AdS/CFT CORRESPONDENCE

AND UNIFICATION

AT ABOUT 4 TEV

Paul H. Frampton

UNC - Chapel Hill

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CONCISE HISTORY OF STRING THEORY

- Began 1968 with Veneziano model.
- 1968-1974 dual resonance models for strong interactions. Replaced by QCD around 1973. DRM book 1974. Hiatus 1974-1984
- 1984 Cancellation of hexagon anomaly.
- 1985 $E(8) \times E(8)$ heterotic strong compactified on Calabi-Yau manifold gives temporary optimism of TOE.
- 1985-1997 Discovery of branes, dualities, M theory.
- 1997 Maldacena AdS/CFT correspondence relating 10 dimensional superstring to 4 dimensional gauge field theory.
- 1997-present Insights into gauge field theory including possible new states beyond standard model. String not only as quantum gravity but as powerful tool in nongravitational physics.

MORE ON STRING DUALITY:

Duality: Quite different looking descriptions of the same underlying theory.

The difference can be quite striking. For example, the AdS/CFT correspondence describes duality between a $d = 4$ SU(N) GFT and a $D = 10$ superstring. Nevertheless, a few non-trivial checks have confirmed this correspondence.

In its most popular version, one takes a Type IIB superstring (closed, chiral) in $d = 10$ and one compactifies on:

$$(AdS)_5 \qquad \times \qquad S^5$$

Perturbative finiteness of

$\mathcal{N} = 4$ SUSY Yang-Mills theory.

- Was proved by Mandelstam, Nucl. Phys. B213, 149 (1983).

- The Malcacena correspondence is primarily aimed at the $N \to \infty$ limit with the 't Hooft parameter of N times the squared gauge coupling held fixed.

- Conformal behavior valid here also for finite N.

II BREAKING SUPERSYMMETRIES

To approach the real world, one needs less or no supersymmetry in the (conformal?) gauge theory.

By factoring out a discrete (abelian) group and composing an orbifold:

$$S^5/\Gamma$$

one may break $\mathcal{N} = 4$ supersymmetry to $\mathcal{N} = 2$, 1, or 0. Of special interest is the $\mathcal{N} = 0$ case.

We may take $\Gamma = Z_p$ which identifies p points in $\mathcal{C}_3$.

The rule for breaking the $\mathcal{N} = 4$ supersymmetry is:

$$\Gamma \subset SU(2) \quad \Rightarrow \quad \mathcal{N} = 2$$

$$\Gamma \subset SU(3) \quad \Rightarrow \quad \mathcal{N} = 1$$

$$\Gamma \not\subset SU(3) \quad \Rightarrow \quad \mathcal{N} = 0$$

In fact to specify the embedding of $\Gamma = Z_p$ we need to identify three integers (a_1, a_2, a_3):

$$\mathcal{C}_3 : \; (X_1, X_2, X_3) \; \stackrel{Z_p}{\rightarrow} \; (\alpha^{a_1} X_1, \alpha^{a_2} X_2, \alpha^{a_3} X_3)$$

with

$$\alpha = exp\left(\frac{2\pi i}{p}\right)$$

What is known to be true - proved both from string theory in

Bershadsky, Kakushadze and Vafa, hep-th/9803076,

and from field theory† in

Bershadsky and Johansen, hep-th/9803349,

- is that to leading order in $(1/N)$ such theories have all $\beta = 0$ $(\beta_g = \beta_Y = \beta_H = 0)$ to all orders of GFT perturbation theory.

This is remarkable from the field theory point of view.

† This proof involves Γ projections of the states and turns out to be disappointingly kinematic.

Without the stimulus of AdS/CFT it would be:

- Difficult to guess any $\mathcal{N}=0$ theory with all $\beta-$functions vanishing to all orders of perturbation theory, even for leading order in 1/N.

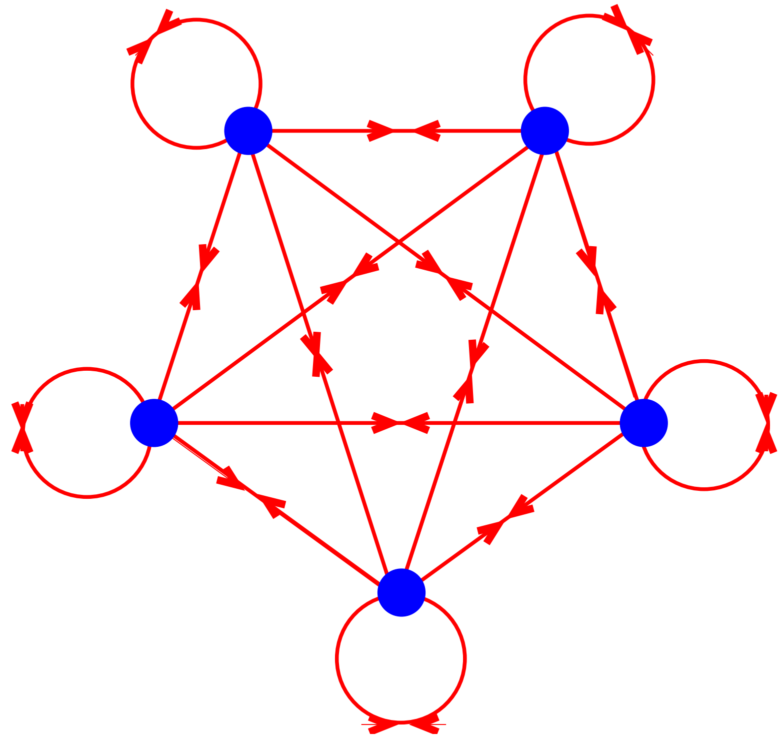
- Because without renormalizarion theorems ($\mathcal{N}=0$) there is an infinite number of constraints on a finite number of representations.

