STRING PHENOMENOLOGY 2004

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HOLOGRAPHY, DIFFEOMORHISMS, AND THE CMB

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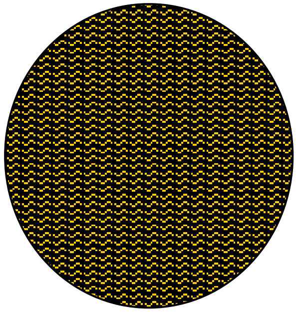
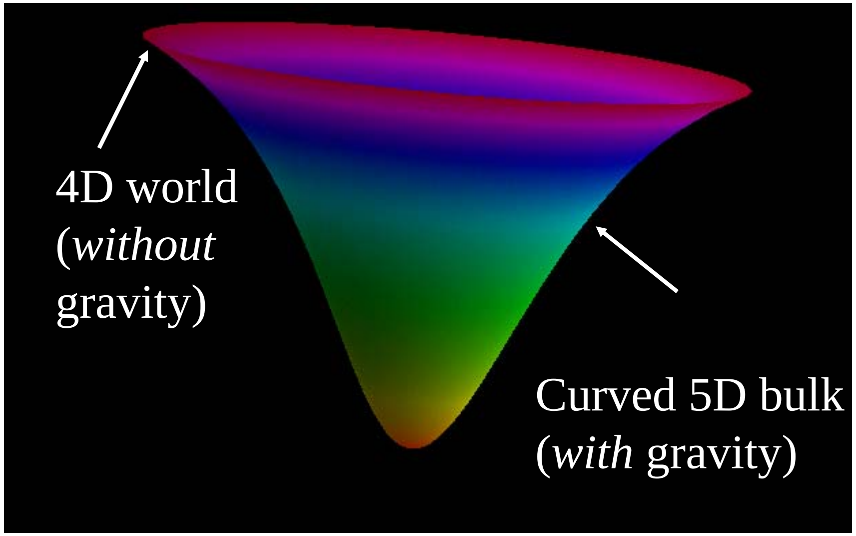


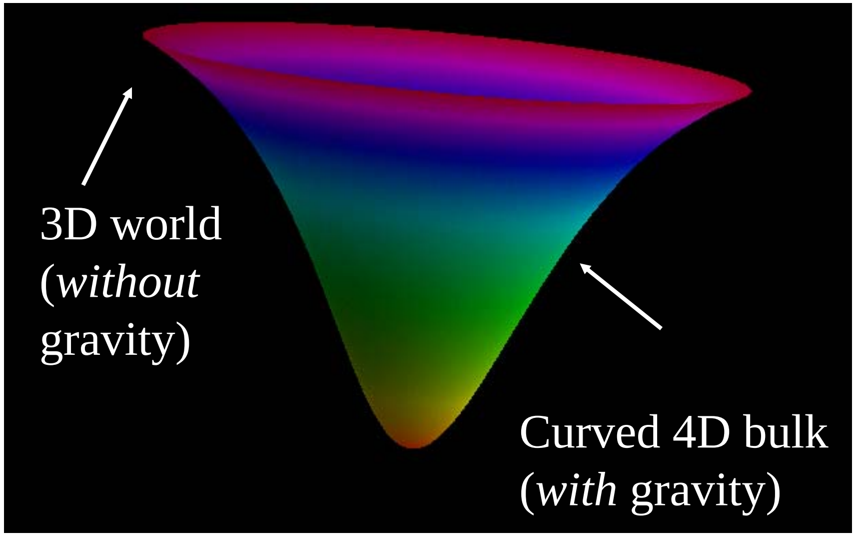
INTRODUCTION: HOLOGRAPHY

- AdS/CFT: equivalence between QFT (without gravity) and gravitational theory with (at least) one more dimension. Maldacena



- Possible relation to our world 1: our 4D (with modest gravity) mapped to 5D auxiliary geometry (with gravity). Randall+Sundrum

