casini-Huerta, 10, Myers-Sinha 10]
- A direct proof when A = round ball [Casini-Hueta-Myers 11]
- Cadney-Linden-Winter inequality (and Monogamy)

  [Hayden-Headrick-Maloney 11]
- Agreements with localization calculations [Jafferis-Rafy 12]

(2-4) General Behavior of HEE [Ryu-TT 06]



$$S_A = \frac{\pi^{d/2} R^d}{2G_N^{(d+2)}\Gamma(d/2)} \left[p_1 \left(\frac{l}{a}\right)^{d-1} + p_3 \left(\frac{l}{a}\right)^{d-3} + \cdots \right.$$

$$\left. \cdots + \begin{cases} p_{d-1}\left(\frac{l}{a}\right) + p_d & (\text{if } d = \text{even}) \\ p_{d-2}\left(\frac{l}{a}\right)^2 + q \log\left(\frac{l}{a}\right) & (\text{if } d = \text{odd}) \end{cases} \right],$$

Area law div.

Central charges

[Ryu-TT 06, Solodukhin 08,10, Lohmayer-Neuberger-Schwimmer-Theisen 09, Dowker 10, Casini-Huerta, 10, Myers-Sinha 10, Casini-Hueta-Myers 11]

where $p_1 = (d-1)^{-1}, p_3 = -(d-2)/[2(d-3)], \ldots.$

$$\ldots\ldots \quad q = (-1)^{(d-1)/2}(d-2)!!/(d-1)!! \quad .$$

A universal quantity which characterizes odd dim. CFT. ⇒F-theorem in 3 dim. CFTs (Renormalized EE)

[Liu-Mezei 12, Myers-Singh 12]

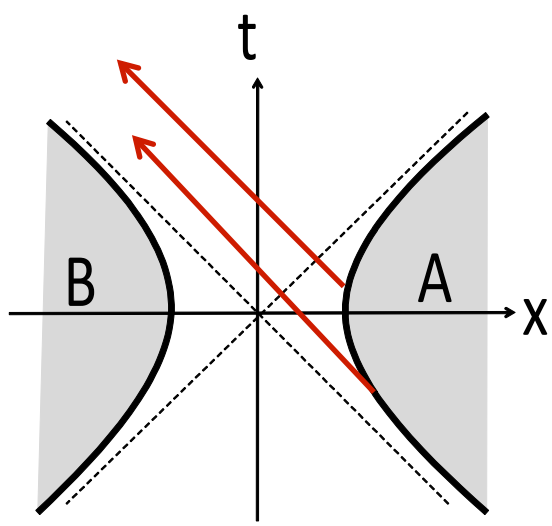
[Jafferis-Klebanov-Pufu-Safdi 11
Proof: Casini-Huerta 12
See also Klebanov-Nishioka-Pufu-Safdi 12, Closset-Dumitrescu-Festuccia-Komargodski-Seiberg 12]

(2-5) Construction of an entangled pair in AdS/BCFT

[Fujita-Tonni-TT 11]

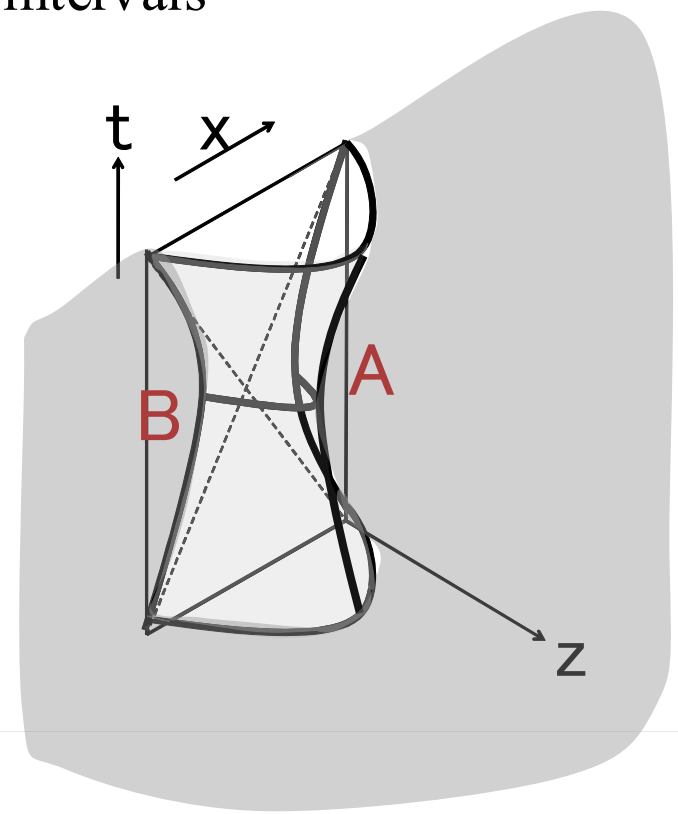
$$\left\{(t,x,z)\in AdS_3 \;\middle|\; -t^2+x^2+\left(z-r_D\sinh\frac{\rho_*}{R}\right)^2 \geq \left(r_D\cosh\frac{\rho_*}{R}\right)^2\right\}$$

$\Leftrightarrow$ A pair of CFTs on semi-infinite intervals



Two BCFTs (z=0)

A and B are causally disconnected :



AdS/BCFT Proposal [TT 11]

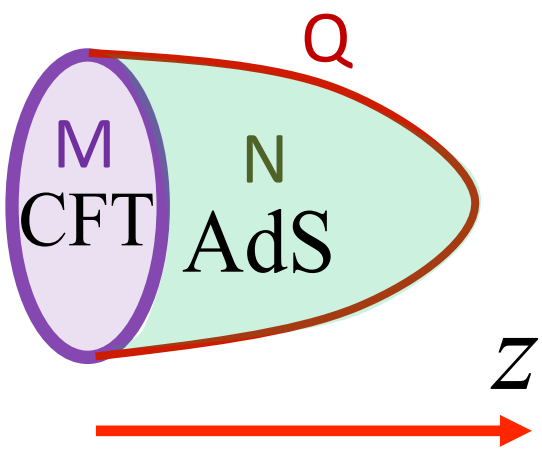
In addition to the standard AdS boundary M, we include an extra boundary Q, such that ∂ Q= ∂ M.

