The Two Higgs-Doublet Model: Past, Present and Future

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University of Michigan

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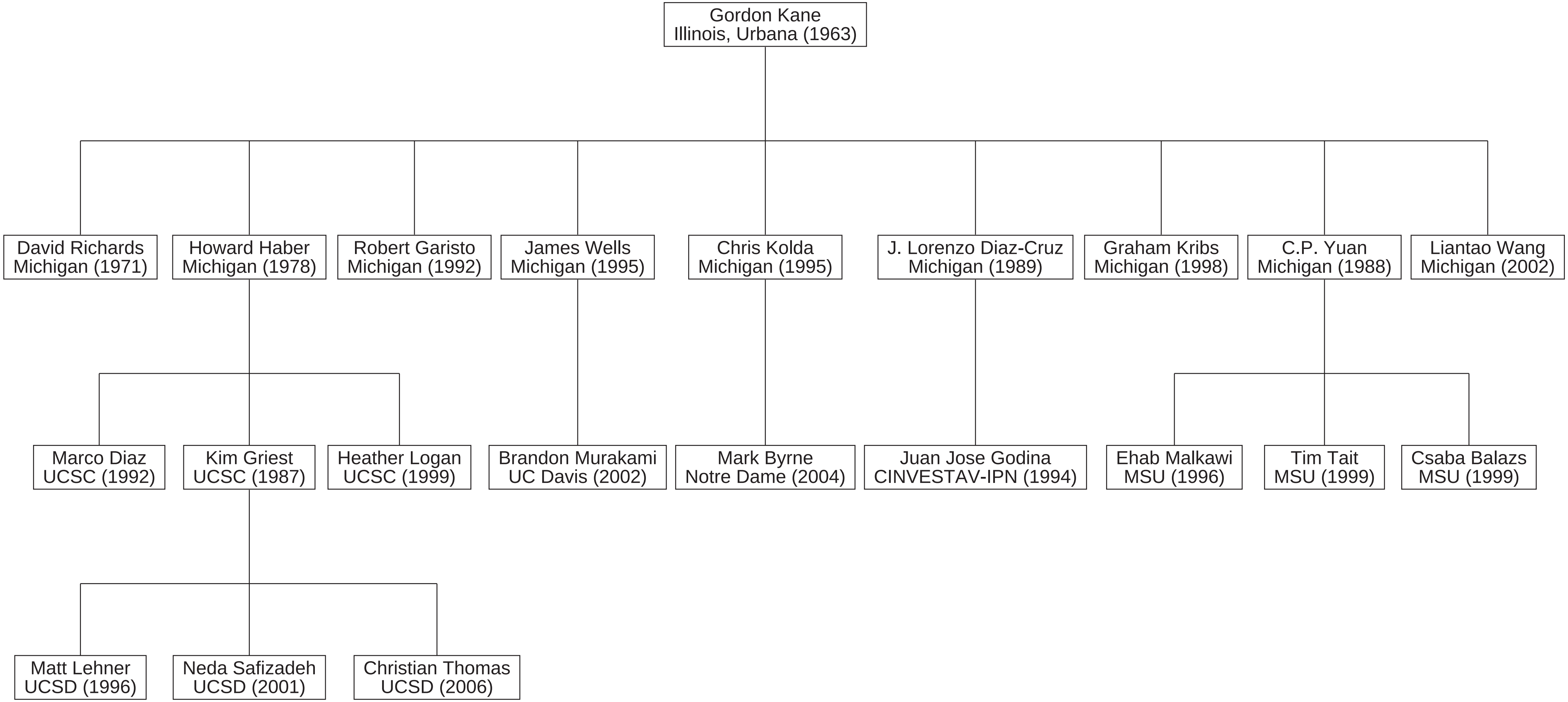
Kane Symposium

On the way to an important meeting at the Aspen Center for Physics.

Gordon Kane's Academic Family Tree (Descendants)

