Constructing 5D orbifold GUTs from heterotic strings

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Outline

- 3 family orbifold GUT on $\mathcal{M}_4 \times S_1/(Z_2 \times Z_2')$

$\Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow \quad \Leftrightarrow$

- heterotic string compactified on $[T_2]^3/Z_6$ + Wilson lines
- $E(6)$ example

  Orbifold breaking E(6) to PS

  PS $\rightarrow$ SM via Higgs

  3rd family in bulk

  1st & 2nd families on SO(10) fixed pts.

  Fermion masses

  Gauge coupling unification & proton decay

- Conclusions

SUSY E(6) on $\mathcal{M}_4 \times S_1/(Z_2 \times Z_2')$
$M_c = (\pi R)^{-1} \ll M_*$

- E(6) breaks to PS by orbifold parities

$$\begin{array}{c} \quad\quad P \quad\quad\quad\quad\quad P' \\ \text{E}(6) \rightarrow \text{SO}(10) \rightarrow \text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \; [= \text{PS}] \end{array}$$

- Gauge and hypermultiplet in bulk –

$$(V,\ \Sigma)\ [\mathbf{78}]\ +\ (\mathbf{27} \oplus \overline{\mathbf{27}})$$

Consider (+ +) modes $\Longrightarrow$ 3rd family and Higgs

$$F_3^c = (\bar{4}, 1, 2), \quad F_3 = (4, 2, 1), \quad \mathcal{H} = (1, 2, 2)$$

$V = \mathbf{78} \rightarrow \mathbf{45} \rightarrow$ adjoint of PS

$\Sigma = \mathbf{78} \rightarrow (\ \mathbf{16} \oplus \overline{\mathbf{16}}\ ) \rightarrow F_3^c + \bar{\chi}^c$

$\mathbf{27} \rightarrow \mathbf{16} \rightarrow F_3$

$\overline{\mathbf{27}} \rightarrow \mathbf{10} \rightarrow \mathcal{H}$

3rd family Yukawa unification
$\lambda_t = \lambda_b = \lambda_\tau = g \equiv \sqrt{4\pi\alpha_G}$

$$g_5\ \overline{\mathbf{27}}\ \Sigma\ \mathbf{27} \rightarrow g\ \mathcal{H}\ F_3^c\ F_3$$

$$g = g_5\ \sqrt{M_c}$$

PS breaks to SM

- PS breaking to SM by Higgs vevs

$$\chi^c = (\bar{4}, 1, 2),\ \bar{\chi}^c = (4, 1, 2)$$

$$\langle \chi^c \rangle = \langle \bar{\chi}^c \rangle = M_b$$

- Need additional bulk states – $3\ (\ \mathbf{27} \oplus \overline{\mathbf{27}}\ )$

$3\ (\ \mathbf{27} \oplus \overline{\mathbf{27}}\ ) \rightarrow\ \ 2\ (\mathbf{16}) \oplus \overline{\mathbf{16}} \rightarrow\ \ 2\ (\chi^c) + \bar{\chi}^c$

Now total $\Longrightarrow\ \ 2(\chi^c + \bar{\chi}^c)$

- Add $\mathrm{T} = (6,\ 1,\ 1)$

$W = \chi^c\ \chi^c\ T\ +\ \bar{\chi}^c\ \bar{\chi}^c\ T$

Gives mass to color triplets and breaks PS to SM

1st & 2nd family ?

5th Dimension

SO(10) brane E(6) bulk SU(6)× SU(2) brane

[————————————————————————]