$$I_E = -\frac{1}{16\pi G_N}\int_N \sqrt{g}(R-2\Lambda-L_{matter}) - \frac{1}{8\pi G_N}\int_Q \sqrt{h}(K-L^Q_{matter}).$$

EOM at boundary leads to the Neumann b.c. on Q :

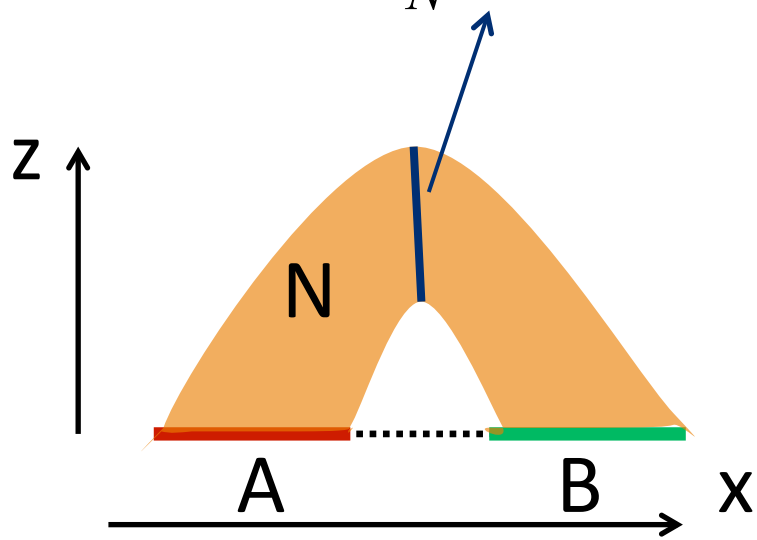
$$K_{ab} - Kh_{ab} = 8\pi G_N T^Q_{ab} \quad .$$

Conformal inv. $\Rightarrow T^Q_{ab} = -Th_{ab}$.



In the BCFT side, these two BCFTs are entangled with each other. The entanglement entropy between them is calculated as

$$S_A = \frac{R}{4G_N}\int_{r_D e^{\rho^*/R}}^{z_{IR}} \frac{dz}{z}.$$



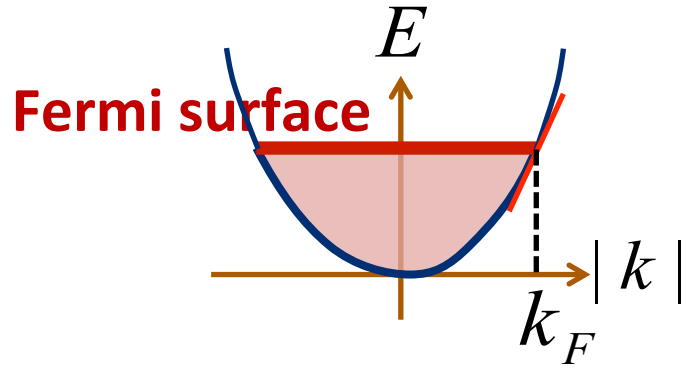
Note: The g-theorem prohibits the entanglement between regions which are causally connected. (No wormholes !)

③ Application of HEE to Condensed Matter

(3-1) EE and Fermi Surfaces

In d+1 dimension, if there exist Fermi surfaces, EE violates the area law logarithmically: [Wolf 05, Gioev-Klich 05]

$$S_A \sim \left(\frac{l}{a}\right)^{d-1} \log \frac{l}{a}.$$



Note1: as typical in **cond-mat**, this argument assumes $a^{-1} \sim k_F$.

Note2: The low-energy excitations around a Fermi surface are approximated by copies of 2d CFTs.

[Interacting case (Non Fermi liquids): Zhang-Grover-Vishwanath 11]

Instead, from **QFT viewpoints** ⇒ $a^{-1} >> k_F$

⇒ We expect

$$S_A = (div.) + \eta \cdot (l \cdot k_F)^{d-1} \log(l \cdot k_F) + \cdots.$$

Below we would like to see if we can realize this behavior in HEE.

We assume that all physical quantities can be calculable in the classical gravity limit (i.e. $\exists O(N^2)$ Fermi surfaces).

[cf. This assumption does not include the probe fermion system: Faulkner-Liu-McGreevy-Vegh 09, Cubrovic-Zaanen-Schalm 09]

(3-2) Holographic Analysis

We assume the 4D background metric:

$$ds^2 = \frac{R_{AdS}^2}{z^2}\left(-f(z)dt^2 + g(z)dz^2 + dx^2 + dy^2\right).$$

(i) We require the **logarithmic behavior** of HEE.

(ii) A sensible gravity theory should satisfy the **null energy condition**: $T_{\mu\nu}N^\mu N^\nu \geq 0$ for any null vector N^μ.

$$\Rightarrow \quad g(z) \propto z^2, \quad f(z) \propto z^{-2m} \quad (m \geq 1).$$

By heating up the system (=creating a black hole), we obtain the specific heat of the 2+1 dim. Fermi surface in the low temp. limit:

$$\boxed{C \propto T^{\alpha} \quad \text{with} \quad \alpha \leq \frac{2}{3}.}$$

[Ogawa-Ugajin-TT 11]

➡ **No Landau Fermi liquids (α=1) are allowed !**
Instead, they are non-Fermi liquids.

Note: this result is **universal** in the same sense as $\frac{\eta}{s} = \frac{1}{4\pi}$.

Our result does not change even if we take into account higher derivatives. [See also Kulaxizi-Parnachev-Schalm 12]

Comments: Interesting coincidences ?