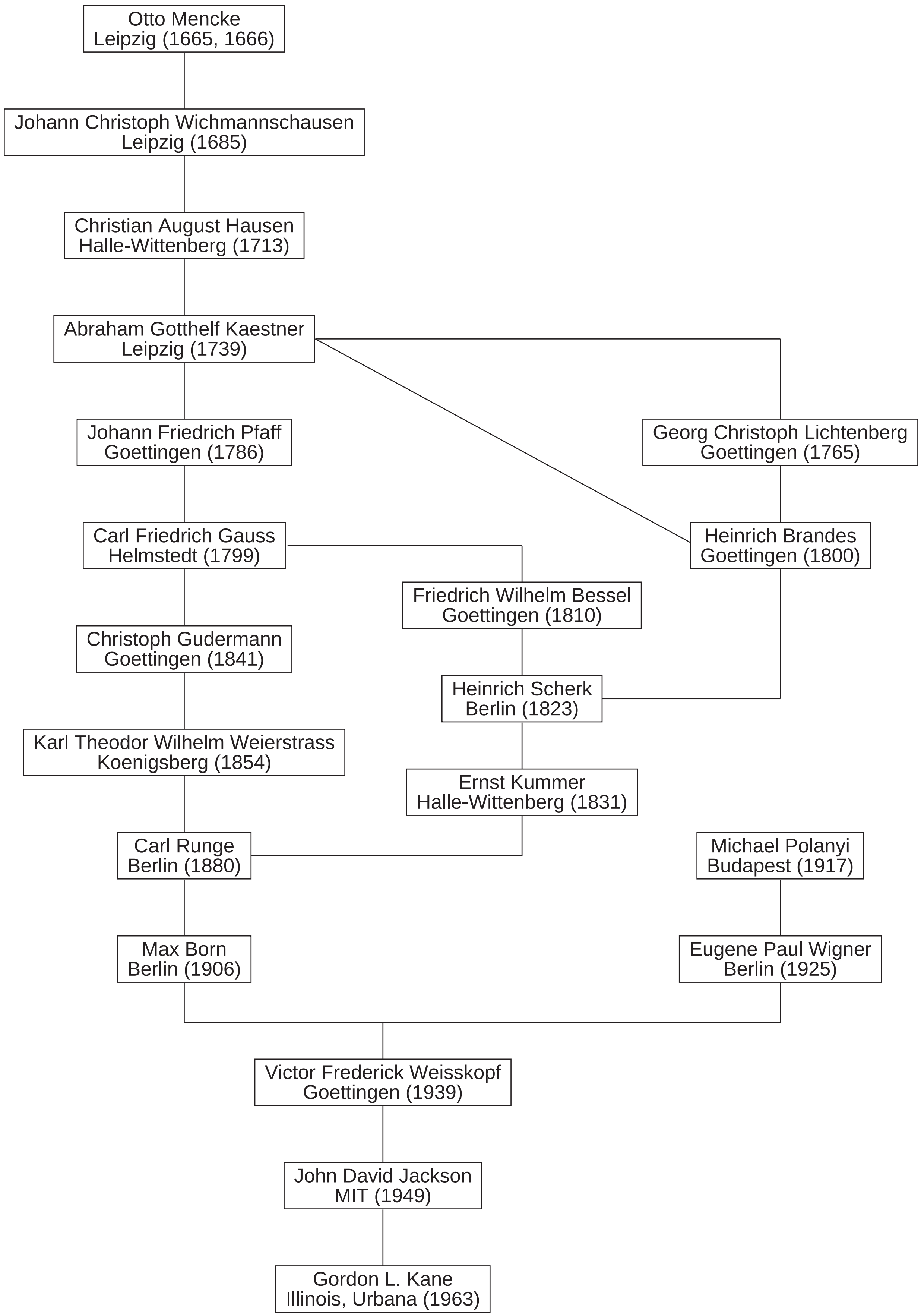


cuts applied to data: all listed entries possess either college/university faculty jobs or permanent jobs closely associated with academic physics/astrophysics as of January, 2007.

sources: SPIRES (http://www.slac.stanford.edu/spires/hep/) and information provided by Gordon Kane's students.

The academic family trees of Gordon Kane have been prepared in honor of his 70th birthday celebration.

Gordon Kane's Academic Family Tree (Ancestors)



sources: The Mathematics Genealogy Project (http://genealogy.math.ndsu.nodak.edu/)
and SPIRES (http://www.slac.stanford.edu/spires/hep/)

Outline

- The early history of the two-Higgs-doublet model (2HDM)
- The paradox of $\tan\beta$
- The general Two-Higgs-Doublet Model (2HDM)
  - Basis-independent techniques
  - Basis-independent (invariant) form of the Higgs couplings
- The Higgs-fermion interaction
  - The significance of $\tan\beta$?
  - Many $\tan\beta$-like parameters?
- Lessons for future work

Highlights of the early history of the 2HDM

• T.D. Lee, *A Theory of Spontaneous T Violation*, Phys. Rev. **D8**, 1226 (1973).

The first motivated 2HDM: an attempt to find a new source of CP-violation.

• S.L. Glashow and S. Weinberg, *Natural Conservation Laws For Neutral Currents*, Phys. Rev. **D15**, 1958 (1977).

To avoid neutral-Higgs-mediated tree-level flavor changing neutral currents (FCNCs), all fermions of a given electric charge can couple to at most one Higgs doublet (in a model with multiple scalar doublets).

• N.G. Deshpande and E. Ma, *Pattern Of Symmetry Breaking With Two Higgs Doublets*, Phys. Rev. **D18**, 2574 (1978).

Parameters of the Higgs potential had to lie in an appropriate region of parameter space to ensure that $\mathrm{U}(1)_{\mathrm{EM}}$ is not broken.

• J.F. Donoghue and L. F. Li, *Properties Of Charged Higgs Bosons*, Phys. Rev. **D19**, 945 (1979).

The inventors of the 2HDM with Type-II Higgs-fermion interactions: one Higgs doublet couples to up-type fermions and the other Higgs doublet couples to down-type fermions.

• H.E. Haber, G.L. Kane and T. Sterling, *The Fermion Mass Scale And Possible Effects Of Higgs Bosons On Experimental Observables*, Nucl. Phys. **B161**, 493 (1979).

The inventors of the 2HDM with Type-I Higgs-fermion interactions: one Higgs doublet couples to both up-type and down-type fermions, and the other Higgs doublet does not couple at all to the fermions.

• L.J. Hall and M.B. Wise, *Flavor Changing Higgs Boson Couplings*, Nucl. Phys. **B187**, 397 (1981).

The inventors of the Type-I and Type-II nomenclature.

• T.P. Cheng and M. Sher, *Mass Matrix Ansatz and Flavor Nonconservation in Models with Multiple Higgs Doublets*, Phys. Rev. **D35**, 3484 (1987).

The first realistic Type-III 2HDM (defined as a 2HDM with all possible Higgs-fermion couplings allowed).

Other important 2HDM milestones

- the axion as the CP-odd scalar of a 2HDM [the Peccei-Quinn mechanism].

- the requirement of a second Higgs doublet in the minimal supersymmetric extension of the Standard Model (MSSM).

In a supersymmetric extension of a one-doublet Standard Model, the corresponding higgsinos are anomalous. Anomalies are canceled if the higgsino doublets come in pairs with opposite sign hypercharges. Influential early papers: Fayet; Inoue *et al.*; Flores and Sher; and Gunion and Haber.

The MSSM Higgs sector

The Higgs sector of the MSSM (at tree-level) is a constrained Type-II 2HDM. One of the key parameters of the model is:

$$\tan\beta \equiv v_u/v_d\,,$$

where v_u $[v_d]$ is the vacuum expectation value of the neutral Higgs boson that couples exclusively to up-type [down-type] fermions.