0 πR

- In bulk or on SU(6)× SU(2) brane — $M_c < M_G$ *

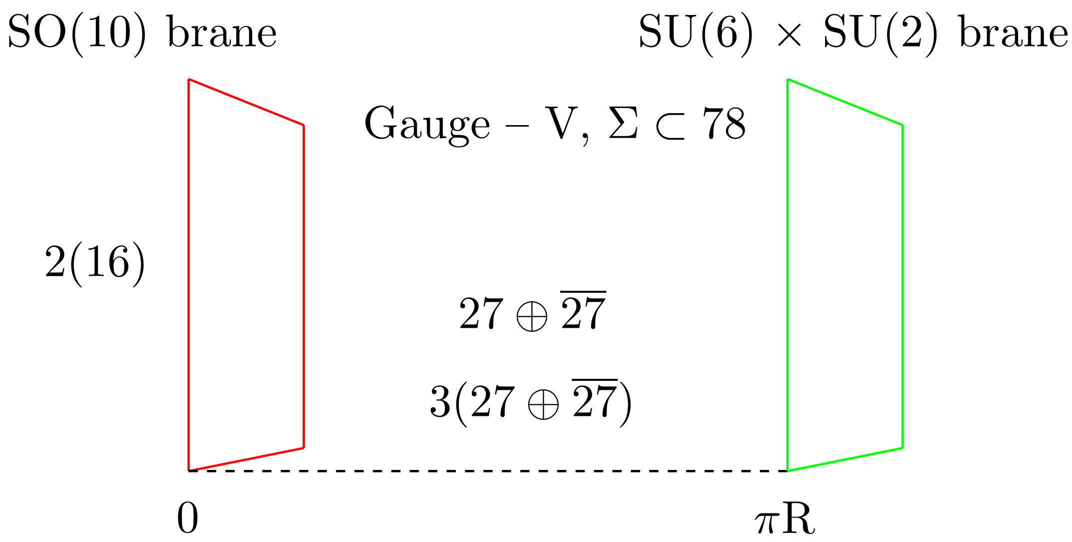
On SU(6)× SU(2) brane, one family $\in$ $[(15,1) \oplus (\bar{6},2)]$

- On SO(10) brane — $M_c \geq M_G$ *

* NO problem with proton decay !

♢ **In string theory, don't get to choose** *easily*

Summary
E(6) orbifold GUT



- 3rd family and Higgs

$\Sigma \rightarrow 16 \oplus \overline{16} \rightarrow (\bar{4},1,2)[= F_3^c] \oplus (4,1,2)[= \bar{\chi}^c]$

$27 \rightarrow 16 \rightarrow (4,2,1)[= F_3]$

$\overline{27} \rightarrow 10 \rightarrow (1,2,2)[= \mathcal{H}]$

- Yukawa unification

$g_5 \sqrt{M_c}\ (\overline{27}\ \Sigma\ 27) \quad \rightarrow \quad g\ (\mathcal{H}\ F_3^c\ F_3)$

$\Longrightarrow \lambda_t = \lambda_b = \lambda_\tau = g = \sqrt{4\pi\alpha_G}$

- PS breaking sector in blue

$3(27 \oplus \overline{27}) \rightarrow 2(16) \oplus \overline{16} \rightarrow 2(\bar{4},1,2)[= 2\ \chi^c] \oplus (4,1,2)[= \bar{\chi}^c]$

Heterotic string compactified on $[T_2]^3/Z_6$

$[T_2]^3 = G(2) \otimes SU(3) \otimes SO(4)$ root lattice

$Z_6 = Z_2 \otimes Z_3$

Orbifold defined by rotation on torus
$\mathbf{Z^i} \to e^{2\pi i v_i}\ \mathbf{Z^i}\ \ (i = 1, 2, 3)$

$v_3 = \frac{1}{3}\ (1, -1, 0), \quad v_2 = \frac{1}{2}\ (1, 0, -1)$

Rotations embedded in $E(8) \times E(8)$ root lattice via shift vectors + Wilson lines

Satisfy NON-TRIVIAL constraints of modular invariance !
(See R.-J. Zhang talk for more details)

