Kane's Birthday Symposium

January 19-20, 2007

The Black Hole Information Paradox

1. What is the Paradox?
2. What is the origin of BH entropy?
3. What is entanglement?
4. Applications to Eternal Spacetimes.
5. Application to Gravitational Collapse.
6. Microscopic dynamics of Hawking Radiation.
7. Conclusions. Open questions.

What is the Paradox?

- Hawking (1975) discovered that BH's radiate.
- Radiation appeared to be thermal.

  Evolution in Quantum Mechanics is Unitary.

$$|\psi(0)\rangle \longrightarrow U(t)|\psi(0)\rangle = |\psi(t)\rangle$$

pure state ⟶ *pure state*

- If universe starts in pure state, forms BH, evaporates by radiating a thermal gas, then

  pure state ⟶ *mixed state*

- Contradiction with QM

Problem of principle, not of practical observation.

Compare to burning a book.

What is the origin of BH entropy?

$$S_{BH} = \frac{k_B c^3}{4\hbar} AREA$$

Beckenstein-
Hawking

AREA of Event Horizon

$$S = \ln \mathcal{N}$$

$\mathcal{N}$= # of dof

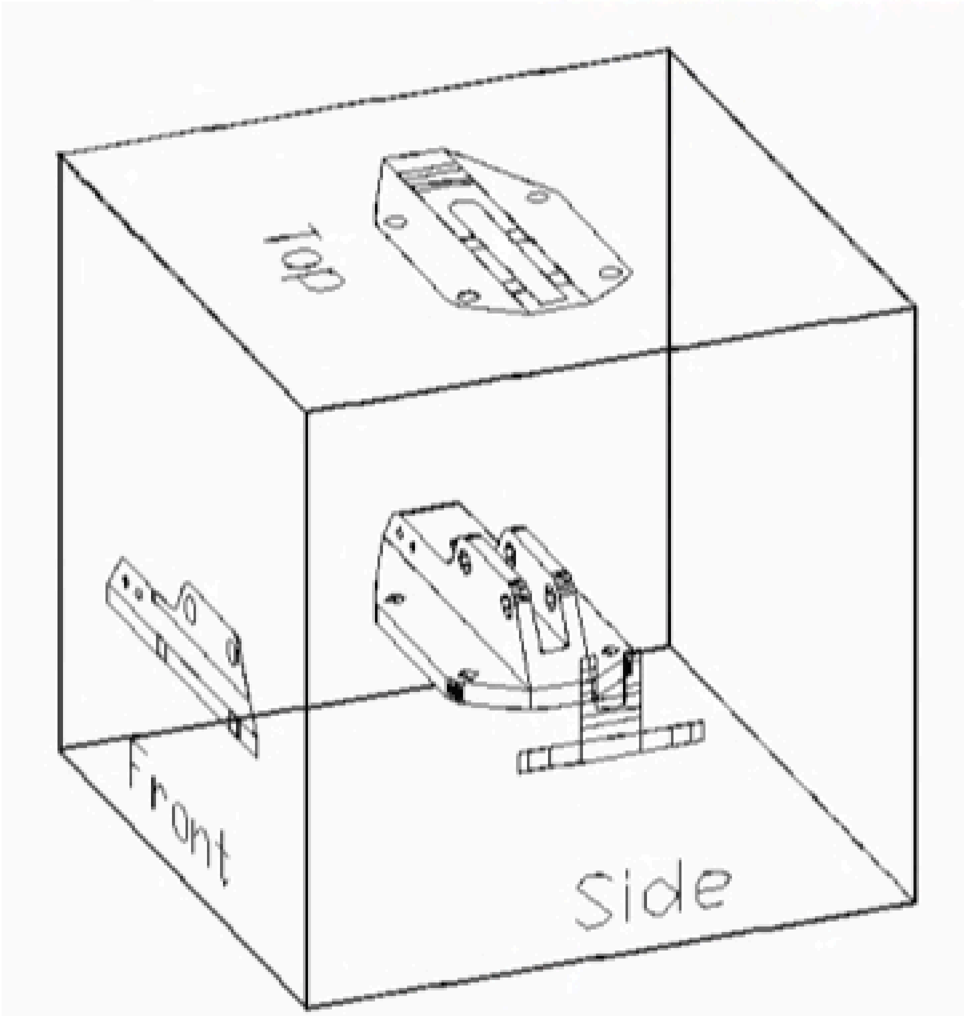
Should be extensive, scale as volume.