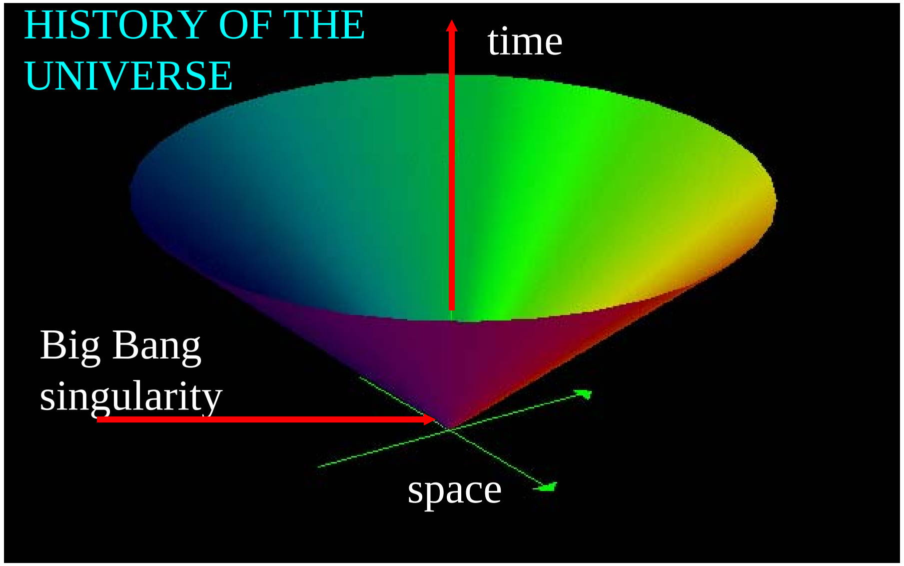
- Useful for analyzing strongly coupled QFT (map to weakly coupled gravity).



- Possible relation to our world 2: strong gravity in our 4D world represented by auxiliary 3D QFT.

RESOLVING SINGULARITIES

- Usual example: internal structure of black hole represented by QFT.
- More urgent (but confusing) example: resolve the big bang singularity.
- Hope: given late time geometry, sum over all possible early geometries; this should be regular.
- This talk: holographic representation of standard cosmological perturbation theory.



$$Z_{\text{cosmo}} = Z_{\text{FRW}} + \sum Z_{\text{quantum}}$$

All geometries, with late time cosmology fixed

OUTLINE

- ***Motivation***
- ***Some background***
- ***Boundary violations of diffeomorphism invariance and counterterms***
- ***Renormalization Group Improved Primordial Density Perturbations***
- ***Summary***

FL, R. Leigh, J.P. van der Schaar, hep-th/0202172

FL and R. McNees, hep-th/0307026, **hep-th/0402050**

THE SETTING

- Standard minimal inflationary theory

$$S = \int_{\mathcal{M}} d^4x \sqrt{g} \left(\frac{1}{16\pi G} R - \frac{1}{2} \nabla^\mu \varphi \nabla_\mu \varphi - V(\varphi) \right) - \frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^3x \sqrt{\tilde{g}}\, K$$

- Focus: fluctuations around background

$$\begin{aligned} \varphi(\vec{x},\tau) &= \varphi(\tau) + \chi(\vec{x},\tau) \\ g_{\mu\nu}(\vec{x},\tau) &= a(\tau)^2 \eta_{\mu\nu} + h_{\mu\nu}(\vec{x},\tau) \end{aligned}$$

- Background satisfies standard FRW equations

$$\varphi'' + 2\mathcal{H}\varphi' + a^2 \partial_\varphi V = 0$$

$$3\left(\frac{\mathcal{H}}{a}\right)^2 = \frac{1}{2}\left(\frac{\varphi'}{a}\right)^2 + V$$

$$\mathcal{H} = a'/a$$

THE ON-SHELL ACTION

- Want holographic description of inflationary epoch.
- In AdS/CFT the generating function of correlators in the boundary theory is given by the on-shell action of the bulk theory

$$Z_{\rm CFT}(\lambda) = Z_{\rm on-shell}(\lambda) \simeq e^{iS_{\rm on\ shell}(\lambda)}$$

Gubser,Klebanov,Polyakov
Witten

- Proposed holographic cosmology: time direction of the universe projected out, leaving 3 spatial dimensions. Wave function of the universe is

$$\Psi[\varphi(\vec{x}), \tilde{g}_{ij}(\vec{x})] \;\; = \;\; Z[\varphi(\vec{x}), \tilde{g}_{ij}(\vec{x})]$$

Maldacena

- Upshot: wave function determined by imposing the equations of motion prior that slice. It is a functional of the 3 dimensional data only. The semiclassical action is the Hamilton-Jacobi functional.