AdS: No curvature singularity in the gravity dual ⇒ α=2/3 [Shaghoulian 11]

CMT: *Spin fluctuations:*

[Moriya, Hertz, Millis …. 70'-90']

N Fermions + U(1) gauge:

⇒ α=2/3 (i.e. z=3)

[Lee 09, Metlitski, and S. Sachdev 10, Mross-McFreevy-Liu-Senthil 10, Lawler-Barci-Fernandez-Fradkin-Oxman 06]

Experiment: $YbRh_2(Si_{1-x}Ge_x)_2$

⇒ α=2/3

Examples of heavy fermions

[Pepin 11]

Compound	$H_c/P_c/x_c$	$\frac{C_\varphi}{T} \to \infty$?	$\rho \sim T^\alpha$
$YbRh_2(Si_{1-x}Ge_x)_2$	$x_c = 0.05$ $H_c^{\parallel c} = 0.66T$ $H_c^{\perp c} = 0.06T$	$T^{-0.34}$	T
$CeCoIn_5$	$H_c = 5T$	$T^{-\alpha}$	T
$Ce(Cu_{1-x}Au_x)_6$	$x_c = 0.016$	$Log\left(\frac{T_\varphi}{T}\right)$	T
$CeCu_{6-x}Ag_x$	$x_c = 0.2$	$Log\left(\frac{T_\varphi}{T}\right)$	$T^{1.1}$
$CeNi_2Ge_2$	$P_c = 0$	$Log\left(\frac{T_\varphi}{T}\right)$	$T^{1.4}$
U_2Pt_2In	$P_c = 0$	$Log\left(\frac{T_\varphi}{T}\right)$	T
$CeCu_2Si_2$	$Pc = 0$	$Log\left(\frac{T_\varphi}{T}\right)$	$T^{1.5}$
$Ce(Ni_{1-x}Pd_x)Ge_2$	$x = 0.065$	$\gamma_0 - T^{1/2}$	$\rho_0 + T^{3/2}$
$YbAgGe$	$H = 4T$	$Log\left(\frac{T_\varphi}{T}\right)$	T
$CeIn_{3-x}Sn_x$	$p_c = 26kbar$	?	$T^{1.6}$
U_2Pd_2In	$P_c < 0$	?	T
$CePd_2Si_2$	$P_c > 0$	?	$T^{1.2}$
$CeRhIn_5$	$P_c \sim 1.6GPa$	?	T
$CeIn_3$	$P_c > 0$	?	$T^{1.5}$
$Ce_{1-x}La_xRu_2Si_2$	$x_c = 0.1$	no	?
$U_3Ni_3Sn_4$	$P_c > 0$	no	?

This background is understood as a **violation of hyperscaling**

⇒ A generalization of Lifshitz spacetime

[Gouteraux-Kiritsis 11 , Huijse-Sachdev-Swingle 11 (hidden Fermi surface), Dong-Harrison-Kachru-Torroba-Wang 12]

$$ds^2_{(d+2)} = r^{-(d-\theta)}\left(-r^{-2(z-1)}dt^2 + dr^2 + \sum_{i=1}^{d} dx_i^2\right) \quad .$$

$$\Rightarrow C \propto S \propto T^{(d-\theta)/z}. \quad (\theta = \text{hyperscaling violation})$$

These backgrounds are solutions of Einstein-Scalar-Maxwell theory:

$$S_{EMS} = \frac{1}{16G_N}\int dx^{d+2}\sqrt{-g}\left[R - 2\Lambda - W(\phi)F_{\mu\nu}F^{\mu\nu} - \partial_\mu\phi\partial^\mu\phi - V(\phi)\right].$$

[Solutions: Gubser-Rocha 09, Charmousis-Goutéraux-Kim-Kiritsis-Meyer 10, Goldstein-Iizuka-Kachru-Prakash-Trivedi-Westphal 10,….
String embeddings of θ=d-1 (log EE) solutions: Dey-Roy, Singh, Narayan 12]

④ Emergent Metric from Quantum Entanglement

(4-1) Basic Outline

In principle, we can obtain a metric from a CFT as follows:

a CFT state ⇒ Information (~EE) = Minimal Areas ⇒ metric

$$|\Psi\rangle \qquad S_A \qquad \mathrm{Area}(\gamma_A) \qquad g_{\mu\nu}$$

One candidate of such frameworks is so called the entanglement renormalization (MERA) [Vidal 05 (for a review see 0912.1651)] as pointed out by [Swingle 09]. [cf. Emergent gravity: Raamsdonk 09, Lee 09]

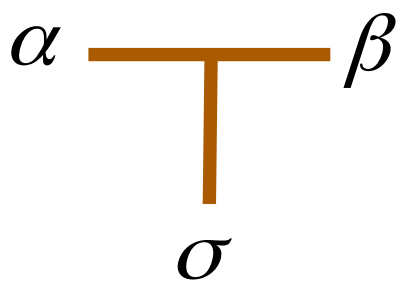
(4-2) Tensor Network (TN)

[See e.g. the review Cirac-Verstraete 09]

Recently, there have been remarkable progresses in numerical algorithms for quantum lattice models, based on so called *tensor product states*.

This leads to various nice variational ansatzs for the ground state wave functions in various quantum many-body systems.

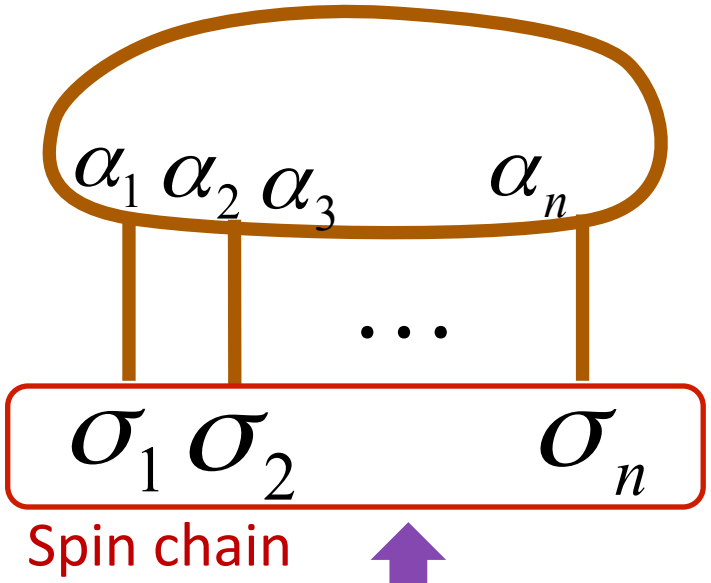
⇒ An ansatz is good if it respects the quantum entanglement of the true ground state.



$$\Leftrightarrow \quad M_{\alpha\beta}(\sigma)$$

Ex. Matrix Product State (MPS)

[DMRG: White 92,…, Rommer-Ostlund 95,..]