True for gases, true for QFT

Why not for BH's?

Stimulated much speculation!

Do BH's have fewer dof?



Holography

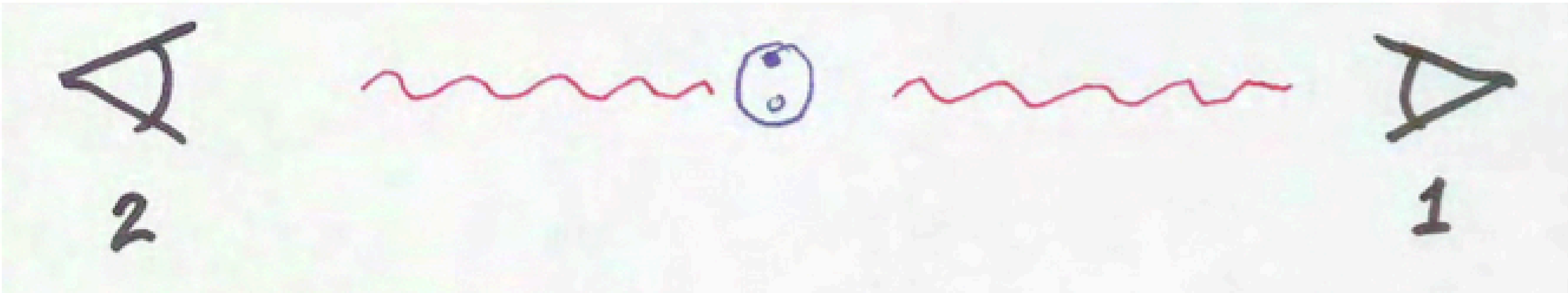
't Hooft
Susskind

(AdS/CFT correspondence)
Is QFT redundant,
when quantum gravity comes into play?

Another point of view

- Hypothesis--entropy of BH’s due to entanglement between states inside and outside.
- Consistent with Event Horizon being a coordinate not a curvature singularity.
- Entropy–a matter of perception (*i.e.*, observation.)

Entanglement



Positronium Annhiliation

$$|0,0)\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1(|\uparrow\rangle_2\Big)$$

Observer 1 can instantly predict what Observer 2 will see. Seems to violate causality.

EPR Paradox — Einstein-Podolsky-Rosen

“Superluminal” “Action at a distance” “Nonlocality”

$$|0,0)\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 (|\uparrow\rangle_2\Big)$$

- Globally, system is in a pure state.

  $$\rho = |0,0\rangle\langle 0,0|$$

- Focus on observer 1, who cannot detect the second photon. All her observations can be inferred from a density matrix obtained by summing over the possibilities for the second photon.

$$\begin{aligned} \rho_1 &= \mathrm{Tr}_2[\rho] \\ &= {}_2\langle\uparrow|0,0\rangle\langle 0,0|\uparrow\rangle_2 + {}_2\langle\downarrow|0,0\rangle\langle 0,0|\downarrow\rangle_2. \end{aligned}$$

With the result:

$$\rho_1 = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \qquad \text{mixed state}$$

Pure state has only eigenvalues 0 and 1,

$$\rho^2 = \rho$$

but a mixed state has

$$\rho^2 \neq \rho$$

von Neumann entropy:

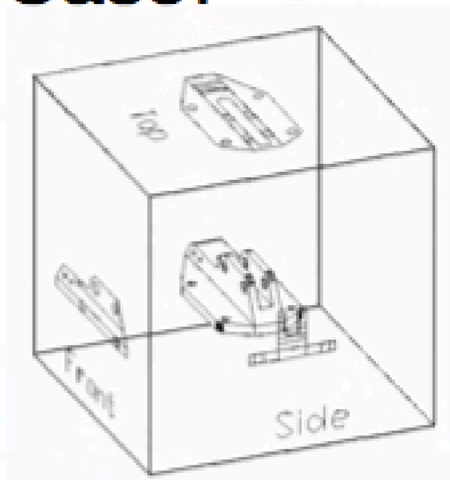
$$S = -\mathrm{Tr}\, \rho \ln \rho$$

Pure State: $S = 0,$

Mixed State: $S_1 = -\mathrm{Tr}\, \rho_1 \ln \rho_1 = \ln 2 \neq 0.$

- Entropy of "Entanglement"
- General Case:

Feynman, *Stat. Mech.*



System (1) — Rest of Universe (2)

$|\phi\rangle_i \quad \mathcal{H}_1 \qquad |\theta_r\rangle \quad \mathcal{H}_2$

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$$

$$|\psi\rangle = \sum_{i,r} C_{ir} |\phi_i\rangle_1 |\theta_r\rangle_2$$

Entangled if no basis set gives $|\phi\rangle_1 |\theta\rangle_2$

Observable: **A** Hermitian Operator

$$\langle \mathbf{A} \rangle \equiv \langle \psi | \mathbf{A} | \psi \rangle$$

Suppose **A** acts only on the ***system***: $\mathcal{H}_1 \to \mathcal{H}_1$

$$\langle \mathbf{A} \rangle = \mathrm{Tr}_1(\mathbf{A}\rho_1) \quad \text{where} \quad \rho_1 = \mathrm{C}\ \mathrm{C}^\dagger \qquad \mathrm{C} \equiv C_{ir}$$

$$\rho_1 = \mathrm{Tr}_2[\rho] \quad \text{where} \quad \rho \equiv |\psi\rangle\langle\psi|$$

ρ_1 is Hermitian, diagonalizable

$$S = \mathrm{Tr}[\rho] = 0$$

$$S_1 = \mathrm{Tr}[\rho]_1 \neq 0$$

Entanglement Entropy

Similarly, $\rho_2 \equiv \mathrm{Tr}_1[\rho] = \mathrm{C}^\dagger \mathrm{C}.$ $(\rho_1 = \mathrm{Tr}_2[\rho])$

ρ_1 & ρ_2 may have different dimensions.

What is relationship of eigenvalues?

Similarly, $\rho_2 \equiv \mathrm{Tr}_1[\rho] = \mathrm{C}^\dagger \mathrm{C}.$ $(\rho_1 = \mathrm{Tr}_2[\rho])$

ρ_1 & ρ_2 may have different dimensions.

What is relationship of eigenvalues?

Nonzero eigenvalues are same!

$$\Longrightarrow S_1 = S_2$$

Alternate definition of entropy

$$S = \ln \mathcal{N}$$

$\mathcal{N}$ = # of states (for some fixed quantities.)

What does all this have to do with black holes?

Classical Event Horizon prevents probing beyond.

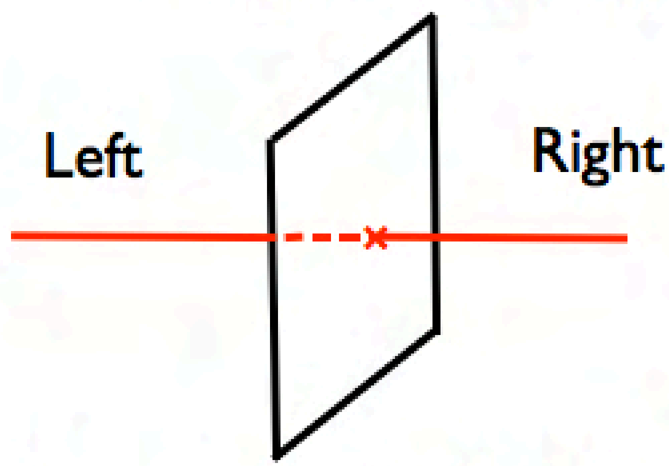
Measurements are classical. Natural split into

(observable region) ***x*** (rest of universe.)

Trace over region that cannot be observed.

Even if universe is in a pure state, an observer sees a density matrix and describes observables by a mixed state.

Volume *vs.* Area Law?