But, one-loop radiative effects generate corrections to the tree-level structure of the model due to SUSY-breaking effects that enter in loops. In particular, for MSSM Higgs couplings to fermions, Yukawa vertex corrections modify the effective Lagrangian that describes the coupling of the Higgs bosons to the third generation quarks:

$$-\mathcal{L}_{\rm eff} = \epsilon_{ij}\left[(h_b+\delta h_b)\bar{b}_R H_d^i Q_L^j + (h_t+\delta h_t)\bar{t}_R Q_L^i H_u^j\right] + \Delta h_b \bar{b}_R Q_L^k H_u^{k*} + \Delta h_t \bar{t}_R Q_L^k H_d^{k*} + {\rm h.c.}$$

Thus, the MSSM Higgs-sector is actually a type-III model.

For example, in some MSSM parameter regimes (corresponding to large $\tan\beta$ and large supersymmetry-breaking scale compared to v), *

$$\Delta h_b \simeq h_b \left[\frac{2\alpha_s}{3\pi}\mu M_{\tilde{g}}\, I(M^2_{\tilde{b}_1}, M^2_{\tilde{b}_2}, M^2_{\tilde{g}}) + \frac{h_t^2}{16\pi^2}\mu A_t\, I(M^2_{\tilde{t}_1}, M^2_{\tilde{t}_2}, \mu^2)\right] .$$

The tree-level relation between m_b and h_b is modified (first pointed out by Hempfling and later emphasized strongly by Carena, Olechowski, Pokorski and Wagner):

$$h_b = \frac{\sqrt{2}m_b}{v\ \cos\beta(1+\Delta_b)}\,,$$

where $\Delta_b \equiv (\Delta h_b/h_b)\tan\beta$. That is, Δ_b is $\tan\beta$-enhanced, and governs the leading one-loop correction to the physical Higgs couplings to third generation quarks. In typical models at large $\tan\beta$, Δ_b can be of order 0.1 or larger and of either sign.

* $I(a,b,c) = [ab\ln(a/b) + bc\ln(b/c) + ca\ln(c/a)]/(a-b)(b-c)(a-c)$.

The paradox of $\tan\beta$

If the 2HDM is realized in nature, it is likely that its effective Lagrangian will consist of all possible dimension-four terms or less, consistent with the electroweak gauge invariance—that is a general type-III model.

The general 2HDM consists of two identical (hypercharge-one) scalar doublets Φ_1 and Φ_2. One can always redefine the basis, so the parameter $\tan\beta \equiv v_2/v_1$ is not meaningful!

Nevertheless, the literature is filled with 2HDM Feynman rules that depend on $\tan\beta$ and many phenomenological proposals to measure it! Hence, the paradox.

The parameter $\tan\beta$ makes sense only if there is a physical principle that distinguishes between Φ_1 and Φ_2. Such a principle is model-dependent. Any experimental study of 2HDM physics should avoid theoretical bias in defining their measurements. The theoretical interpretation should be a consequence of the observations.

To determine the relevant physical quantities for measurements, one must develop "basis-independent" techniques. Inspired by a beautifully written chapter on the 2HDM by G. Branco, L. Lavoura and J.P. Silva, in *CP Violation* (Oxford University Press, Oxford, UK, 1999), my collaborators (S. Davidson, J.F. Gunion and D. O'Neil) and I set out to develop the basis independent formalism of the 2HDM in order to identify the relevant invariant (basis-independent) quantities.

In particular, O'Neil and I were able to write down a complete set of Feynman rules that completely avoid the parameter $\tan\beta$, while describing all the CP-violating and flavor-violating phenomena in an elegant form.

The General Two-Higgs-Doublet Model

Consider the 2HDM potential in a *generic* basis:

$$\mathcal{V} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \mathrm{h.c.}] + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2$$

$$+ \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+ \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \mathrm{h.c.} \right\}$$

A basis change consists of a U(2) transformation $\Phi_a \longrightarrow U_{a\bar{b}} \Phi_b$ (and $\Phi_{\bar{a}}^\dagger = \Phi_{\bar{b}}^\dagger U_{b\bar{a}}^\dagger$). Rewrite $\mathcal{V}$ in a U(2)-covariant notation:

$$\boxed{\mathcal{V} = Y_{a\bar{b}} \Phi_{\bar{a}}^\dagger \Phi_b + \tfrac{1}{2} Z_{a\bar{b}c\bar{d}} (\Phi_{\bar{a}}^\dagger \Phi_b)(\Phi_{\bar{c}}^\dagger \Phi_d)}$$

where $Z_{a\bar{b}c\bar{d}} = Z_{c\bar{d}a\bar{b}}$ and hermiticity implies $Y_{a\bar{b}} = (Y_{b\bar{a}})^*$ and $Z_{a\bar{b}c\bar{d}} = (Z_{b\bar{a}d\bar{c}})^*$. The barred indices help keep track of which indices transform with U and which transform with $U^\dagger$. For example, $Y_{a\bar{b}} \longrightarrow U_{a\bar{c}} Y_{c\bar{d}} U_{d\bar{b}}^\dagger$ and $Z_{a\bar{b}c\bar{d}} \longrightarrow U_{a\bar{e}} U_{f\bar{b}}^\dagger U_{c\bar{g}} U_{h\bar{d}}^\dagger Z_{e\bar{f}g\bar{h}}$.