MATTER REPRESENTATIONS

- The Z_p discrete group identifies p points in $\mathcal{C}_3$.

- The N converging D3-branes meet on all p copies, giving a gauge group: $SU(N) \times SU(N) \times ...... \times SU(N)$.

- The matter (spin-1/2 and spin-0) which survives is invariant under a product of a gauge transformation and a Z_p transformation.



One can draw p points and arrows for a_1, a_2, a_3.

$$e.g. \qquad Z_5 \ \ (1,3,0)$$

Quiver diagram (Douglas-Moore).

Scalar representation is:

$$\sum_{k=1}^{3} \sum_{i=1}^{p} (N_1, \bar{N}_{i \pm a_k})$$

For fermions, one must construct the **4** of R-parity $SU(4)$:

From the $a_k = (a_1, a_2, a_3)$ one constructs the 4-spinor $A_\mu = (A_1, A_2, A_3, A_4)$:

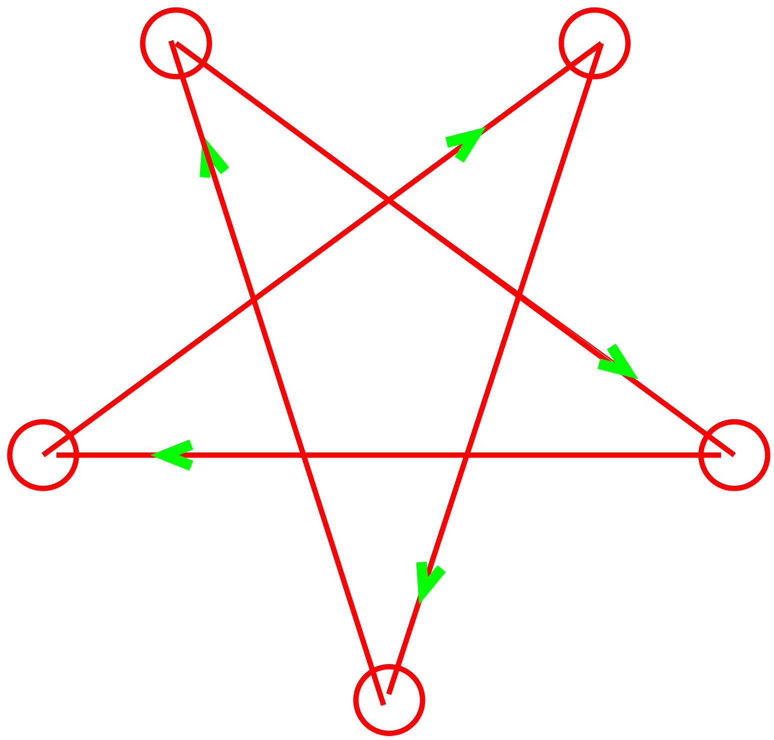
$$A_1 = \frac{1}{2}(a_1 + a_2 + a_3)$$

$$A_2 = \frac{1}{2}(a_1 - a_2 - a_3)$$

$$A_3 = \frac{1}{2}(-a_1 + a_2 - a_3)$$

$$A_4 = \frac{1}{2}(-a_1 - a_2 + a_3)$$

These transform as $exp\left(\frac{2\pi i}{p}A_\mu\right)$ and the invariants may again be derived (by a different diagram):



$$e.g. A_\mu = 2; \quad p = 5.$$

These lines are oriented.

One finds for the fermion representation

$$\sum_{\mu=1}^{4} \sum_{i=1}^{p} (N_i, \bar{N}_{i+A_\mu})$$

Nevertheless, since 4 global supersymmetries give conformality including for finite N (all $\beta-$functions vanish)

- To all orders of perturbation theory even for finite N of $SU(N)$ we can be more ambitious and ask for finiteness without any global supersymmetry and finite N of $SU(N)$.