$$\alpha_i = 1, 2, \ldots, \chi,$$

$$\sigma_i = \uparrow \text{ or } \downarrow .$$

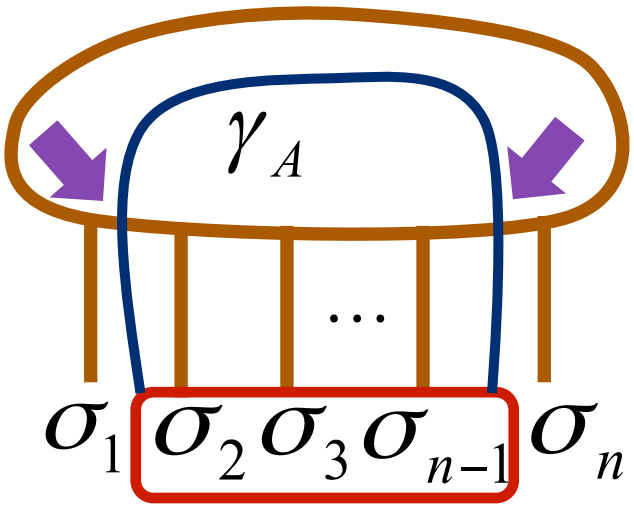
$$\updownarrow$$

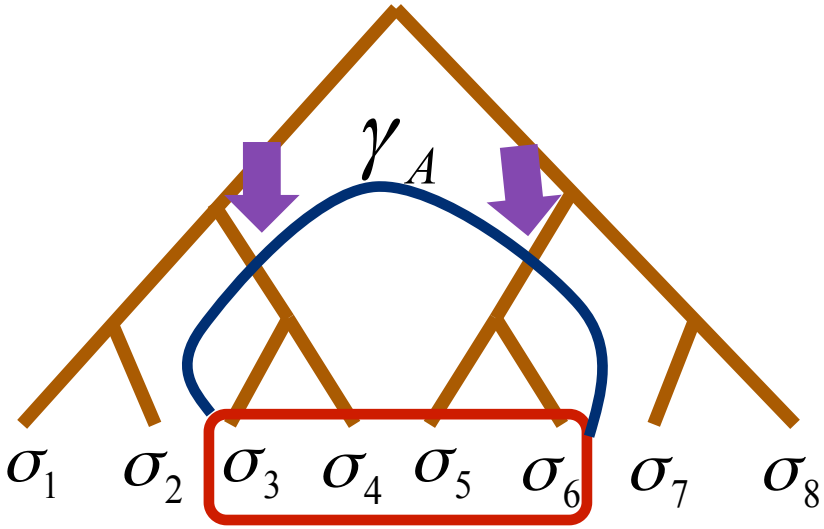
$$|\Psi\rangle = \sum_{\sigma_1,\sigma_2,\cdots,\sigma_n} \mathrm{Tr}[M(\sigma_1)M(\sigma_2)\cdots M(\sigma_n)]\,|\sigma_1,\sigma_2,\cdots,\sigma_n\rangle$$

n *Spins*

MPS and TTN are not good near quantum critical points (CFTs) because their entanglement entropies are too small:

$$S_A \le 2\log\chi \quad (<< \log L \sim S_A^{CFT}).$$





In general,

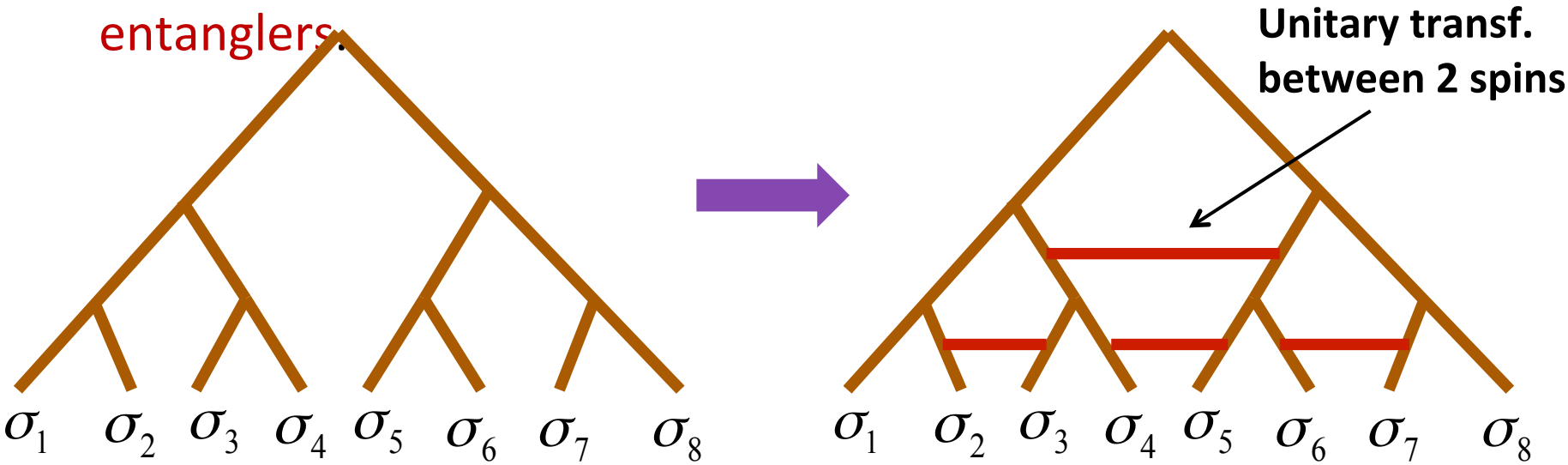
$$\boxed{\begin{aligned} &S_A \sim N_{\text{int}} \cdot \log\chi, \\ &N_{\text{int}} \equiv \min[\#\,\text{Intersections of } \gamma_A]. \end{aligned}}$$

(4-3) AdS/CFT and (c)MERA

<u>MERA</u> (Multiscale Entanglement Renormalization Ansatz):