The most general $\mathrm{U(1)_{EM}}$-conserving vacuum expectation value (vev) is:

$$\langle\Phi_a\rangle = \frac{v}{\sqrt{2}}\left(\begin{array}{c} 0 \\ \widehat{v}_a \end{array}\right), \qquad \text{with} \qquad \widehat{v}_a \equiv e^{i\eta}\left(\begin{array}{c} c_\beta \\ s_\beta\, e^{i\xi} \end{array}\right),$$

where $v \equiv 2m_W/g = 246$ GeV. The overall phase η is arbitrary (and can be removed with a $\mathrm{U(1)_Y}$ hypercharge transformation). If we define the hermitian matrix $V_{a\bar{b}} \equiv \hat{v}_a \hat{v}^*_{\bar{b}}$, then the scalar potential minimum condition is given by the invariant condition:

$$\mathrm{Tr}\,(VY) + \tfrac{1}{2}v^2 Z_{a\bar{b}c\bar{d}} V_{b\bar{a}} V_{d\bar{c}} = 0\,.$$

The orthonormal eigenvectors of $V_{a\bar{b}}$ are $\hat{v}_b$ and $\widehat{w}_b \equiv \widehat{v}^*_{\bar{c}}\,\epsilon_{cb}$ (with $\epsilon_{12} = -\epsilon_{21} = 1$, $\epsilon_{11} = \epsilon_{22} = 0$). Note that $\hat{v}^*_{\bar{b}}\hat{w}_b = 0$. Under a U(2) transformation, $\hat{v}_a \longrightarrow U_{a\bar{b}}\hat{v}_b$, but:

$$\widehat{w}_a \longrightarrow (\det\, U)^{-1}\, U_{a\bar{b}}\, \widehat{w}_b\,,$$

where $\det\, U \equiv e^{i\chi}$ is a pure phase. That is, $\widehat{w}_a$ is a pseudo-vector with respect to U(2). One can use $\widehat{w}_a$ to construct a proper second-rank tensor: $W_{a\bar{b}} \equiv \hat{w}_a \hat{w}^*_{\bar{b}} \equiv \delta_{a\bar{b}} - V_{a\bar{b}}$.

Remark: $\mathrm{U(2)} \cong \mathrm{SU(2)}\times\mathrm{U(1)_Y}/\mathbb{Z}_2$. The parameters m^2_{11}, m^2_{22}, m^2_{12}, and $\lambda_1, \ldots, \lambda_7$ are invariant under $\mathrm{U(1)_Y}$ transformations, but are modified by a "flavor"-SU(2) transformation; whereas $\hat{v}$ transforms under the full U(2) group.

A list of invariant and pseudo-invariant quantities

$$Y_1 \equiv \mathrm{Tr}\,(YV)\,, \qquad Y_2 \equiv \mathrm{Tr}\,(YW)\,,$$

$$Z_1 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{a}}V_{d\bar{c}}\,, \qquad Z_2 \equiv Z_{a\bar{b}c\bar{d}}\,W_{b\bar{a}}W_{d\bar{c}}\,,$$

$$Z_3 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{a}}W_{d\bar{c}}\,, \qquad Z_4 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{c}}W_{d\bar{a}}$$

are invariants, whereas the following (potentially complex) pseudo-invariants

$$Y_3 \equiv Y_{a\bar{b}}\,\widehat{v}^*_{\bar{a}}\,\widehat{w}_b\,, \qquad Z_5 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}^*_{\bar{a}}\,\widehat{w}_b\,\widehat{v}^*_{\bar{c}}\,\widehat{w}_d\,,$$

$$Z_6 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}^*_{\bar{a}}\,\widehat{v}_b\,\widehat{v}^*_{\bar{c}}\,\widehat{w}_d\,, \qquad Z_7 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}^*_{\bar{a}}\,\widehat{w}_b\,\widehat{w}^*_{\bar{c}}\,\widehat{w}_d\,.$$

transform as

$$[Y_3, Z_6, Z_7] \longrightarrow (\det\,U)^{-1}[Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \longrightarrow (\det\,U)^{-2} Z_5\,.$$

Physical quantities must be invariants. For example, the charged Higgs boson mass is $m^2_{H^\pm} = Y_2 + \frac{1}{2}Z_3 v^2$. Pseudo-invariants are useful because one can always combine two such quantities to create an invariant.

The invariants and pseudo-invariants in the generic basis are given by:

$$
\begin{aligned}
Y_1 &= m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - \mathrm{Re}(m_{12}^2 e^{i\xi}) s_{2\beta}\,, \\
Y_2 &= m_{11}^2 s_\beta^2 + m_{22}^2 c_\beta^2 + \mathrm{Re}(m_{12}^2 e^{i\xi}) s_{2\beta}\,, \\
Y_3\, e^{i\xi} &= \tfrac{1}{2}(m_{22}^2 - m_{11}^2) s_{2\beta} - \mathrm{Re}(m_{12}^2 e^{i\xi}) c_{2\beta} - i\,\mathrm{Im}(m_{12}^2 e^{i\xi})\,, \\
Z_1 &= \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}\left[c_\beta^2 \mathrm{Re}(\lambda_6 e^{i\xi}) + s_\beta^2 \mathrm{Re}(\lambda_7 e^{i\xi})\right]\,, \\
Z_2 &= \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 - 2 s_{2\beta}\left[s_\beta^2 \mathrm{Re}(\lambda_6 e^{i\xi}) + c_\beta^2 \mathrm{Re}(\lambda_7 e^{i\xi})\right]\,, \\
Z_3 &= \tfrac{1}{4} s_{2\beta}^2 \left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \lambda_3 - s_{2\beta} c_{2\beta} \mathrm{Re}[(\lambda_6 - \lambda_7) e^{i\xi}]\,, \\
Z_4 &= \tfrac{1}{4} s_{2\beta}^2 \left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \lambda_4 - s_{2\beta} c_{2\beta} \mathrm{Re}[(\lambda_6 - \lambda_7) e^{i\xi}]\,, \\
Z_5\, e^{2i\xi} &= \tfrac{1}{4} s_{2\beta}^2 \left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \mathrm{Re}(\lambda_5 e^{2i\xi}) + i c_{2\beta} \mathrm{Im}(\lambda_5 e^{2i\xi})\,, \\
&\quad - s_{2\beta} c_{2\beta} \mathrm{Re}[(\lambda_6 - \lambda_7) e^{i\xi}] - i s_{2\beta} \mathrm{Im}[(\lambda_6 - \lambda_7) e^{i\xi})]\,, \\
Z_6\, e^{i\xi} &= -\tfrac{1}{2} s_{2\beta}\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta} - i\mathrm{Im}(\lambda_5 e^{2i\xi})\right] + c_\beta c_{3\beta} \mathrm{Re}(\lambda_6 e^{i\xi})\,, \\
&\quad + s_\beta s_{3\beta} \mathrm{Re}(\lambda_7 e^{i\xi}) + i c_\beta^2 \mathrm{Im}(\lambda_6 e^{i\xi}) + i s_\beta^2 \mathrm{Im}(\lambda_7 e^{i\xi})\,, \\
Z_7\, e^{i\xi} &= -\tfrac{1}{2} s_{2\beta}\left[\lambda_1 s_\beta^2 - \lambda_2 c_\beta^2 + \lambda_{345} c_{2\beta} + i\mathrm{Im}(\lambda_5 e^{2i\xi})\right] + s_\beta s_{3\beta} \mathrm{Re}(\lambda_6 e^{i\xi}) \\
&\quad + c_\beta c_{3\beta} \mathrm{Re}(\lambda_7 e^{i\xi}) + i s_\beta^2 \mathrm{Im}(\lambda_6 e^{i\xi}) + i c_\beta^2 \mathrm{Im}(\lambda_7 e^{i\xi})\,.
\end{aligned}
$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \mathrm{Re}(\lambda_5\, e^{2i\xi})$.