Brustein & Yarom

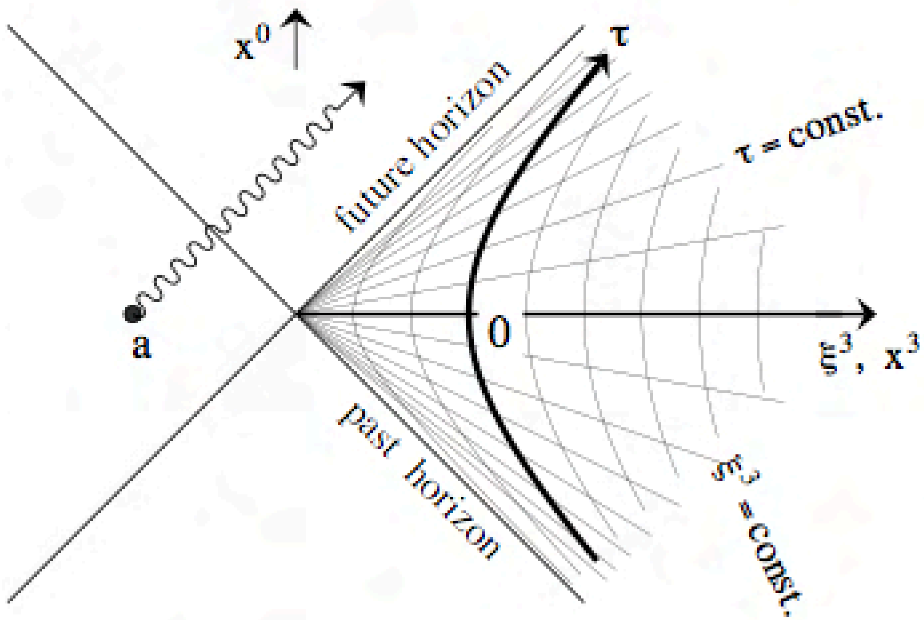
Consider scalar field in Minkowski vacuum.

Divide flat space in half.

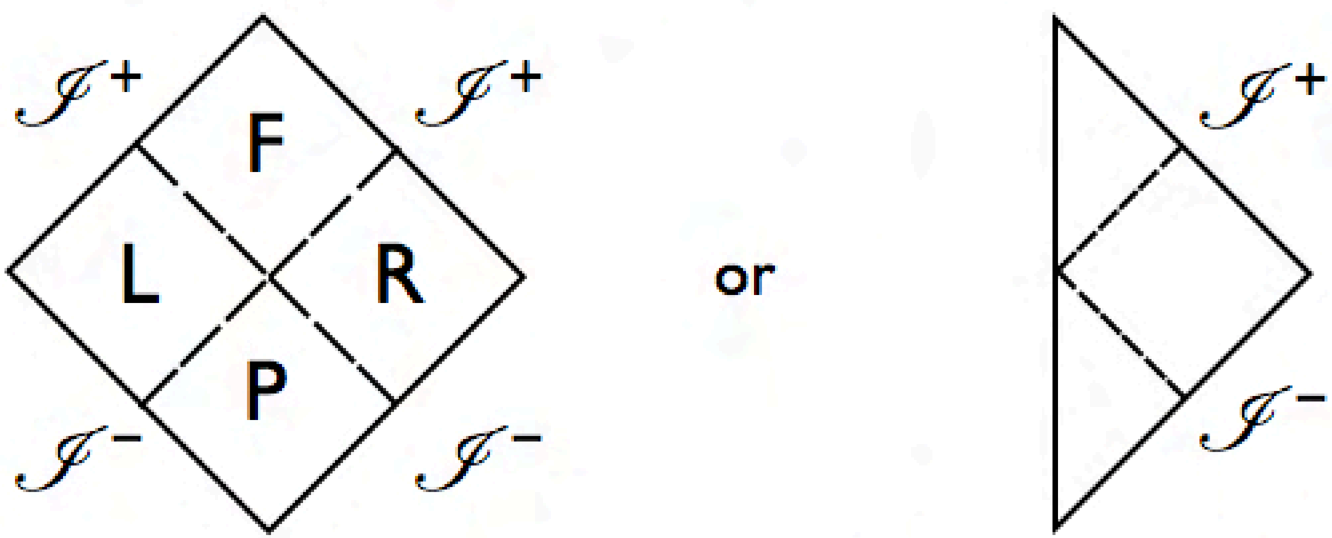
Vacuum fluctuations are correlated.