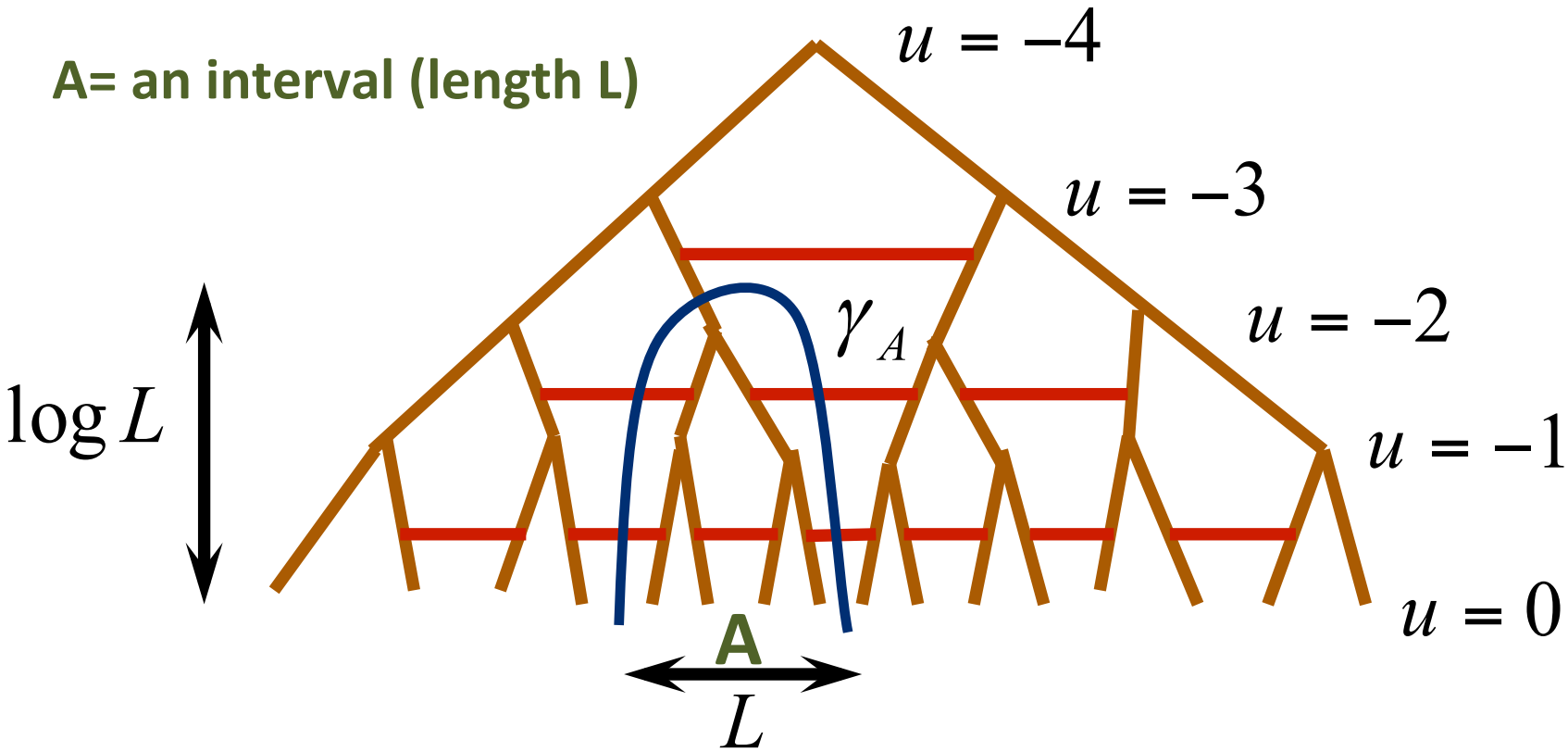
An efficient variational ansatz to find CFT ground states have been developed recently. [Vidal 05 (for a review see 0912.1651)].

To respect its large entanglement in a CFT, we add (dis) entanglers.



Calculations of EE in 1+1 dim. MERA

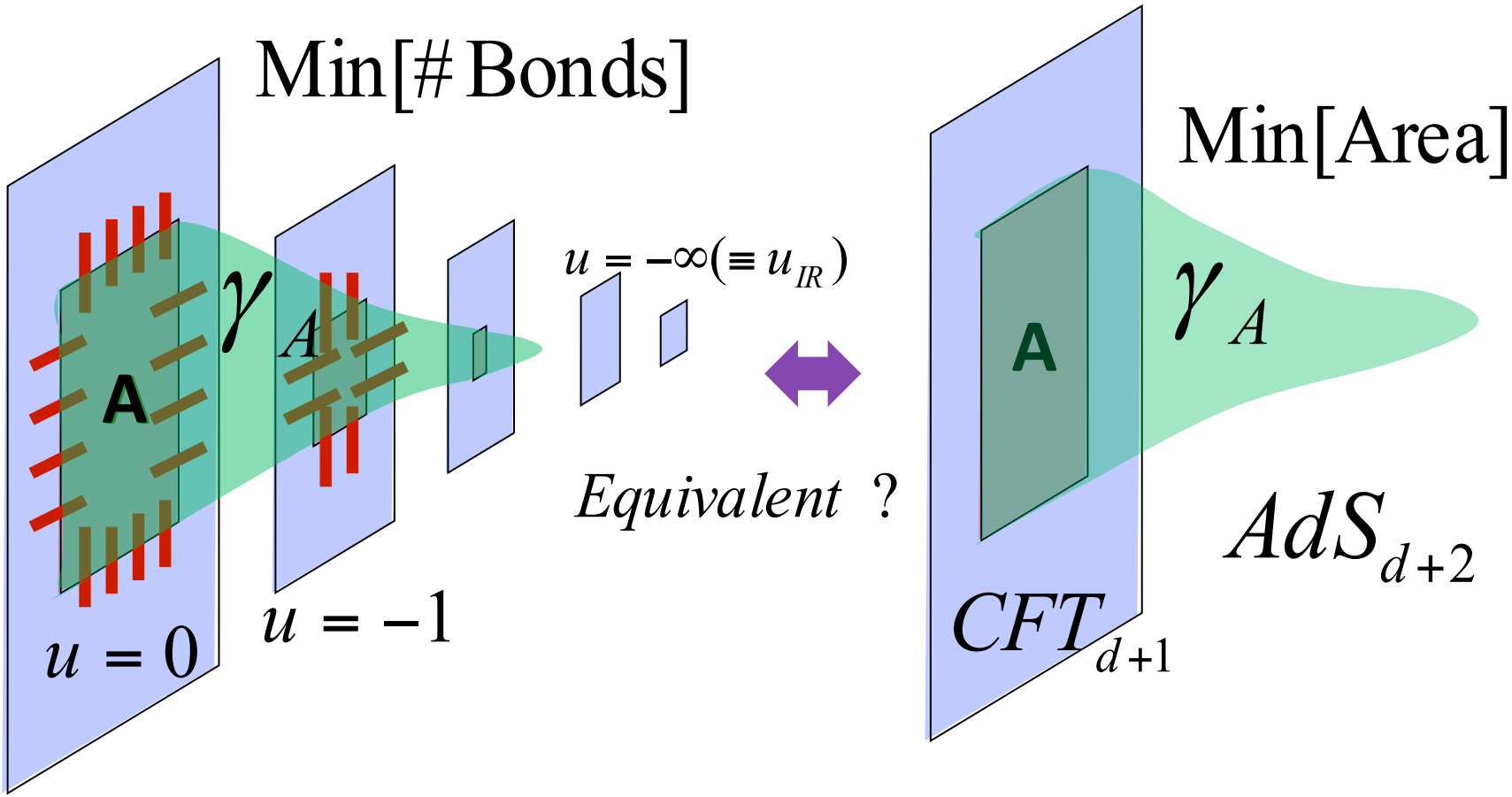
A= an interval (length L)



$$S_A \propto \mathrm{Min}[\#\mathrm{Bonds}] \propto \log L$$

$\Rightarrow$ agrees with 2d CFTs.