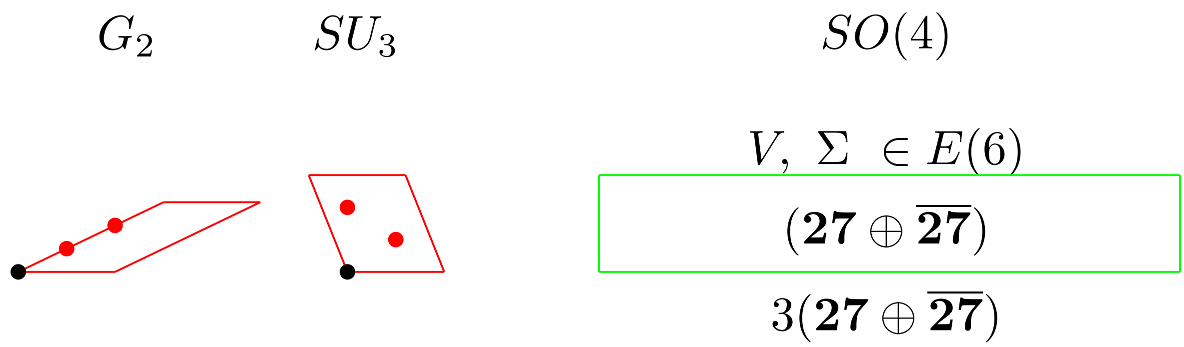
Consider first $[T_2]^3/Z_3$ +
W_3 in SU(3) torus
SO(4) torus $R^{-1} \gg l_s = M_*^{-1}$

$v_3 = \frac{1}{3}\ (1, -1, 0)$

$V_3 = \frac{1}{3}\ (2, 2, 2, 0, 0, 0, 0, 0) \oplus (\cdots)$

$W_3 = \frac{1}{3}\ (1, -1, 0, 0, 0, 0, 0, 0) \oplus (\cdots)$

$\Longrightarrow \quad N = 2$ SUSY in 5D bulk



$G_2 \oplus SU(3) \oplus SO(4)$ lattice with Z_3 fixed points. The fields $V,\ \Sigma,$ and $(\mathbf{27} \in U_1 \oplus \overline{\mathbf{27}} \in U_2)$ are bulk states from the untwisted sector. On the other hand, $3 \times (\mathbf{27} \oplus \overline{\mathbf{27}})$ are "bulk" states located on the $T_{(0,1)}/T_{(0,2)}$ twisted sector (G_2, SU(3)) fixed points.

Applying Z_2 orbifold + Wilson line in 5th Dimension

Breaks N = 2 to N = 1 SUSY

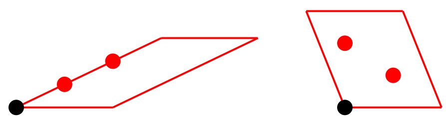
Defines parities $P, \ P'$

$$P \Leftrightarrow (v_6; V_6; W_3) \qquad\qquad P' \Leftrightarrow (v_6; V_6 + W_2; W_3)$$

$$V_6 = V_2 - V_3 \ , \qquad\qquad W_2 = \frac{1}{2}\,(1,0,0,0,0,1,1,1) \oplus (\cdots)$$

Consider massless states, i.e. $(\,+\,+\,)$ modes of $(P \ \ P')$ from bulk and $T_{(0,1)}/T_{(0,2)}$ twisted sector $(G_2$, SU(3)) fixed points.

Find



$$V \in PS \ \ (F_3^c + \bar{\chi}^c) \in \Sigma$$

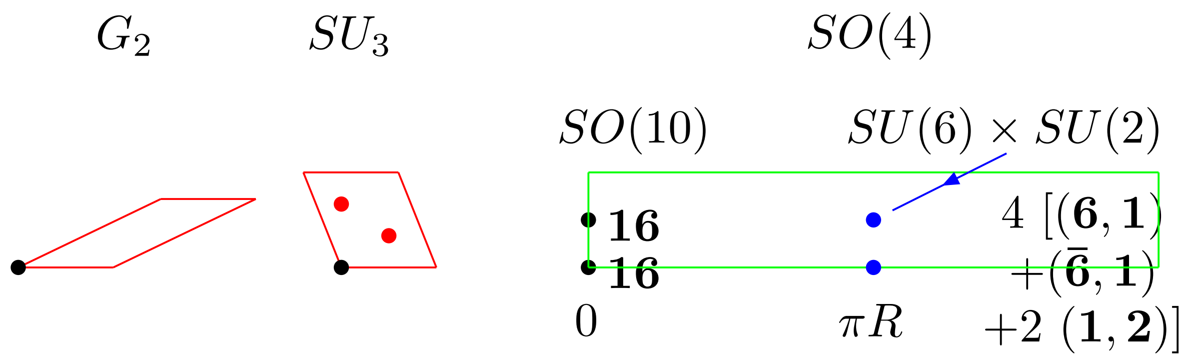
$$F_3 \in \mathbf{27} + \mathcal{H} \in \overline{\mathbf{27}}$$