THE EFFECTIVE SCALAR FIELD

- This talk will focus on the scalar fluctuations.
- There is an obvious scalar fluctuation but also 4 components of the metric:

$$h_{\mu\nu} = a(\tau)^2 \begin{pmatrix} 2\,\Phi(\vec{x},\tau) & \partial_i B(\vec{x},\tau) \\ \partial_i B(\vec{x},\tau) & 2(\psi(\vec{x},\tau)\,\delta_{ij} - \partial_i\partial_j E(\vec{x},\tau)) \end{pmatrix}$$

- Explicit coordinate systems (gauge choices): specify 2 of the 5 scalar fields.
- Remaining 3 scalar components are related by 2 constraints

$$\begin{aligned} \psi' + \mathcal{H}\Phi + \frac{1}{2}\varphi'\chi &= 0 \\ \Phi - \mathcal{H}(B+E') - (B+E')' &= \psi + \mathcal{H}(B+E') \end{aligned}$$

Bardeen

- The five scalars then reduce to one physical scalar as expected.
- To analyze diffeomorphism invariance, keep things general for as long as possible.

QUADRATIC ACTION BEFORE GAUGE FIXING

$$
\begin{aligned}
\delta^2 S \;\; = & \int_{\mathcal{M}_0} d^4x \sqrt{g}\, \frac{1}{a^2} \left[2\,\Phi\, \vec{\partial}^2 (\psi + \mathcal{H}(B + E')) - 4\,\mathcal{H}\,\Phi \left(\psi' + \mathcal{H}\,\Phi + \frac{1}{2}\varphi'\chi \right) \right. \\
- & \; 2\,\mathcal{H}\,\Phi\,\psi' - 2\,\mathcal{H}'\,\Phi^2 + \varphi'\,\Phi\,\chi' - \varphi''\,\Phi\,\chi - 2\,\partial_i B\,\partial_i \left(\psi' + \mathcal{H}\,\Phi + \frac{1}{2}\varphi'\chi \right) \\
+ & \; 2\,(3\psi - E) \left(\frac{\partial}{\partial\tau} + 2\mathcal{H} \right) \left(\psi' + \mathcal{H}\,\Phi + \frac{1}{2}\varphi'\chi \right) \\
- & \; ((2\psi - E)\,\vec{\partial}^2 + \partial_i\partial_j E\,\partial_i\partial_j) \left(\psi - \Phi + B' + E'' + 2\,\mathcal{H}\,(B + E') \right) \\
- & \; \chi\,\chi'' - 2\,\mathcal{H}\,\chi\,\chi' - a^2\,\partial_\varphi^2 V\,\chi^2 + \chi\,\vec{\partial}^2 (\chi + \varphi'(B + E')) \\
- & \left. 2\,(\varphi'' + 2\,\mathcal{H}\,\varphi')\,\Phi\,\chi - \varphi'\,\chi\,\Phi' - 3\,\varphi'\,\chi\,\psi \; \right] \\
& + \int_{\partial\mathcal{M}_0} d^3x \sqrt{\tilde{g}} \left[\frac{1}{a}\chi\,\chi' + \frac{\varphi'}{a}\chi\,\Phi + 3\,\frac{\varphi'}{a}\chi\,\psi - \frac{\varphi'}{a}\chi\,\vec{\partial}^2 E - \frac{6}{a}\psi\,\psi' - 6\,\frac{\mathcal{H}}{a}\psi^2 \right. \\
& \quad -12\,\frac{\mathcal{H}}{a}\psi\,\vec{\partial}^2 E - 6\,\frac{\mathcal{H}}{a}\psi\,\Phi + 2\,\frac{\mathcal{H}}{a}\Phi\,\vec{\partial}^2 E + \frac{2}{a}\psi\,\vec{\partial}^2 B + \frac{\mathcal{H}}{a}\vec{\partial}^2 E\,\vec{\partial}^2 E \\
& \quad \left. + \frac{2}{a} \left(\psi'\,\vec{\partial}^2 E + \psi\,\vec{\partial}^2 E' \right) \right]
\end{aligned}
$$