A conjectued relation to AdS/CFT [Swingle 09]



$$\text{Metric} = ds^2 + \frac{e^{2u}}{\varepsilon^2}(-dt^2 + d\vec{x}^2) = \frac{dz^2 - dt^2 + d\vec{x}^2}{z^2},$$

$$\textit{where} \quad z = \varepsilon \cdot e^{-u} \quad .$$

Now, to make the connection to AdS/CFT clearer, we would like to consider the MERA for quantum field theories.

Continuous MERA (cMERA)

[Haegeman-Osborne-Verschelde-Verstraete 11]

$$\underbrace{|\Psi(u)\rangle}_{\substack{\text{True ground state}\\ \text{(highly entangled)}}} = P\cdot\exp\left(-i\int_{u_{IR}}^{u} ds[K(s)+L]\right)\cdot \underbrace{|\Omega\rangle}_{\substack{\text{IR state}\\ \text{(no entanglement)}}} ,$$

$$\Rightarrow \text{Real space renormalization flow : length scale} \sim \varepsilon\cdot e^{-u}.$$

K(u) : disentangler, L: scale transformation

Conjecture

$$d+1 \text{ dim. cMERA} = \text{gravity on AdS}_{d+2} \qquad z = \varepsilon\cdot e^{-u}.$$

(4-4) Emergent Metric from cMERA [Nozaki-Ryu-TT 12]

We focus on gravity duals of translational invariant static states, which are not conformal in general.

We conjecture that the metric in the extra direction is given by using the Bures metric (or Fisher information metric):

$$\boxed{g_{uu} du^2 = N \cdot \left(1 - \left| \left\langle \Psi(u) \,|\, e^{iLdu} \,|\, \Psi(u+du) \right\rangle \right|^2 \right) .}$$

$$N^{-1} \equiv \int dx^d \cdot \int_0^{\Lambda e^u} dk^d =$$ The total volume of phase space at energy scale u.

Bures Metric

The **Bures distance** between two states is defined by

$$D(\psi_1, \psi_2) = 1 - \left|\langle \psi_1 \,|\, \psi_2 \rangle\right|^2 .$$

More generally, for two mixed states ρ1 and ρ2,

$$D(\rho_1, \rho_2) = 1 - \mathrm{Tr}\sqrt{\sqrt{\rho_1}\,\rho_2\,\sqrt{\rho_1}}\,.$$

When the state depends on the parameters {ξi},

the **Bures metric (Fisher information metric)** is defined as

$$D\left[\psi(\xi), \psi(\xi + d\xi)\right] = g_{ij}\, d\xi^i\, d\xi^j .$$

⇒ Reparameterization invariant (in our case: coordinate u)

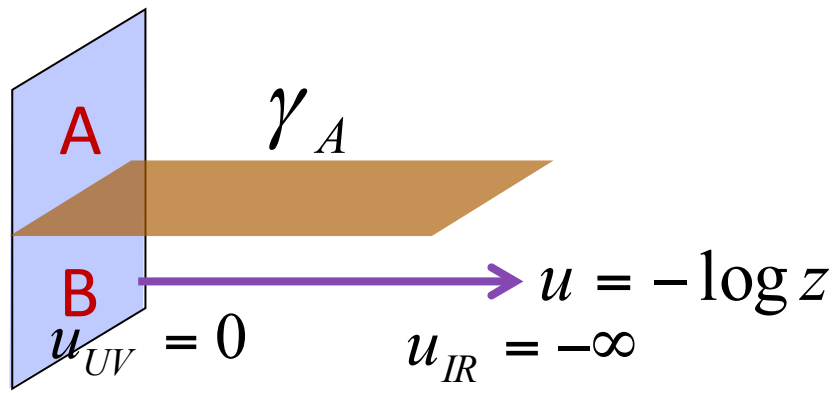
The operation e^{iLdu} removes the coarse-graining procedure to extract the strength of unitary transformations (disentanglers).

⇒ **Our metric = the density of disentanglers**

= the metric guu in the gravity dual

Understandable from the HEE:

$$S_A \sim \int_{u_{IR}}^{0} du \sqrt{g_{uu}} \cdot e^{(d-1)u}$$



(4-5) Emergent Metric in a (d+1) dim. Free Scalar Theory

Hamiltonian: $H = \frac{1}{2}\int dk^d [\pi(k)\pi(-k) + (k^2 + m^2)\phi(k)\phi(-k)].$

Ground state $|\Psi\rangle$: $a_k|\Psi\rangle = 0.$

Moreover, we introduce the `IR state' $|\Omega\rangle$ which has no real space entanglement.

$$a_x|\Omega\rangle = 0, \qquad a_x = \sqrt{M}\phi(x) + \frac{i}{\sqrt{M}}\pi(x),$$

i.e. $|\Omega\rangle = \prod_x |0\rangle_x$ $\qquad a_x^+ = \sqrt{M}\phi(x) - \frac{i}{\sqrt{M}}\pi(x).$

$$\Rightarrow \quad S_A = 0.$$

For a free scalar theory, the ground state corresponds to

$$\hat{K}(u) = \frac{i}{2}\int dk^d \left[\chi(u)\Gamma\left(ke^{-u}/M\right)a_k^+ a_{-k}^+ + (h.c.)\right],$$

where $\Gamma(\mathrm{x})$ is a cut off function : $\Gamma(\mathrm{x}) = \theta(1\text{-}|\,\mathrm{x}\,|)$.

$$\chi(s) = \frac{1}{2}\cdot\frac{e^{2u}}{e^{2u} + m^2/M^2}, \quad (\text{for } m = 0,\ \chi(\mathrm{u}) = 1/2.)$$

For the excited states, $\chi(s)$ becomes time-dependent.