The Higgs mass-eigenstate basis

The three physical neutral Higgs boson mass-eigenstates are determined by diagonalizing a 3×3 squared-mass matrix that is defined in a basis in which only one of the neutral Higgs bosons has a vacuum expectation value (the so-called "Higgs basis"). The diagonalizing matrix is a 3×3 real orthogonal matrix that depends on three angles: θ_{12}, θ_{13} and θ_{23}. Under a U(2) transformation,

$$\theta_{12}\,,\,\theta_{13}\text{ are invariant, and}\quad e^{i\theta_{23}} \to (\det U)^{-1} e^{i\theta_{23}}\,.$$

One can express the mass eigenstate neutral Higgs directly in terms of the original shifted neutral fields, $\overline{\Phi}^0_a \equiv \Phi^0_a - v\widehat{v}_a/\sqrt{2}$:

$$h_k = \frac{1}{\sqrt{2}}\left[\overline{\Phi}^{0\,\dagger}_{\bar{a}}(q_{k1}\widehat{v}_a + q_{k2}\widehat{w}_a e^{-i\theta_{23}}) + (q^*_{k1}\widehat{v}^*_{\bar{a}} + q^*_{k2}\widehat{w}^*_{\bar{a}} e^{i\theta_{23}})\overline{\Phi}^0_a\right]\,,$$

for $k = 1, \ldots, 4$, where $h_4 = G^0$. The *invariant* quantities q_{kj} are given by:

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}
4	i	0

Since $\widehat{w}_a e^{-i\theta_{23}}$ is a *proper* U(2)-vector, we see that the mass-eigenstate fields are indeed U(2)-invariant fields. Inverting the previous result yields:

$$\Phi_a = \begin{pmatrix} G^+\widehat{v}_a + H^+\widehat{w}_a \\ \dfrac{v}{\sqrt{2}}\widehat{v}_a + \dfrac{1}{\sqrt{2}}\displaystyle\sum_{k=1}^{4}\left(q_{k1}\widehat{v}_a + q_{k2}e^{-i\theta_{23}}\widehat{w}_a\right)h_k \end{pmatrix}.$$

If $\mathrm{Im}\,(Z_5^* Z_6^2) = 0$, then the neutral scalar squared-mass matrix can be transformed into block diagonal form, containing the squared-mass of a CP-odd neutral Higgs mass-eigenstate and a 2×2 sub-matrix that yields the squared-masses of two CP-even neutral Higgs mass-eigenstates.

If $\mathrm{Im}\ (Z_5^* Z_6^2) \neq 0$, we can write $Z_6 \equiv |Z_6| e^{i\theta_6}$. Then the neutral scalar mass-eigenstates do not possess definite CP quantum numbers, and the three invariant mixing angles θ_{12}, θ_{13} and $\phi_6 \equiv \theta_6 - \theta_{23}$ are non-trivial.

The angles θ_{13} and ϕ_6 are determined modulo π from

$$\tan\theta_{13} = \frac{\mathrm{Im}(Z_5\, e^{-2i\theta_{23}})}{2\,\mathrm{Re}(Z_6\, e^{-i\theta_{23}})}\,, \qquad \tan 2\theta_{13} = \frac{2\,\mathrm{Im}(Z_6\, e^{-i\theta_{23}})}{Z_1 - A^2/v^2}\,,$$

where $A^2 \equiv Y_2 + \frac{1}{2}[Z_3 + Z_4 - \mathrm{Re}(Z_5 e^{-2i\theta_{23}})]v^2$. These equations exhibit multiple solutions (modulo π) corresponding to different orderings of the h_k masses. Finally,

$$\tan 2\theta_{12} = \frac{2\cos 2\theta_{13}\,\mathrm{Re}(Z_6\, e^{-i\theta_{23}})}{c_{13}\,[c_{13}^2(A^2/v^2 - Z_1) + \cos 2\theta_{13}\,\mathrm{Re}(Z_5\, e^{-2i\theta_{23}})]}\,.$$

For a given solution of θ_{13} and ϕ_6, the two solutions for θ_{12} (modulo π) correspond to the two possible relative mass orderings of h_1 and h_2.