GAUGE DEPENDENCE

- Now go on-shell (impose e.o.m.). Bulk terms vanish after partial integration, but some boundary action remains.
- Note: the coordinate system remains completely general; no gauge was picked.
- Surprise: the result is not gauge invariant, depends on gauge.
- Example: the on-shell Lagrangian does not vanish even for pure gauge (when the "fluctuation" is a pure coordinate artifact).

$$\begin{aligned} \Phi &= \mathcal{H}\,\delta\tau + (\delta\tau)' \\ \psi &= -\mathcal{H}\,\delta\tau \\ B &= -\varepsilon' + \delta\tau \\ E &= \varepsilon \\ \chi &= -\varphi'\,\delta\tau \end{aligned}$$

$$\begin{aligned} \delta^2_\epsilon S &= \int_{\partial\mathcal{M}_0} d^3x \sqrt{\tilde{g}}\, \frac{1}{a} \Big[(4\,\mathcal{H}^3 - 8\,\mathcal{H}\,\mathcal{H}' - \mathcal{H}'')\delta\tau^2 - 2\mathcal{H}\,\delta\tau\,\vec{\partial}^2\delta\tau + \\ &\quad + 2\,\mathcal{H}\,\vec{\partial}^2\epsilon\,\vec{\partial}^2\epsilon + (16\,\mathcal{H}^2 - 4\,\mathcal{H}')\,\delta\tau\,\vec{\partial}^2\epsilon \Big] \end{aligned}$$

BOUNDARY COUNTERTERMS

- If we add a *local* boundary term to the original Lagrangian, the bulk dynamics remain the same. The general *ansatz* is

$$S_{\mathrm{CT}}(\tau_0) = \int_{\partial\mathcal{M}_0} d^3x \sqrt{\tilde{g}} \Big(U(\varphi) + M(\varphi) \vec{D}\varphi \cdot \vec{D}\varphi + C(\varphi)\tilde{R} \Big)$$

DeBoer, 2Verlinde

- Diffeomorphism invariance of the total action is restored precisely if these counterterms satisfy

$$\begin{aligned} 0 &= \frac{3}{4}U^2 - \frac{1}{2}(\partial_\varphi U)^2 - V \\ 0 &= 1 + U\,C - 2\,\partial_\varphi U\,\partial_\varphi C \\ 0 &= 1 - U\,M - 4\partial_\varphi U\,\partial_\varphi C + 2\,\partial_\varphi U\,\partial_\varphi M + 4\partial_\varphi^2 U\,M \end{aligned}$$

- The full Lagrangian including counterterms is manifestly gauge invariant

$$S[v] = \int_{\mathcal{M}_0} d^4x \left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu v \partial_\nu v + \frac{1}{2}\frac{z''}{z}v^2 \right] + \int_{\partial\mathcal{M}_0} d^3x \left[-\frac{1}{2}\frac{z'}{z}v^2 - \frac{1}{a}M\vec{\partial}v \cdot \vec{\partial}v \right]$$

$$v = a\left(\chi - \frac{\varphi}{\mathcal{H}}\psi\right) \qquad\qquad z = a\,\frac{\varphi'}{\mathcal{H}}$$

Mukhanov's gauge invariant variable

ORIGIN OF COUNTERTERMS

- In AdS/CFT holography the boundary theory is a local QFT. The counterterms are the usual ones that cancel UV divergences.

Henningson+Skenderis
Balasubramanian+Kraus

- The divergence is IR in bulk: the volume element of AdS diverges when the boundary is taken to infinity.
- The cosmological spacetime is quasi-deSitter. The volume divergence at late times gives an IR divergence.
- In dS/CFT the counterterms arise from a hypothetical local QFT dual. Strominger
- **Our interpretation**: the introduction of a boundary violates diffeomorphism invariance. Local counterterms restore it.
- Our interpretation is intrinsic to gravity. It is natural for cut-off spacetimes (such as brane worlds).