One might be tempting to guess

Density of *bonds*

$$\boxed{ds^2_{Gravity} = g_{uu}du^2 + \frac{e^{2u}}{\varepsilon^2}\cdot d\vec{x}^2 - g_{tt}dt^2} \Rightarrow \sqrt{g_{uu}} \propto |\,\chi(u)\,| \ \ ?$$

Indeed, the previous proposal for guu lead to $g_{uu} = \chi(u)^2$.

Explicit metric

$$\boxed{ds^2_{Gravity} = g_{uu}du^2 + \frac{e^{2u}}{\varepsilon^2}\cdot d\vec{x}^2 - g_{tt}dt^2}$$

(i) Massless scalar (E=k)

$$g_{uu} = \frac{1}{4} \quad \Rightarrow \text{ the pure } AdS$$

(ii) Lifshitz scalar ($E=k^{\nu}$)

$$g_{uu} = \frac{\nu^2}{4} \quad \Rightarrow \text{ the Lifshitz geometry}$$

(iii) Massive scalar

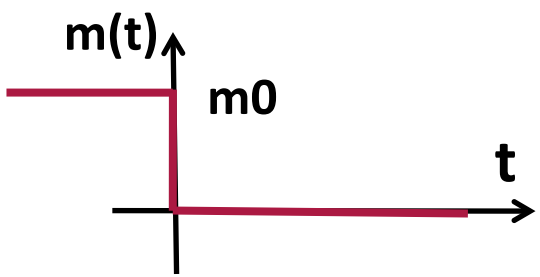
$$g_{uu} = \frac{e^{4u}}{4(e^{2u} + m^2/\Lambda^2)^2}.$$

$$\Rightarrow ds^2 = \frac{dz^2}{z^2} + \left(\frac{1}{\Lambda^2 z^2} - \frac{m^2}{\Lambda^2}\right)d\vec{x}^2 + g_{tt}dt^2$$

Capped off in the IR z<1/m

(4-6) Excited states after quantum quenches

$$(A_k a_k + B_k a_{-k}^{+})|\Psi\rangle = 0, \qquad (|A_k|^2 - |B_k|^2 = 1).$$



$$A_k = \frac{1}{2}\left(\left(\frac{k^2 + m0^2}{k^2}\right)^{1/4} + \left(\frac{k^2}{k^2 + m0^2}\right)^{1/4}\right)\cdot e^{ikt},$$

$$B_k = \frac{1}{2}\left(\left(\frac{k^2 + m0^2}{k^2}\right)^{1/4} - \left(\frac{k^2}{k^2 + m0^2}\right)^{1/4}\right)\cdot e^{-ikt}.$$

To realized these states, we need to extend the ansatz such that

$$\hat{K}(u) = \frac{i}{2}\int dk^d \Gamma\left(ke^{-u}/M\right)\left[g(u)a_k^{+}a_{-k}^{+} + g^{*}(u)a_k a_{-k}\right],$$

⇒ SU(1,1) Bogoliubov transf. Mk(u)

$$(A_k(u), B_k(u)) = (\alpha_k, \beta_k)\cdot M_k(u).$$

For a given UV state $|\Psi\rangle$ or equally $M_k(0)$,
the intermediate state $|\Psi(u)\rangle$ or $M_k(u)$ is determined
up to an ambiguity.

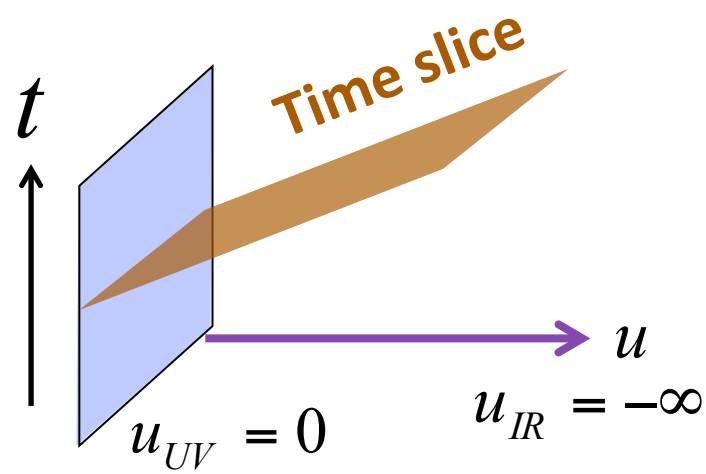
This stems from the phase factor ambiguity of wave function:

$$(A_k a_k + B_k a_{-k}^{+})|\Psi\rangle = 0 \quad \Rightarrow \quad e^{i\theta_k(t)} \cdot (A_k a_k + B_k a_{-k}^{+})|\Psi\rangle = 0 \ .$$

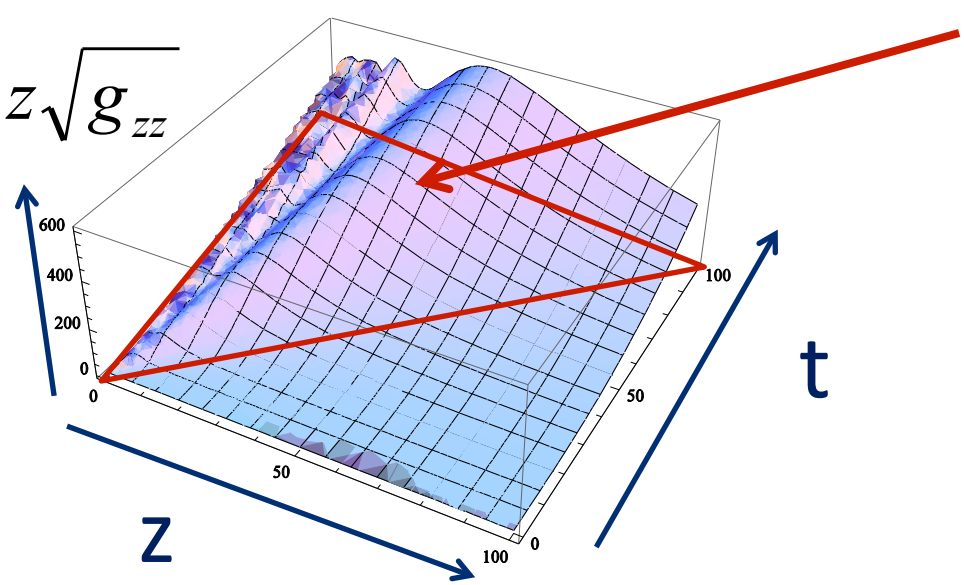
Our conjecture:

the phase ambiguity $\theta_k(t)$
$\Leftrightarrow$ the choice of the time slice

$$F(t,u) = const.$$



Time dependent metric from the Quantum Quench



Light cone

looks like a propagation of gravitational wave.

