## Neutrino masses respecting string constraints



- Introduction
- Neutrino preliminaries
- The GUT seesaw
- Neutrinos in string constructions
- The triplet model
(Work in progress, in collaboration with J. Giedt, G. Kane, B. Nelson.)


## Neutrino mass

- Nonzero mass may be first break with standard model
- Enormous theoretical effort: GUT, family symmetries, bottom up
- Majorana masses may be favored because not forbidden by SM gauge symmetries
- GUT seesaw (heavy Majorana singlet). Usually ordinary hierarchy.
- Higgs triplets ("type II seesaw"), often assuming GUT, LeftRight relations
- Very little work from string constructions, even though probably Planck scale
- Key ingredients of most bottom up models forbidden in known constructions (heterotic or intersecting brane)
- Large representations difficult to achieve (bifundamentals, singlets, or adjoints)
- String symmetries/constraints restrict couplings, e.g., diagonal Majorana masses
- Very nonstandard triplet or singlet seesaws, favoring inverted hierarchy, extended seesaw, or small Dirac masses from HDO.
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## Models and spectra

- Weyl fermion
- Minimal (two-component) fermionic degree of freedom
- $\psi_{L} \leftrightarrow \psi_{R}^{c}$ by CPT
- Active Neutrino (a.k.a. ordinary, doublet)
- in $S U(2)$ doublet with charged lepton $\rightarrow$ normal weak interactions
- $\nu_{L} \leftrightarrow \nu_{R}^{c}$ by CPT
- Sterile Neutrino (a.k.a. singlet, right-handed)
- $S U(2)$ singlet; no interactions except by mixing, Higgs, or BSM
- $N_{R} \leftrightarrow N_{L}^{c}$ by CPT
- Almost always present: Are they light? Do they mix?
- Dirac Mass
- Connects distinct Weyl spinors (usually active to sterile): $\left(m_{D} \bar{\nu}_{L} N_{R}+h . c.\right)$
- 4 components, $\Delta L=0$
$-\Delta I=\frac{1}{2} \rightarrow$ Higgs doublet
- Why small? LED? HDO?
- Variant: couple active to antiactive, e.g., $m_{D} \bar{\nu}_{e L} \nu_{\mu R}^{c} \Rightarrow L_{e}-$ $L_{\mu}$ conserved; $\Delta I=1$

$$
\begin{array}{c|c}
\nu_{L} & v=\langle\phi\rangle \\
\boldsymbol{h} & \boldsymbol{v} \\
\boldsymbol{N}_{\boldsymbol{R}} & \boldsymbol{m}_{\boldsymbol{D}}=\boldsymbol{h} \boldsymbol{v}
\end{array}
$$

- Majorana Mass
- Connects Weyl spinor with itself: $\frac{1}{2}\left(m_{T} \bar{\nu}_{L} \nu_{R}^{c}+h . c.\right)$ (active); $\frac{1}{2}\left(m_{S} \bar{N}_{L}^{c} N_{R}+h . c.\right)$ (sterile)
- 2 components, $\Delta L= \pm 2$
- Active: $\Delta I=1 \rightarrow$ triplet or seesaw
- Sterile: $\Delta I=0 \rightarrow$ singlet or bare mass

- Mixed Masses
- Majorana and Dirac mass terms
- Seesaw for $m_{S} \gg m_{D}$
- Ordinary-sterile mixing for $m_{S}$ and $m_{D}$ both small and comparable (or $m_{S} \ll m_{d}$ (pseudo-Dirac))


## $3 \nu$ Patterns

- Solar: LMA (SNO,
Kamland)
$-\Delta m_{\odot}^{2} \sim 8 \times 10^{-5} \mathrm{eV}^{2}$, nonmaximal
- Atmospheric:
$\left|\Delta m_{\text {Atm }}^{2}\right| \sim 2 \times 10^{-3}$ $\mathrm{eV}^{2}$, near-maximal mixing
- Reactor: $U_{e 3}$ small

- Mixings: let $\nu_{ \pm} \equiv \frac{1}{\sqrt{2}}\left(\nu_{\mu} \pm \nu_{\tau}\right)$ :

$$
\begin{aligned}
\nu_{3} & \sim \nu_{+} \\
\nu_{2} & \sim \cos \theta_{\odot} \nu_{-}-\sin \theta_{\odot} \nu_{e} \\
\nu_{1} & \sim \sin \theta_{\odot} \nu_{-}+\cos \theta_{\odot} \nu_{e}
\end{aligned}
$$



- Hierarchical pattern
* Analogous to quarks, charged leptons
* $\beta \beta_{0 \nu}$ rate very small
- Inverted quasi-degenerate pattern
* $\boldsymbol{\beta} \beta_{0 \nu}$ if Majorana
* SN1987A energetics (if $U_{e 3} \neq 0$ )?
* May be radiative unstable