$$2(\chi^c) + \bar{\chi}^c \in 3(\mathbf{27} \oplus \overline{\mathbf{27}})$$

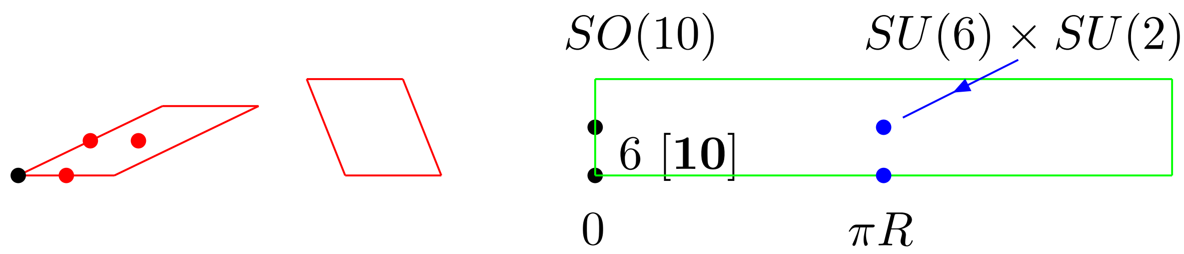
Table 1: Parities of the bulk states in model A1.

States	P	P'	States	P	P'
$V(\mathbf{15},\mathbf{1},\mathbf{1})$	$+$	$+$	$\Sigma(\mathbf{15},\mathbf{1},\mathbf{1})$	$-$	$-$
$V(\mathbf{1},\mathbf{3},\mathbf{1})$	$+$	$+$	$\Sigma(\mathbf{1},\mathbf{3},\mathbf{1})$	$-$	$-$
$V(\mathbf{1},\mathbf{1},\mathbf{3})$	$+$	$+$	$\Sigma(\mathbf{1},\mathbf{1},\mathbf{3})$	$-$	$-$
$V(\mathbf{6},\mathbf{2},\mathbf{2})$	$+$	$-$	$\Sigma(\mathbf{6},\mathbf{2},\mathbf{2})$	$-$	$+$
$V(\mathbf{4},\mathbf{2},\mathbf{1})$	$-$	$+$	$\Sigma(\mathbf{4},\mathbf{2},\mathbf{1})$	$+$	$-$
$V(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})$	$-$	$+$	$\Sigma(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})$	$+$	$-$
$V(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	$-$	$-$	U_3 $\Sigma(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	$+$	$+$
$V(\mathbf{4},\mathbf{1},\mathbf{2})$	$-$	$-$	U_3 $\Sigma(\mathbf{4},\mathbf{1},\mathbf{2})$	$+$	$+$
U_1 $H(\mathbf{4},\mathbf{2},\mathbf{1})$	$+$	$+$	$H^c(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})$	$-$	$-$
$H(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	$+$	$-$	$H^c(\mathbf{4},\mathbf{1},\mathbf{2})$	$-$	$+$
$H(\mathbf{6},\mathbf{1},\mathbf{1})$	$-$	$+$	$H^c(\mathbf{6},\mathbf{1},\mathbf{1})$	$+$	$-$
$H(\mathbf{1},\mathbf{2},\mathbf{2})$	$-$	$-$	U_2 $H^c(\mathbf{1},\mathbf{2},\mathbf{2})$	$+$	$+$
$H(\mathbf{4},\mathbf{2},\mathbf{1})_+$	$+$	$-$	$H^c(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})_+$	$-$	$+$
$H(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_+$	$+$	$+$	$H^c(\mathbf{4},\mathbf{1},\mathbf{2})_+$	$-$	$-$
$H(\mathbf{6},\mathbf{1},\mathbf{1})_+$	$-$	$-$	$H^c(\mathbf{6},\mathbf{1},\mathbf{1})_+$	$+$	$+$
$H(\mathbf{1},\mathbf{2},\mathbf{2})_+$	$-$	$+$	$H^c(\mathbf{1},\mathbf{2},\mathbf{2})_+$	$+$	$-$
$H(\mathbf{4},\mathbf{2},\mathbf{1})_-$	$-$	$+$	$H^c(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})_-$	$+$	$-$
$H(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_-$	$-$	$-$	$H^c(\mathbf{4},\mathbf{1},\mathbf{2})_-$	$+$	$+$
$H(\mathbf{6},\mathbf{1},\mathbf{1})_-$	$+$	$+$	$H^c(\mathbf{6},\mathbf{1},\mathbf{1})_-$	$-$	$-$
$H(\mathbf{1},\mathbf{2},\mathbf{2})_-$	$+$	$-$	$H^c(\mathbf{1},\mathbf{2},\mathbf{2})_-$	$-$	$+$