We can also (analytically) confirm the linear growth: SA∝t.
This is consistent with the known CFT (2d) .

[Calabrese-Cardy 05]

[HEE under quantum quenches: Arrastia-Aparicio-Lopez 10 , Albash-Johnson 10, Balasubramanian-Bernamonti-de Boer-Copland-Craps-Keski-Vakkuri-Müller-Schäfer-Shigemori-Staessens 10, 11,….]

(4-7) Towards Holographic Dual of Flat Space

If we consider the (almost) flat metric

$$ds^2 = e^{2u} du^2 + e^{2u} dx^2 \Rightarrow g_{uu} = e^{2u},$$

the corresponding dispersion relation reads

$$\chi(u) = \frac{1}{2} \cdot \left(\frac{k \partial_k E_k}{E_k} \right) \Bigg|_{k = \Lambda e^u} = e^u \qquad \Rightarrow E_k = e^k.$$

This leads to the highly non-local Hamiltonian:

$$H = \int dx^d \phi(x) e^{\sqrt{-\partial^2}} \phi(x).$$

[cf. Li-TT 10]

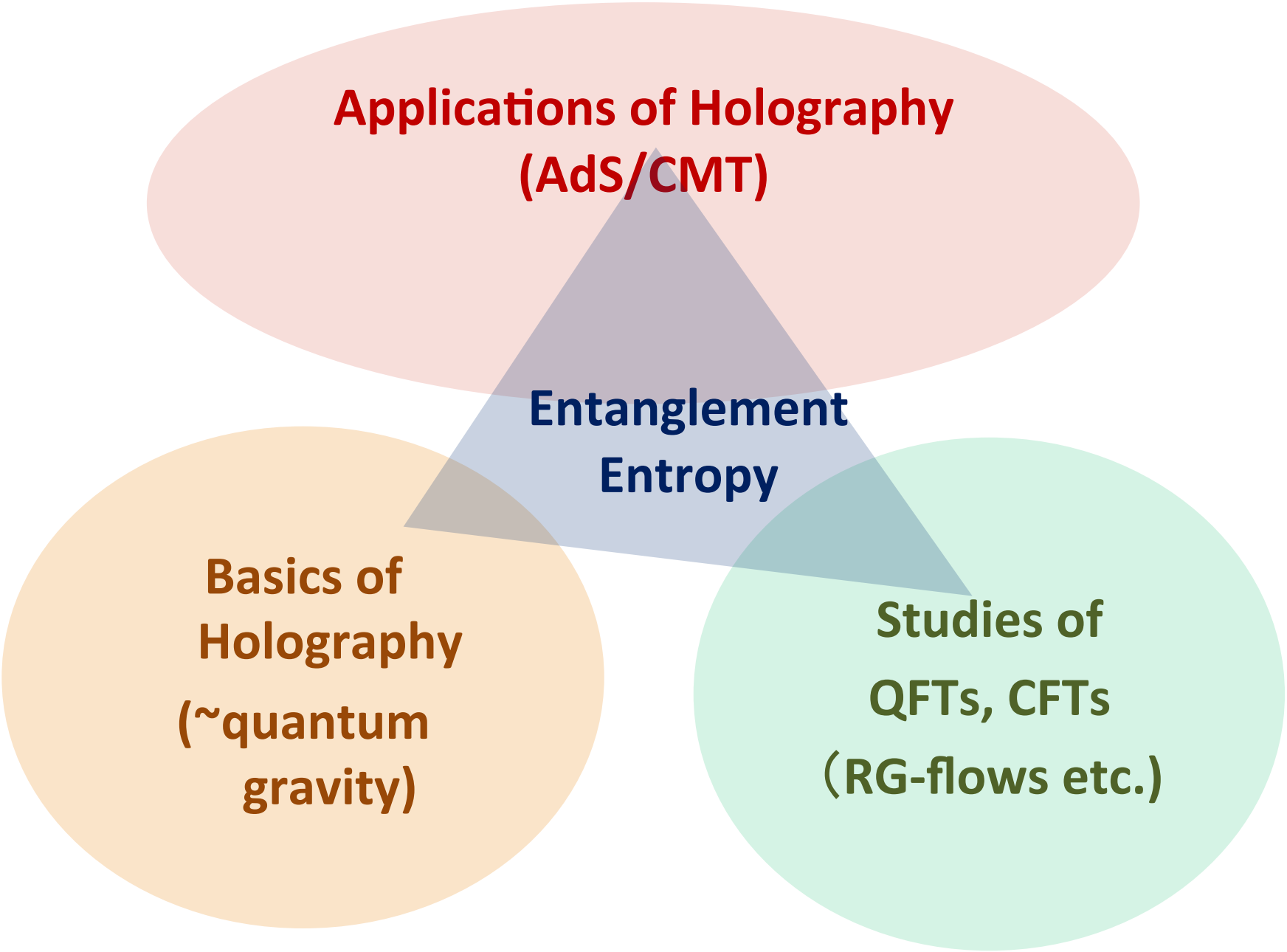
⑤ Conclusions

The idea of the entanglement renormalization can be a basic mechanism of the AdS/CFT correspondence.

We explored this connection by examining cMERA and proposed a metric in the extra dimension purely in terms of QFT data.

Many future problems:

- How to calculate gtt ? Boosting the subsystem ? Finite temp.?
- The effect of Large N limit in cMERA ?
  (largen N limit ⇔locality⇒saturation of entropy bound ?)
- Time slices and diff. inv. in cMERA ?
- Free field theories ⇒ Higher spin gauge theory?


Applications of Holography
(AdS/CMT)
Entanglement
Entropy
Basics of
Holography
(~quantum
gravity)
Studies of
QFTs, CFTs
（RG-flows etc.)