## The GUT Seesaw

- Elegant mechanism for small Majorana masses
- Leptogenesis
- Expect small mixings in simplest versions (can evade by lopsided $e / d$, Majorana textures, etc.)
- Large Majorana often forbidden, e.g., by extra $U(1)$ 's
- Direct Majorana masses and large scales forbidden in some string constructions
- GUTs, adjoint Higgs, large Higgs hard to accomodate in simplest heterotic constructions
- LSND: active-sterile difficult in simple versions
- Therefore, explore alternatives, e.g., with small Dirac and/or Majorana masses
- Small Majorana from loops, $\boldsymbol{R}_{p}$ violation, TeV seesaw, or triplet
- Small Dirac from large extra dimension or by higher dimensional operators in intermediate scale models (e.g. $\left.U(1)^{\prime}\right)$
- Variant ordinary and triplet seesaws motivated by string constructions


## Neutrinos in string constructions

Key ingredients of most GUT/bottom up models forbidden or different in known constructions (heterotic or intersecting brane)

- Bifundamentals, singlets, or adjoints; not large representations
- String symmetries/constraints may forbid couplings allowed by 4d symmetries
- Diagonal superpotential terms (e.g., diagonal Majorana masses) usually absent
- GUT Yukawa relations broken
- Non-zero superpotential terms may be equal (gauge couplings)
- Hierarchies from HDO (heterotic), intersection triangles (intersecting brane)


## Dirac masses

- Can achieve small Dirac masses (neutrino or other) by higher dimensional operators or by large intersection areas

$$
\begin{gathered}
L_{\nu} \sim\left(\frac{S}{M_{P l}}\right)^{p} L N_{L}^{c} H_{2}, \quad\langle S\rangle \ll M_{P l} \\
\Rightarrow m_{D} \sim\left(\frac{\langle S\rangle}{M_{P l}}\right)^{p}\left\langle H_{2}\right\rangle
\end{gathered}
$$

- Large $p \Rightarrow\langle S\rangle$ close to $M_{P l}$ (e.g., anomalous $\left.U(1)^{\prime}\right)$
- Small $p \Rightarrow$ intermediate scale $\ll M_{P l}$
- Intermediate scale in (non-anomalous) $U(1)^{\prime}$ from $D$ and (almost) $F$ flat direction:

Two SM singlets charged under $U(1)^{\prime}$. If no $F$ terms,

$$
V\left(S_{1}, S_{2}\right)=m_{1}^{2}\left|S_{1}^{2}\right|+m_{2}^{2}\left|S_{2}^{2}\right|+\frac{g^{\prime 2} Q^{\prime 2}}{2}\left(\left|S_{1}^{2}\right|-\left|S_{2}^{2}\right|\right)^{2}
$$

Break at EW scale for $m_{1}^{2}+m_{2}^{2}>0$, at intermediate scale for $m_{1}^{2}+m_{2}^{2}<0$ (stabilized by loops or HDO)

## The ordinary seesaw

- Active neutrinos $\nu_{L}, N_{R}$ (3 flavors each)

$$
L=\frac{1}{2}\left(\begin{array}{ll}
\bar{\nu}_{L} & \bar{N}_{L}^{c}
\end{array}\right)\left(\begin{array}{cc}
\mathrm{m}_{T} & \mathrm{~m}_{D} \\
\mathrm{~m}_{D}^{T} & \mathrm{~m}_{S}
\end{array}\right)\binom{\nu_{R}^{c}}{\boldsymbol{N}_{R}}+\mathrm{hc}
$$

- $m_{T}=m_{T}^{T}=$ triplet Majorana mass matrix (Higgs triplet)
- $m_{D}=$ Dirac mass matrix (Higgs doublet)
- $m_{S}=m_{S}^{T}=$ singlet Majorana mass matrix (Higgs singlet); eg, 126 of $S O(10)$
- Ordinary (type I) seesaw: $m_{T}=0$ and (eigenvalues) $m_{S} \gg m_{D}$ :

$$
m_{\nu}^{\mathrm{eff}}=-m_{D} m_{S}^{-1} m_{D}^{T}
$$

with

$$
U_{P M N S}=U_{e}^{\dagger} U_{\nu}
$$

- Most models assume either
- $U_{e} \sim I$ in basis with manifest symmetries for $m_{D, S} \Rightarrow$ large mixings in $U_{\nu}$
- Large $U_{e}$ mixings from lopsided $m_{e}$ in basis with $m_{D, S} \sim$ diagonal (harder to achieve in $S O(10)$ than $S U(5)$ )
- $S O(10)$ models usually yield ordinary hierarchy
- String constructions: may be able to generate large effective $m_{S}$ from

$$
W_{\nu} \sim c_{i j} \frac{S^{q+1}}{M_{P l}^{q}} N_{i} N_{j} \quad \Rightarrow\left(m_{S}\right)_{i j} \sim c_{i j} \frac{\langle S\rangle^{q+1}}{M_{P l}^{q}}
$$