Calculate $\rho_R \propto Area$

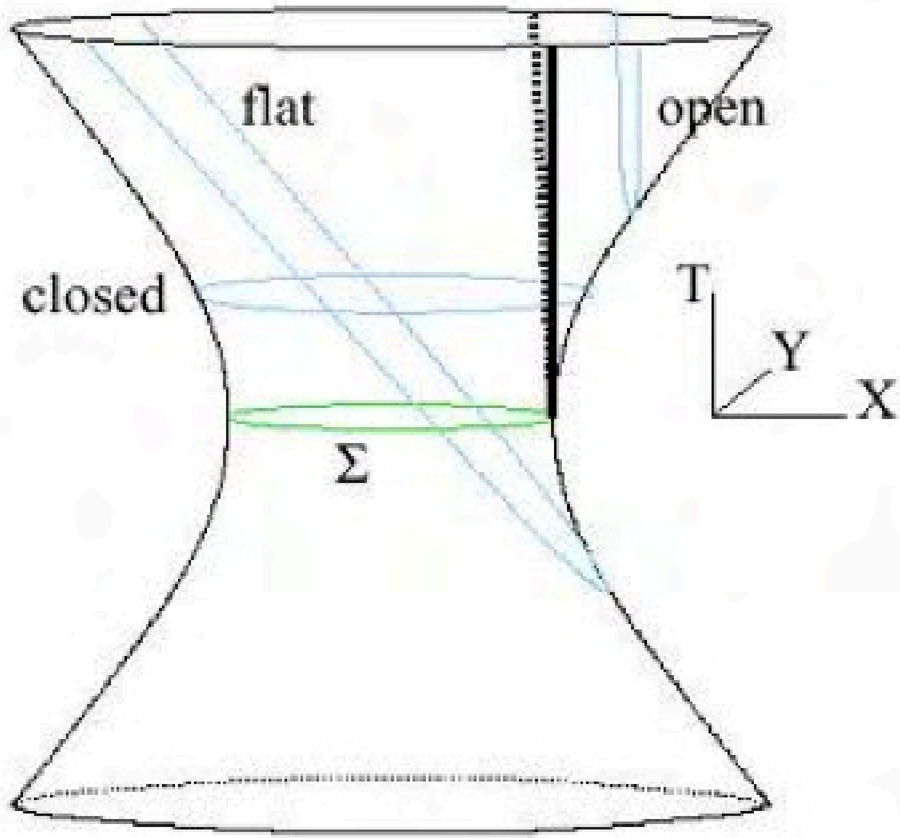
Stationary Spacetimes with Horizons



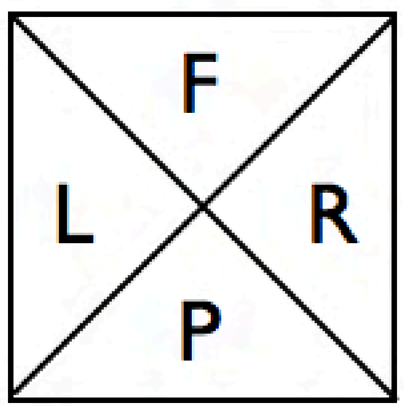
Minkowski Space vs.
Rindler Space



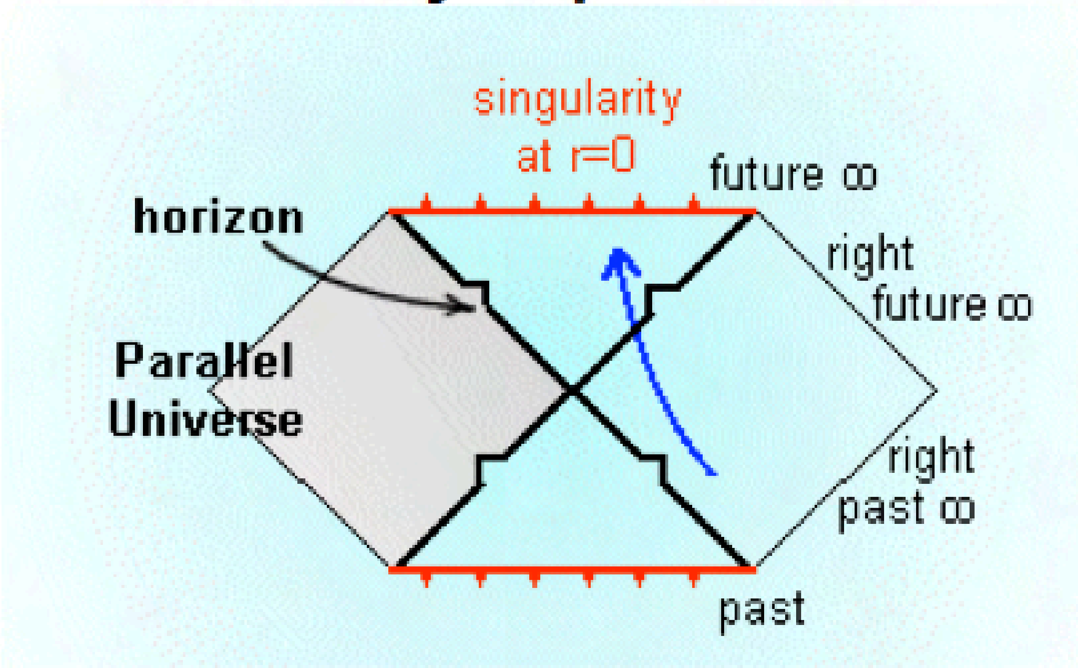
Carter-Penrose
Diagram



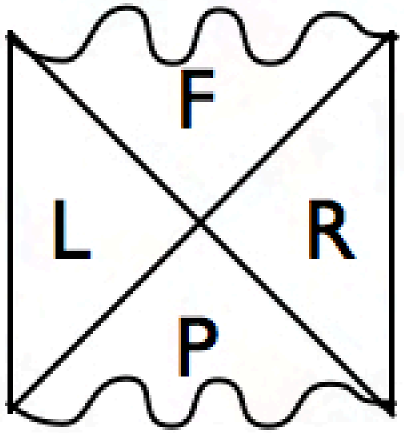
de Sitter Space



Stationary Spacetimes with Horizons-2



Schwarzschild BH



Schwarzschild BH
in anti-de Sitter space

All have similar vacuum structure.