The gauge boson–Higgs boson interactions

$$\mathscr{L}_{VVH} = \left(g m_W W_\mu^+ W^{\mu\,-} + \frac{g}{2c_W} m_Z Z_\mu Z^\mu\right) \mathrm{Re}(q_{k1}) h_k + e m_W A^\mu (W_\mu^+ G^- + W_\mu^- G^+)$$

$$- g m_Z s_W^2 Z^\mu (W_\mu^+ G^- + W_\mu^- G^+)\,,$$

$$\mathscr{L}_{VVHH} = \left[\tfrac{1}{4} g^2 W_\mu^+ W^{\mu\,-} + \frac{g^2}{8c_W^2} Z_\mu Z^\mu\right] \mathrm{Re}(q_{j1}^* q_{k1} + q_{j2}^* q_{k2})\, h_j h_k$$

$$+ \left[\tfrac{1}{2} g^2 W_\mu^+ W^{\mu\,-} + e^2 A_\mu A^\mu + \frac{g^2}{c_W^2}\left(\tfrac{1}{2} - s_W^2\right)^2 Z_\mu Z^\mu + \frac{2ge}{c_W}\left(\tfrac{1}{2} - s_W^2\right) A_\mu Z^\mu\right] (G^+ G^- + H^+ H^-)$$

$$+ \left\{ \left(\tfrac{1}{2} e g A^\mu W_\mu^+ - \frac{g^2 s_W^2}{2c_W} Z^\mu W_\mu^+\right) (q_{k1} G^- + q_{k2}\, e^{-i\theta_{23}} H^-) h_k + \text{h.c.} \right\},$$

$$\mathscr{L}_{VHH} = \frac{g}{4c_W} \mathrm{Im}(q_{j1} q_{k1}^* + q_{j2} q_{k2}^*) Z^\mu h_j \overleftrightarrow{\partial}_\mu\, h_k - \tfrac{1}{2} g \left\{ i W_\mu^+ \left[q_{k1} G^- \overleftrightarrow{\partial}^\mu\, h_k + q_{k2} e^{-i\theta_{23}} H^- \overleftrightarrow{\partial}^\mu\, h_k \right] + \text{h.c.} \right\}$$

$$+ \left[i e A^\mu + \frac{ig}{c_W}\left(\tfrac{1}{2} - s_W^2\right) Z^\mu \right] (G^+ \overleftrightarrow{\partial}_\mu\, G^- + H^+ \overleftrightarrow{\partial}_\mu\, H^-)\,.$$

The cubic and quartic Higgs couplings

$$
\begin{aligned}
\mathscr{L}_{3h} = & -\tfrac{1}{2}v\, h_j h_k h_\ell \Big[q_{j1}q^*_{k1}\mathrm{Re}(q_{\ell 1})Z_1 + q_{j2}q^*_{k2}\,\mathrm{Re}(q_{\ell 1})(Z_3+Z_4) + \mathrm{Re}(q^*_{j1}q_{k2}q_{\ell 2}Z_5\, e^{-2i\theta_{23}}) \\
& \qquad + \mathrm{Re}\Big([2q_{j1}+q^*_{j1}]q^*_{k1}q_{\ell 2}Z_6\, e^{-i\theta_{23}}\Big) + \mathrm{Re}(q^*_{j2}q_{k2}q_{\ell 2}Z_7\, e^{-i\theta_{23}})\Big] \\
& -v\, h_k G^+G^- \Big[\mathrm{Re}(q_{k1})Z_1 + \mathrm{Re}(q_{k2}\, e^{-i\theta_{23}}Z_6)\Big] + v\, h_k H^+H^- \Big[\mathrm{Re}(q_{k1})Z_3 + \mathrm{Re}(q_{k2}\, e^{-i\theta_{23}}Z_7)\Big] \\
& -\tfrac{1}{2}v\, h_k \Big\{ G^-H^+\, e^{i\theta_{23}} \left[q^*_{k2}Z_4 + q_{k2}\, e^{-2i\theta_{23}}Z_5 + 2\mathrm{Re}(q_{k1})Z_6\, e^{-i\theta_{23}}\right] + \mathrm{h.c.}\Big\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}_{4h} = & -\tfrac{1}{8} h_j h_k h_l h_m \Big[q_{j1}q_{k1}q^*_{\ell 1}q^*_{m1}Z_1 + q_{j2}q_{k2}q^*_{\ell 2}q^*_{m2}Z_2 + 2q_{j1}q^*_{k1}q_{\ell 2}q^*_{m2}(Z_3+Z_4) \\
& + 2\mathrm{Re}(q^*_{j1}q^*_{k1}q_{\ell 2}q_{m2}Z_5\, e^{-2i\theta_{23}}) + 4\mathrm{Re}(q_{j1}q^*_{k1}q^*_{\ell 1}q_{m2}Z_6\, e^{-i\theta_{23}}) + 4\mathrm{Re}(q^*_{j1}q_{k2}q_{\ell 2}q^*_{m2}Z_7\, e^{-i\theta_{23}})\Big] \\
& -\tfrac{1}{2} h_j h_k G^+G^- \Big[q_{j1}q^*_{k1}Z_1 + q_{j2}q^*_{k2}Z_3 + 2\mathrm{Re}(q_{j1}q_{k2}Z_6\, e^{-i\theta_{23}})\Big] \\
& -\tfrac{1}{2} h_j h_k H^+H^- \Big[q_{j2}q^*_{k2}Z_2 + q_{j1}q^*_{k1}Z_3 + 2\mathrm{Re}(q_{j1}q_{k2}Z_7\, e^{-i\theta_{23}})\Big] \\
& -\tfrac{1}{2} h_j h_k \Big\{ G^-H^+\, e^{i\theta_{23}} \left[q_{j1}q^*_{k2}Z_4 + q^*_{j1}q_{k2}Z_5\, e^{-2i\theta_{23}} + q_{j1}q^*_{k1}Z_6\, e^{-i\theta_{23}} + q_{j2}q^*_{k2}Z_7\, e^{-i\theta_{23}}\right] + \mathrm{h.c.}\Big\} \\
& -\tfrac{1}{2}Z_1 G^+G^-G^+G^- - \tfrac{1}{2}Z_2 H^+H^-H^+H^- - (Z_3+Z_4)G^+G^-H^+H^- \\
& -\tfrac{1}{2}(Z_5 H^+H^+G^-G^- + Z_5^* H^-H^-G^+G^+) - G^+G^-(Z_6 H^+G^- + Z_6^* H^-G^+) - H^+H^-(Z_7 H^+G^- + Z_7^* H^-G^+)\,.
\end{aligned}
$$