$T_{(1,2)}$ and $T_{(1,0)}$ twisted sectors



$G_2 \oplus SU(3) \oplus SO(4)$ lattice with Z_6 fixed points. The $T_{(1,2)}$ twisted sector states sit at these fixed points.



$G_2 \oplus SU(3) \oplus SO(4)$ lattice with Z_2 fixed points. The $T_{(1,0)}$ twisted sector states sit at these fixed points.

D_4 family symmetry !

Note *unrequested*
$[(6,1,1) \oplus (1,2,2)] \oplus [(4,1,1) \oplus (1,2,1)] \oplus (1,1,2) \oplus (1,1,1)$

Gauge coupling unification & Proton decay

- 5D RG equations

M_s = string scale, M_b = PS breaking scale,
M_c = 5D compactification scale

$$\begin{aligned}\frac{2\pi}{\alpha_i(\mu)} &= \frac{2\pi}{\alpha_s} + b_i^{MSSM} \ln \frac{M_b}{\mu} + (b_{++}^{PS} + b_{brane}^{PS})_i \ln \frac{M_s}{M_b} \\ &- \frac{1}{2}(b_{++}^{PS} + b_{--}^{PS})_i \ln \frac{M_s}{M_c} + b^G \left(\frac{M_s}{M_c} - 1 \right)\end{aligned}$$

- 4D RG equations

$M_G \approx 3 \times 10^{16}$ GeV, $\alpha_G^{-1} \approx 24$
and included threshold correction at M_G.

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha_G} + b_i^{MSSM} \ln \frac{M_G}{\mu} + 6\, \delta_{i3}$$

- $2\pi/\alpha_s = \pi/4\ (M_{Planck}/M_s)^2$ **

Find —

$M_b = e^{-3/2}\ M_G \sim 7 \times 10^{15}$ GeV,
$M_s(MAX) = e^2\ M_G \sim 2 \times 10^{17}$ GeV
** $\Longrightarrow \alpha_s^{-1} \sim 450$, α_s too small — PROBLEM !

NO solution !!

Give up perturbative heterotic string boundary conditions

Eleven-dimensional Hořava-Witten

$$\frac{2\pi}{\alpha_s} = \frac{1}{2(4\pi)^{5/3} M\rho} \left(\frac{M_{\rm Pl}}{M}\right)^2$$

M — eleven-dimensional Newton's constant by $\kappa^{2/3} = M^{-3}$

ρ — size of the eleventh dimension

Now find solution

$M_s \simeq M = 2M_G$,
$M_c \simeq M_b = e^{-3/2} M_G$
$M\rho \simeq 2$

◇ Enhanced proton decay rate — dimension-six operators

$$\tau(p \to e^+\pi^0) = \quad 1.2 \times 10^{34} \left(\frac{0.015\ \text{GeV}^3}{\beta_{\rm lattice}}\right)^2 \ \text{yrs}$$