- Can one have such terms simultaneously with Dirac couplings, consistent with flatness and other constraints? (Under investigation for $Z_{3}$ orbifold.)
- $c_{i i}=0$ in all known examples $\Rightarrow$

$$
m_{S}=\left(\begin{array}{ccc}
0 & m_{12} & m_{13} \\
m_{12} & 0 & m_{23} \\
m_{13} & m_{23} & 0
\end{array}\right)
$$

- Very different from standard seesaw textures
- Case with three large eigenvalues requires complicated $m_{D}$ and/or $m_{e}$
- $2 \times 2$ case could resemble special pseudo-Dirac inverse hierarchy model found for triplets
- Extended seesaw with greater than 3 N fields? (Coriano, Faraggi; F., Thormeier)


## Triplet models

- Introduce Higgs triplet $T=\left(T^{++} T^{+} T^{0}\right)^{T}$ with weak hypercharge $Y=1$
- Majorana masses $m_{T}$ generated from $L_{\nu}=\lambda_{i j}^{T} L_{i} T L_{j}$ if $\left\langle T^{0}\right\rangle \neq 0$
- Old Roncadelli-Gelmini model: $\left\langle T^{0}\right\rangle \ll$ EW scale with explicit $L$ violation
- Excluded by $Z \rightarrow$ Majoron + scalar (equivalent to $\Delta N_{\nu}=2$ )
- Modern triplet models (type II seesaw) break $L$ explicity by THH couplings, giving large Majoron mass (Lazarides, Shafi, Wetterich, Mohapatra, Senjanovic, Schechter, Valle, Ma, Hambye, Sarkar, Rossi, ...)
- Often considered in $S O(10)$ or LR context, with both ordinary and triplet mechanisms competing and with related parameters, but can consider independently.
- General SUSY case

$$
\begin{aligned}
W_{\nu}= & \lambda_{i j}^{T} L_{i} T L_{j}+\lambda_{1} H_{1} T H_{1}+\lambda_{2} H_{2} \bar{T} H_{2} \\
& +M_{T} T \bar{T}+\mu H_{1} H_{2}
\end{aligned}
$$

$T, \bar{T}$ are triplets with $Y= \pm 1, M_{T} \sim 10^{12}-10^{14} \mathrm{GeV}$. Typically,

$$
\begin{gathered}
\left\langle T^{0}\right\rangle \sim-\lambda\left\langle H_{2}^{0}\right\rangle^{2} / m_{T} \Rightarrow \\
\mathrm{~m}_{i j}^{\nu}=-\lambda_{i j}^{T} \lambda_{2} \frac{v_{2}^{2}}{M_{T}}
\end{gathered}
$$

## String constructions

- Expect $\lambda_{i j}^{T}=0$ for $i=j$ (off-diagonal) $\Rightarrow m_{i i}^{\nu}=0$
- Also, need multiple Higgs doublets $H_{1,2}$ with $\lambda_{1,2}$ off diagonal
- Partial explanation: $S U(2)$ triplet with $Y \neq 0$ requires higher level embedding, e.g., of $S U(2) \subset S U(2) \times S U(2)$ (Have $Z_{3}$ constructions with some but not all of the features.)

$$
W \sim \lambda_{1 j}^{T} L_{1}(2,1) T(2,2) L_{j}(1,2), j=2,3
$$

yields

$$
m^{\nu}=\left(\begin{array}{ccc}
0 & a & b \\
a & 0 & 0 \\
b & 0 & 0
\end{array}\right)
$$

- Typical string case: $|a|=|b|$
- HDO (or $S U(2) \subset S U(2) \times S U(2) \times S U(2))$ can give $m_{23}^{\nu} \neq 0$
- For

$$
m^{\nu}=\left(\begin{array}{ccc}
0 & a & b \\
a & 0 & c \\
b & c & 0
\end{array}\right)
$$

can take $a, b, c$ real w.l.o.g. by redefinition of fields (not true for general $m^{\nu}$ )