Example: Higgs self-couplings

Lightest neutral Higgs boson cubic self-coupling:

$$g(h_1h_1h_1) = -3v\{\,Z_1c_{12}^3c_{13}^3 + (Z_3+Z_4)c_{12}c_{13}|s_{123}|^2 + c_{12}c_{13}\,\mathrm{Re}(s_{123}^2Z_5e^{2i\theta_{23}})$$
$$-3c_{12}^2c_{13}^2\,\mathrm{Re}(s_{123}Z_6e^{i\theta_{23}}) - |s_{123}|^2\,\mathrm{Re}(s_{123}Z_7e^{i\theta_{23}})\}$$

Lightest neutral Higgs boson quartic self-coupling:

$$g(h_1h_1h_1h_1) = -3\{\,Z_1c_{12}^4c_{13}^4 + Z_2|s_{123}|^4 + 2(Z_3+Z_4)c_{12}^2c_{13}^2|s_{123}|^2$$
$$+2c_{12}^2c_{13}^2\,\mathrm{Re}(s_{123}^2Z_5e^{2i\theta_{23}}) - 4c_{12}^3c_{13}^3\,\mathrm{Re}(s_{123}Z_6e^{i\theta_{23}})$$
$$-4c_{12}c_{13}|s_{123}|^2\,\mathrm{Re}(s_{123}Z_7e^{i\theta_{23}})\}$$

where $s_{123} \equiv s_{12} + ic_{12}s_{13}$.

Note that these quantities depend on U(2)-invariants. In particular $Z_5e^{-2i\theta_{23}}$, $Z_6e^{-i\theta_{23}}$ and $Z_7e^{-i\theta_{23}}$ are U(2)-invariants!

The Higgs-fermion Yukawa couplings

The Yukawa Lagrangian can be written in terms of the quark mass-eigenstate fields as:

$$-\mathscr{L}_{\rm Y} = \overline{U}_L \widetilde{\Phi}^0_{\bar{a}} \eta^U_a U_R + \overline{D}_L K^\dagger \widetilde{\Phi}^-_{\bar{a}} \eta^U_a U_R + \overline{U}_L K \Phi^+_a \eta^{D\,\dagger}_{\bar{a}} D_R + \overline{D}_L \Phi^0_a \eta^{D\,\dagger}_{\bar{a}} D_R + {\rm h.c.}\,,$$

where $\widetilde{\Phi}_{\bar{a}} \equiv (\widetilde{\Phi}^0\,,\,\widetilde{\Phi}^-) = i\sigma_2 \Phi^*_{\bar{a}}$ and K is the CKM mixing matrix. The $\eta^{U,D}$ are 3×3 Yukawa coupling matrices. We can construct invariant and pseudo-invariant matrix Yukawa couplings:

$$\kappa^Q \equiv \widehat{v}^*_{\bar{a}} \eta^Q_a\,, \qquad\qquad \rho^Q \equiv \widehat{w}^*_{\bar{a}} \eta^Q_a\,,$$

where $Q = U$ or D. Inverting these equations yields: $\eta^Q_a = \kappa^Q \widehat{v}_a + \rho^Q \widehat{w}_a$. Under a U(2) transformation, κ^Q is invariant, whereas $\rho^Q \to (\det\, U)\rho^Q$.

By construction, κ^U and κ^D are proportional to the (real non-negative) diagonal quark mass matrices M_U and M_D, respectively. In particular,

$$M_U = \frac{v}{\sqrt{2}} \kappa^U = {\rm diag}(m_u\,,\, m_c\,,\, m_t)\,, \qquad M_D = \frac{v}{\sqrt{2}} \kappa^{D\,\dagger} = {\rm diag}(m_d\,,\, m_s\,,\, m_b)\,.$$

The matrices ρ^U and ρ^D are independent complex 3×3 matrices.

The final form for the Yukawa couplings of the mass-eigenstate Higgs bosons and the Goldstone bosons to the quarks is:

$$
\begin{aligned}
-\mathscr{L}_Y = {} & \frac{1}{v}\overline{D}\left\{ M_D(q_{k1}P_R + q_{k1}^* P_L) + \frac{v}{\sqrt{2}}\left[q_{k2}\,[e^{i\theta_{23}}\rho^D]^\dagger P_R + q_{k2}^*\, e^{i\theta_{23}}\rho^D P_L\right]\right\} D h_k \\
& + \frac{1}{v}\overline{U}\left\{ M_U(q_{k1}P_L + q_{k1}^* P_R) + \frac{v}{\sqrt{2}}\left[q_{k2}^*\, e^{i\theta_{23}}\rho^U P_R + q_{k2}\,[e^{i\theta_{23}}\rho^U]^\dagger P_L\right]\right\} U h_k \\
& + \left\{ \overline{U}\left[K[\rho^D]^\dagger P_R - [\rho^U]^\dagger K P_L\right] D H^+ + \frac{\sqrt{2}}{v}\overline{U}\left[K M_D P_R - M_U K P_L\right] D G^+ + \text{h.c.}\right\}.
\end{aligned}
$$

By writing $[\rho^Q]^\dagger H^+ = [\rho^Q e^{i\theta_{23}}]^\dagger [e^{i\theta_{23}} H^+]$, we see that the Higgs-fermion Yukawa couplings depend only on invariant quantities: the diagonal quark mass matrices, $\rho^Q e^{i\theta_{23}}$, and the invariant angles θ_{12} and θ_{13}.

The couplings of the neutral Higgs bosons to quark pairs are generically CP-violating as a result of the complexity of the q_{k2} and the fact that the matrices $e^{i\theta_{23}}\rho^Q$ are not generally hermitian or anti-hermitian. $\mathscr{L}_{\mathrm{Y}}$ also exhibits Higgs-mediated flavor-changing neutral currents (FCNCs) at tree-level by virtue of the fact that the ρ^Q are not flavor-diagonal. Thus, for a phenomenologically acceptable theory, the off-diagonal elements of ρ^Q must be small.