Hartle-Hawking

Vacuum state $|0\rangle$

$$H = H_R - H_L$$

$$|0\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\frac{\beta E_i}{2}} |E_i\rangle_R |E_i\rangle_L$$

$Z = \mathrm{Tr}\exp(-\beta \mathrm{H})$ partition function

Calculate density matrix for causal region.

$$S \propto \frac{1}{G_N} Area$$

Brustein, Yarom, M.E.

Gravitational Collapse? Easier in principle.

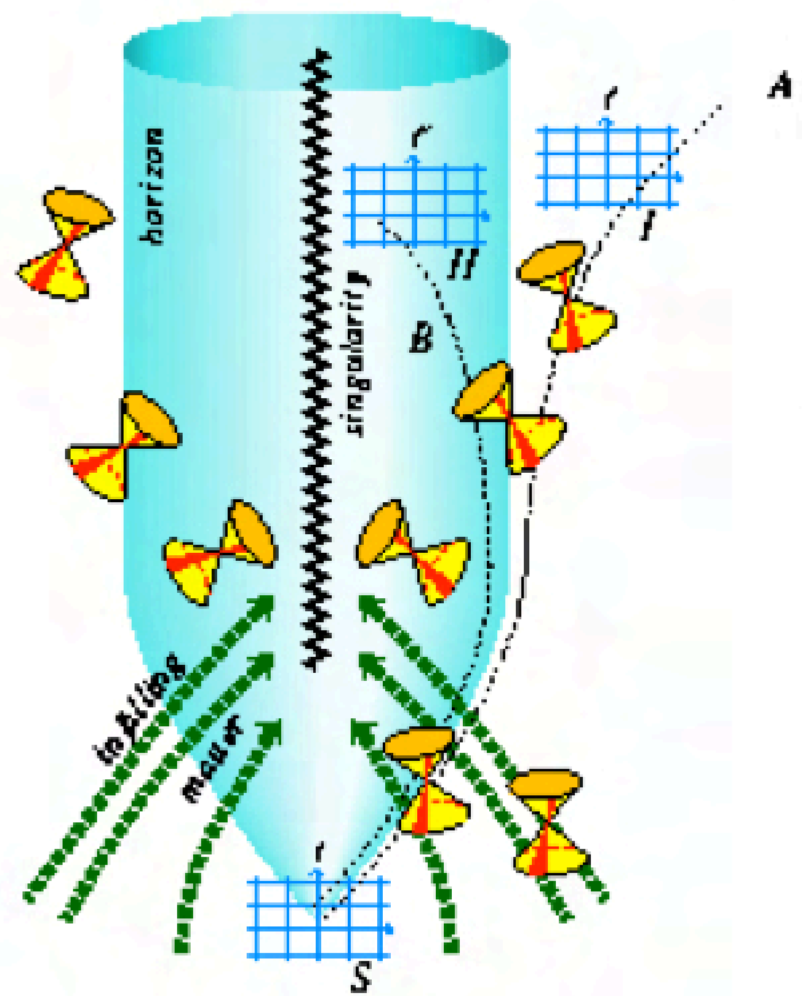
Start in pure state, no horizons.