HOLOGRAPHIC RENORMALIZATION

- Want to analyze consequences of diffeomorphism symmetry for the correlation functions in the theory. Total action is invariant

$$\delta_\epsilon S_{\rm tot} \;=\; \int d^3x \left[\delta_\epsilon \tilde{g}_{ij}\,\frac{\delta S_{\rm tot}}{\delta \tilde{g}_{ij}} + \delta_\epsilon \varphi\,\frac{\delta S_{\rm tot}}{\delta \varphi}\right] = 0$$

- Consider time reparametrization and choose a convenient gauge

$$\left(a\,\frac{\partial}{\partial a} + \beta(\varphi)\,\frac{\partial}{\partial \varphi} + 2\gamma(\varphi)\right) S^{(2)}_{\rm tot}[a,\varphi;\vec{x},\vec{y}] \;=\; 0$$

$$\beta(\varphi) \;=\; \frac{1}{\mathcal{H}}\,\frac{\partial \varphi}{\partial \tau} \qquad\qquad \gamma(\varphi) \equiv \frac{\partial \beta(\varphi)}{\partial \varphi}$$

- This is the Callan-Symanzik equation for the two point function. The scalar field is the coupling, the beta-function is given in terms of background geometry.

- Holographic interpretation: shifts in time act as Weyl rescaling on the boundary because of cosmological expansion.

SOLVING THE CS-EQUATION

- Correlation functions are related to the action through

$$\langle \tilde{\chi}_f(\vec{k})\, \tilde{\chi}_f(-\vec{k}) \rangle \;=\; \int \mathcal{D}\chi_f[\vec{q}]\;\; \chi_f(\vec{k})\chi_f(-\vec{k})\; |\Psi[\chi_f(\vec{q})]|^2 = \frac{1}{2\mathrm{Im}\tilde{S}^{(2)}_{\mathrm{tot}}[a,\varphi,k]}$$

- The corresponding CS equation can be solved by introducing the running coupling

$$\beta(\varphi)\frac{\partial\bar{\varphi}}{\partial\varphi} \;=\; \beta(\bar{\varphi})$$

- The two form correlation function takes the form

$$\langle \tilde{\chi}_f(\vec{k})\, \tilde{\chi}_f(-\vec{k}) \rangle \;=\; \frac{1}{2k^3\tilde{F}_0\left(\bar{\varphi}\left(k/a,\varphi\right)\right)}\exp\left[-\int_{aM}^{k} d\log\left(\frac{k'}{aM}\right)\;2\,\gamma(\bar{\varphi}(k'/a,\varphi))\right]$$

- The method describes the scaling effectively, but the overall amplitude F requires a separate computation. This is analogous to the standard RG.
- Our result resums large logarithms. It is more accurate for models with structure.

CMB POWER SPECTRUM

- It is conventional to present results in terms of the power spectrum

$$P_s(k) \;=\; \frac{k^3}{2\pi^2}\,\langle \tilde{\psi}_{\rm com}(\vec{k})\,\tilde{\psi}_{\rm com}(-\vec{k})\rangle \quad \propto \quad \left(\frac{k}{aH}\right)^{n_s-1}$$

- Our result for the power spectrum

$$P_s(k) \;=\; \frac{H(\varphi)^2}{(2\pi\beta(\varphi))^2}\,\mathcal{A}_s(\bar{\varphi}(k/a,\varphi))\exp\left(-\int_{aM}^{k} d\log(\frac{k'}{aM})\left(\beta(\bar{\varphi}(k'/a,\varphi))^2 + 2\,\gamma(\bar{\varphi}(k'/a,\varphi))\right)\right)$$

- gives the spectral index

$$n_s - 1 \;=\; k\frac{\partial}{\partial k}\log P_s(k) = -\beta(\varphi)^2 - 2\,\gamma(\varphi) \;=\; 1 - 4\,\bar{\epsilon} + 2\,\bar{\eta}$$

- This expression agrees with standard inflationary theory to linear order.
- Scaling violations in the CMB are *very* similar to the familiar ones in QFT.

OPEN QUESTIONS

Cf. Silverstein

- Does a holographic dual description of inflation exist?
- If so, cosmological evolution (change of scale) is an RG-flow and the inflationary epoch is governed by some IR-fixed point.
- Is this useful for addressing fine-tuning in inflation?
- Does it resolve the cosmological singularity?
- Does it single out inflationary models with specific signatures?

Certainly red spectrum $n_s<1$
Perhaps near-vanishing tensor amplitudes

SUMMARY

- Diffeomorphism invariance is not automatic when there is a boundary term: counterterms are needed.
- The counterterms are determined explictly; the quadratic action for fluctuations in the presence of a boundary follows.
- CMB is interpreted as scaling violations in a boundary theory.
- Standard gauge invariant perturbation theory in cosmology and standard holography (adS/CFT) have much in common!