Super-Kamiokande bound 5.7×10^{33} years @ 90% CL

Effective fermion mass operators

$$F_3\ \mathcal{H}\ F_3^c$$

$$F_n\ \mathcal{H}\ (\langle\bar{\chi}^c\ \chi^c\rangle_1\ \langle\bar{\chi}^c\ \chi^c\rangle_2\ \phi_n + \tilde{\phi}_n)\ F_3^c$$
$$F_3\ \mathcal{H}\ \phi'_n\ F_n^c$$

$$F_n\ \mathcal{H}\ (\langle\bar{\chi}^c\ \chi^c\rangle_2\ \tilde{S}_0 + S_0)\ F_n^c$$

$$F_0\ \mathcal{H}\ (\langle\bar{\chi}^c\ \chi^c\rangle_2\ \tilde{S}_1 + S_1)\ F_1^c + (0 \leftrightarrow 1)$$

where $n = 0,\ 1$

$S_n = S_n^{(10)(22)(22)(23)}, \quad \tilde{S}_n = S_n^{(3)(9)(12)}, \quad \phi_n = S_n^{(3)(12)}$

$\tilde{\phi}_n = S_n^{(9)(10)(22)(22)(23)}, \quad \phi'_n = S_n^{(10)(26)}$

$\langle\bar{\chi}^c\ \chi^c\rangle_1 = \langle U_3(\mathbf{4},\mathbf{1},\mathbf{2})\ T_2(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})\rangle$

$\langle\bar{\chi}^c\ \chi^c\rangle_2 = \langle T_4(\mathbf{4},\mathbf{1},\mathbf{2})\ T_2(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})\rangle$

- Fermion mass matrix

$$\mathcal{M} = \begin{pmatrix} (\langle\bar{\chi}^c\ \chi^c\rangle_2\ \tilde{S}_n + S_n) & (\langle\bar{\chi}^c\ \chi^c\rangle_1\ \langle\bar{\chi}^c\ \chi^c\rangle_2\ \phi_n + \tilde{\phi}_n) \\ \phi'_n & 1 \end{pmatrix}$$

Problems and Virtues

- Virtues
  - $E(6) \rightarrow SO(10) \rightarrow PS$
  - Three families (**16** of SO(10)) + 5 Higgs pairs (**10**) + PS breaking sector
  - D_4 Family symmetry $\Longrightarrow$ hierarchy of masses and mixing

    $D_4 \equiv \{\ S_2 : (0 \leftrightarrow 1),\ Z_2 : 0 \rightarrow 0\ (1 \rightarrow -1)\ \}$
  - Baryon and Lepton # violation in effective low energy field theory

    $F\ F\ F^c\ \langle \chi^c\ S^n \rangle \Longrightarrow Q\ L\ \bar{D} + L\ L\ \bar{E}$

    $F^c\ F^c\ F^c\ \langle \chi^c\ S^n \rangle \Longrightarrow \bar{U}\ \bar{D}\ \bar{D}$

    $F\ F\ F\ F\ \langle S^n \rangle \Longrightarrow Q\ Q\ Q\ L\ ?$

    NOT found to order S^8 – inconsistent with string selection rules !
- Problem
  - Effective dimension 4 R parity violating operators via color triplet mixing

    $(6,1,1)\ F^c\ \langle \chi^c\ S^n \rangle \Longrightarrow M_b\ T\ \bar{D}$

    Combined with $\langle S^m \rangle\ (6,1,1)\ (6,1,1) \Longrightarrow M_s\ T\ \bar{T}$

    Implies – massless $\bar{D}^0 \sim \bar{D} + \frac{M_b}{M_s}\ \bar{T}$

    $(6,1,1)\ F^c\ F^c \Longrightarrow \bar{D}^0 \bar{D} \bar{U}$

    $(6,1,1)\ F\ F \Longrightarrow \bar{D}^0\ Q\ L$

    B and L – R parity violating terms O(M_b/M_s) — TOO large !

Conclusions

- Obtained UV completion of E(6) orbifold GUT in 5D

- Obtained cubic and higher order *effective Yukawa couplings*

- Need more study of baryon and lepton number violation

- We have two other three family models — one with SO(10) in bulk

- Just the beginning

Expand search to $Z_N \otimes Z_2$ orbifolds +
1 (or 2) Wilson lines in SO(4) direction
$\Longrightarrow$ Effective 5 or 6 D orbifold GUTs

- Promising new directions for 3 family models !