- $\operatorname{Tr} m^{\nu}=0$ and $m^{\nu}=m^{\nu \dagger} \Rightarrow m_{1}+m_{2}+m_{3}=0$
- $\left|\Delta m_{\text {Atm }}^{2}\right| \sim 2 \times 10^{-3} \mathrm{eV}^{2}, \Delta m_{\odot}^{2} \sim 8 \times 10^{-5} \mathrm{eV}^{2} \Rightarrow$ two solutions
- For $\Delta m_{\odot}^{2}=0$
(a) $m_{i} \propto 1,-\frac{1}{2},-\frac{1}{2}$ (ordinary, with shifted masses)
(b) $m_{i} \propto 1,-1,0$ (inverted)
- With $\Delta m_{\odot}^{2} \neq 0$
(a) $m_{i}=0.054,-0.026,-0.026 \mathrm{eV}\left(\sum\left|m_{i}\right|=0.107 \mathrm{eV}\right.$ (cosmology))
(b) $m_{i}=0.046,-0.045,-0.001 \mathrm{eV}\left(\sum\left|m_{i}\right|=0.092 \mathrm{eV}\right.$ (cosmology))

$$
m_{a}^{\nu} \sim\left(\begin{array}{ccc}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{array}\right) \quad m_{b}^{\nu} \sim\left(\begin{array}{ccc}
0 & a & b \\
a & 0 & 0 \\
b & 0 & 0
\end{array}\right)
$$

- (a) leads to unrealistic mixing matrix $\Rightarrow$ consider (b)


## A special texture

- The $L_{e}-L_{\mu}-L_{\tau}$ conserving texture

$$
m^{\nu} \sim\left(\begin{array}{ccc}
0 & a & b \\
a & 0 & 0 \\
b & 0 & 0
\end{array}\right)
$$

has been considered phenomenologically by many authors (Zee; Barbieri, Hall, Smith, Strumia, Weiner; King, Singh; Ohlsson; Barbieri, Hambye, Romanino; Lebed, Martin; Babu, Mohapatra; Lavignac, Masina, Savoy; Feruglio, Strumia, Vissani; Altarelli, Feruglio, Masina)

$$
m^{\nu} \sim\left(\begin{array}{ccc}
0 & a & b \\
a & 0 & 0 \\
b & 0 & 0
\end{array}\right)
$$

- New aspects
- Strong string motivation
- Motivation for special case $|a|=|b|$
- Most likely perturbation in 23 element from HOT
- Diagonalization: $\tan \theta_{\mathrm{Atm}}=b / a \Rightarrow$ need $|b|=|a|$ for maximal
- $\tan ^{2} \theta_{\odot}=1$ (maximal) (experiment $\tan ^{2} \theta_{\odot}=0.40_{-0.07}^{+0.09}$ )
- Majorana mass matrix

$$
m^{\nu} \sim\left(\begin{array}{ccc}
0 & 1 & -1 \\
1 & 0 & 0 \\
-1 & 0 & 0
\end{array}\right)
$$

- Inverted hierarchy
- Bimaximal mixing for $U_{e}=I$ :

$$
U_{\nu} \sim\left(\begin{array}{rrr}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}}
\end{array}\right)
$$

- Perturbations on $m^{\nu}$ cannot give both $\Delta m_{\odot}^{2}$ and $\frac{\pi}{4}-\theta_{\odot} \sim \theta_{C} \sim$ 0.23 without fine-tuning between terms, e.g.,

$$
\frac{1}{4 \sqrt{2}} \frac{\Delta m_{\odot}^{2}}{\Delta m_{\mathrm{Atm}}^{2}}=-\frac{\epsilon_{23}}{4} \sim 0.007 \neq \frac{\pi}{4}-\theta_{\odot} \sim 0.23
$$

- However, $U_{e} \neq I$ with small angles (comparable to CKM) can can give agreement with experiment (Frampton, Petcov, Rodejohann; Romanino; Altarelli, Feruglio, Masina)

$$
U_{e}^{\dagger} \sim\left(\begin{array}{ccc}
1 & -s_{12}^{e} & 0 \\
s_{12}^{e} & 1 & 0 \\
0 & 0 & 1
\end{array}\right)
$$

yields

$$
\begin{aligned}
\theta_{\odot} & \sim \frac{\pi}{4}-\frac{s_{12}^{e}}{\sqrt{2}}=0.56_{-0.04}^{+0.05} \\
\left|U_{e 3}\right|^{2} & \sim \frac{\left(s_{12}^{e}\right)^{2}}{2} \sim(0.023-0.081), 90 \%(\exp :<0.03) \\
m_{\beta \beta} & \sim m_{2}\left(\cos ^{2} \theta_{\odot}-\sin ^{2} \theta_{\odot}\right) \sim 0.020 \mathrm{eV}
\end{aligned}
$$

## In progress

- Detailed $Z_{3}$ constructions for higher level embeddings (triplets) and for heavy Majorana neutrinos
- Implications for $m_{e}, m_{q}$
- Implications of additional Higgs
- RGE effects
- Leptogenesis


## Conclusions

- Neutrino mass likely due to large or Planck scale effects, but little work in string context
- Specific orbifold string constructions (heterotic, intersecting brane) not consistent with common GUT and bottom up assumptions for $m_{\nu}$
- Preliminary conclusion: inverted hierarchy (pseudo Dirac), extended seesaw, or small Dirac favored
- Inverted hierarchy (e.g., from triplet) very predictive