Matter collapses; horizon forms.

Hawking radiation occurs.

Maybe BH evaporates.

No simple, solvable model yet.



If unitary evolution, then

pure state $\longrightarrow$ *pure state*

$$S_{in} = S_{out} \qquad \text{always.}$$

If BH evaporates, then $S = 0$

What happens at singularity?

Nobody knows!

Hawking's thermal state must be an illusion,
but what goes wrong?

Is there a dynamical way to calculate Hawking effect?

Somebody knows!

Is there a dynamical way to calculate Hawking effect?

Classically, light cannot escape from a BH,
but there is no locally observable horizon.

Schwarzschild BH:

Kraus–Wilczek

Choose **nonsingular** coordinates.

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + 2\sqrt{\frac{2M}{r}}\,dt\,dr + dr^2 + r^2 d\Omega^2$$

Painlevè
Gullstrand

"Stationary" but not "static."

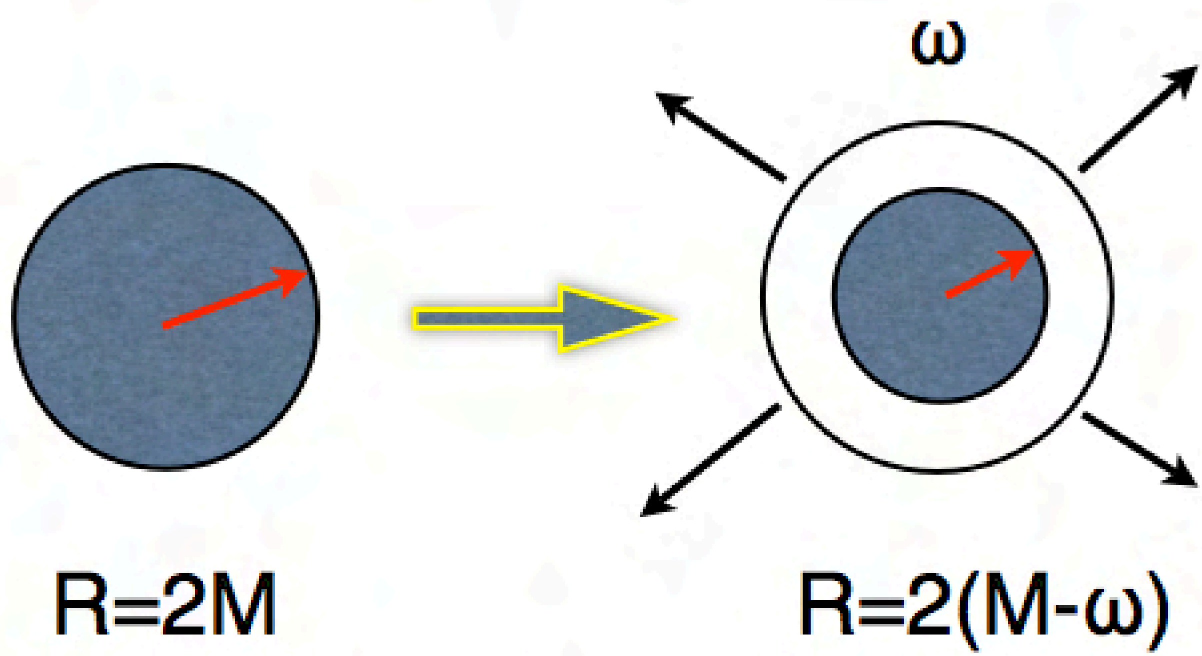
Kraus–Wilczek derived feedback on metric of particle emission.

Emit energy ω in S-wave; replace M by $M - \omega$ above.

Tunneling through the horizon.

Decay Rate $\Gamma \propto e^{-2\Im m(\mathcal{S})}$ $\mathcal{S} \equiv Action$

WKB approx.



Parikh–Wilczek

$$\Gamma \propto e^{-8\pi\omega(M-\frac{\omega}{2})} = e^{\Delta S_{BH}}$$

Small ω, agrees with Hawking

Conclusions

- Hawking Radiation consistent with QM

1. Area law not in conflict with QFT.
2. No loss of information--like burning a book.

Questions

- Nature of correlations?
- What happens at the singularity?
- Does BH evaporate?
- Entanglement can explain the "nonlocality."
- How to reconcile entanglement picture with microstate counting in string theory? (Strominger-Vafa)