The significance of $\tan\beta$

So far, $\tan\beta$ has been completely absent from the Higgs couplings. This must be so, since $\tan\beta$ is basis-dependent in a general 2HDM. However, a particular 2HDM may single out a preferred basis, in which case $\tan\beta$ would be promoted to an observable. To simplify the discussion, we focus on a one-generation model, where the Yukawa coupling matrices are simply numbers.

As an example, the MSSM Higgs sector is a type-II 2HDM, *i.e.*, $\eta_1^U = \eta_2^D = 0$. A basis-independent condition for type-II is: $\eta_{\bar{a}}^{D\,*}\eta_a^U = 0$. In the preferred basis, $\hat{v} = (\cos\beta\,,\ \sin\beta\, e^{i\xi})$ and $\hat{w} = (-\sin\beta e^{-i\xi}\,,\ \cos\beta)$. Evaluating $\kappa^Q = \hat{v}^* \cdot \eta^Q$ and $\rho^Q = \hat{w}^* \cdot \eta^Q$ in the preferred basis, it follows that:

$$e^{-i\xi}\tan\beta = -\frac{\rho^{D\,*}}{\kappa^D} = \frac{\kappa^U}{\rho^U}\,,$$

where $\kappa^Q = \sqrt{2}m_Q/v$. These two definitions are consistent if $\kappa^D\kappa^U + \rho^{D\,*}\rho^U = 0$ is satisfied. But this is equivalent to the type-II condition, $\eta_{\bar{a}}^{D\,*}\eta_a^U = 0$.

Since ρ^Q is a pseudo-invariant, we can eliminate ξ by rephasing Φ_2. Hence,

$$\tan\beta = \frac{|\rho^D|}{\kappa^D} = \frac{\kappa^U}{|\rho^U|}\,,$$

with $0 \le \beta \le \pi/2$. Indeed, $\tan\beta$ is now a physical parameter, and the $|\rho^Q|$ are no longer independent:

$$|\rho^D| = \frac{\sqrt{2}m_d\tan\beta}{v}\,, \qquad |\rho^U| = \frac{\sqrt{2}m_u\cot\beta}{v}\,.$$

In the more general (type-III) 2HDM, $\tan\beta$ is not a meaningful parameter. Nevertheless, one can introduce three $\tan\beta$-like parameters:†

$$\tan\beta_d \equiv \frac{|\rho^D|}{\kappa^D}\,, \qquad \tan\beta_u \equiv \frac{\kappa^U}{|\rho^U|}\,, \qquad \tan\beta_e \equiv \frac{|\rho^E|}{\kappa^E}\,,$$

the last one corresponding to the Higgs-lepton interaction. In a type-III 2HDM, there is no reason for the three parameters above to coincide.

† Interpretation: In the Higgs basis, up and down-type quarks interact with both Higgs doublets. But, clearly there exists some basis (*i.e.*, a rotation by angle β_u from the Higgs basis) for which only one of the two up-type quark Yukawa couplings is non-vanishing. This defines the physical angle β_u.

The MSSM Higgs sector is a type-III 2HDM

Recall the effective one-loop Higgs-fermion Yukawa couplings in the MSSM are of the form:

$$-\mathcal{L}_{\rm eff} = \epsilon_{ij}\left[(h_b + \delta h_b)\bar{b}_R H_d^i Q_L^j + (h_t + \delta h_t)\bar{t}_R Q_L^i H_u^j\right] + \Delta h_b \bar{b}_R Q_L^k H_u^{k*} + \Delta h_t \bar{t}_R Q_L^k H_d^{k*} + \text{h.c.}$$

For illustrative purposes, we neglect CP violation in the following simplified discussion. Keeping only the leading $\tan\beta$-enhanced terms, $\Delta_b \equiv (\Delta h_b/h_b)\tan\beta$,

$$\tan\beta_b \equiv \frac{v\rho^D}{\sqrt{2}\,m_b} \simeq \frac{\tan\beta}{1+\Delta_b}\,, \qquad \tan\beta_t \equiv \frac{\sqrt{2}\,m_t}{v\rho^U} \simeq \frac{\tan\beta}{1-\tan\beta\,(\Delta h_t/h_t)}\,.$$

Thus, supersymmetry-breaking loop-effects can yield observable differences between $\tan\beta$-like parameters that are defined in terms of basis-independent quantities. In particular, the leading one-loop $\tan\beta$-enhanced corrections are automatically incorporated into:

$$g_{Ab\bar{b}} = \frac{m_b}{v}\tan\beta_b\,, \qquad g_{At\bar{t}} = \frac{m_t}{v}\cot\beta_t\,.$$

Lessons for future work

- If phenomena consistent with the 2HDM are found, we will not know a priori the underlying structure that governs the model. In this case, one needs a model-independent analysis of the data that allows for the most general CP-violating Model-III.

- Instead of claiming that you have measured $\tan\beta$ (unless you wish to test a specific theoretical framework), measure the physical parameters of the model. Examples include the $\tan\beta$-like parameters introduced in the one-generation model. (For three generations, the formalism becomes more complicated. However, one has good reason to assume that the third generation quark–Higgs Yukawa couplings dominate.)

- Which $\tan\beta$-like parameters will be measured in precision Higgs studies at the ILC? How can one best treat the full three-generation model at one-loop order?

- Even in the MSSM where $\tan\beta$ at tree-level is physically well-defined, the scheme presented here might be useful in achieving a more direct connection between model parameters and physical observables (when radiative corrections are incorporated).

And finally . . .

Happy 70th Birthday, Gordy!!!

I am looking forward to a time in the not too distant future, when we will celebrate together the discovery of the Higgs boson(s)!

(We've waited long enough, don't you think?)