2 - LOOP BETA FUNCTIONS

We know that if Γ is absent the resultant $\mathcal{N} = 4$ SUSY SU(N) GFT has $\beta_g = \beta_Y = \beta_H = 0$ to all orders (the proof of Mandelstam, 1983). When supersymmetries are broken, one must check in more detail:

$$\beta_g = \beta_g^{(1)} + \beta_g^{(2)}$$

$$\beta_g^{(1)} = -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3}C_2(G) - \frac{4}{3}\kappa S_2(F) - \frac{1}{6}S_2(S)\right]$$

Here the quadratic Casimir is $C_2(G) = N$. The Dynkin indices are $S_2(F) = 4N$ and $S_2(S) = 6N$ for the fermion ($\kappa = 1/2$ for Weyl spinors) and scalar representations respectively.
Thus $\beta_g^{(1)} = 0$.

The general expression for $\beta_g^{(2)}$ is:

$$\beta_g^{(2)} = -\frac{g^5}{(4\pi)^4}\left[\frac{34}{3}(C_2(G))^2 - \right.$$

$$\kappa\left(4C_2(F) + \frac{20}{3}C_2(G)\right)S_2(F) -$$

$$\left.\left(2C_2(S) + \frac{1}{3}C_2(G)\right)S_2(S) + \frac{2\kappa Y_4(F)}{g^2}\right]$$

In $\beta_g^{(2)}$ the 1st, 3rd and 5th of the six terms are the same in all theories, namely:

$$\frac{34N^2}{3} - \frac{40N^2}{3} - 2N^2 = -4N^2$$

In the 2nd and 4th terms there is an implicit sum over irreducible representations. In the 6th term are Yukawa couplings $Y_4(F)$.

HOW MANY CANDIDATES FOR $\mathcal{N} = 0$ $d = 4$ CONFORMAL THEORIES?

$$Z_p \qquad (a_1, a_2, a_3)$$

$$a_1 \leq a_2 \leq a_3$$

Let us define $\nu_k(p)$ to be the number of different theories with k non-zero a_i.

For $\nu_1(p)$ we note that $(0, 0, a_3)$ is equivalent to $(0, 0, p - a_3)$ and hence

$$\nu_1(p) = \lfloor p/2 \rfloor$$

where $\lfloor x \rfloor$ is the largest integer not greater than x.

Consider:

$$\nu_2(p)$$

where there is the equivalence of:

$$(0, a_2, a_3) \equiv (0, p - a_3, p - a_2)$$

One finds that for p = even

$$\nu_2(p) = 2 \sum_{r=1}^{\lfloor (p-2)/2 \rfloor} r = \frac{1}{4} p(p-2)$$

while for p = odd

$$\nu_2(p) = 2 \sum_{r=1}^{\lfloor (p-2)/2 \rfloor} r + \lfloor p/2 \rfloor = \frac{1}{4}(p-1)^2$$

Finally consider $\nu_3(p)$. This counting is only slightly more intricate.

One uses:

$$\nu_3(p) = \frac{1}{2}[\bar{\nu}(p) - \nu_p(p) + \nu_{SE}(p)]$$

(discuss)

Define $\nu_{TOTAL}(p) = \nu_1(p) + \nu_2(p) + \nu_3(p)$. The results depend on what is the remainder when p is divided by 6.

If p = 6k, $\nu_{TOTAL}(p) = \frac{p}{12}(p^2 + 2p + 2)$.
If p = 6k + 1 or 6k + 5, then $\nu_{TOTAL}(p) = \frac{1}{12}(p-1)(p+1)(p+2)$.
If p = 6k+2 or 6k + 4, $\nu_{TOTAL}(p) = \frac{1}{12}(p^3 + 2p^2 + 2p + 4)$.
Finally if p = 6k+3, $\nu_{TOTAL} = \frac{1}{12}(p^3 + 2p^2 - p - 6)$.

p	$\nu_1(p)$	$\nu_2(p)$	$\nu_3(p)$	$\nu_{TOTAL}(p)$	$\sum \nu_{TOTAL}$	$\nu_{alive}(p)$	$\sum \nu_{alive}(p)$
2	1	0	1	2	2	0	0
3	1	1	1	3	5	1	1
4	2	2	5	9	14	1	2
5	2	4	8	14	28	4	6
6	3	6	16	25	53	5	11
7	3	9	24	36	89	9	20
8	4	12	39	55	144	10	30
9	4	16	53	73	217	16	46
10	5	20	77	102	319	18	64
11	5	25	100	130	449	25	89
12	6	30	134	170	619	27	116
13	6	36	168	210	829	36	152
14	7	42	215	264	1093	39	191
15	7	49	261	317	1410	49	240
16	8	56	323	387	1797	52	292
17	8	64	384	456	2253	64	356
18	9	72	462	543	2796	68	424
19	9	81	540	630	3426	81	505
20	10	90	637	737	4163	85	590

p	$\nu_1(p)$	$\nu_2(p)$	$\nu_3(p)$	$\nu_{TOTAL}(p)$	$\sum \nu_{TOTAL}$	$\nu_{alive}(p)$	$\sum \nu_{alive}(p)$
21	10	100	733	843	5006	100	690
22	11	110	851	972	5978	105	795
23	11	121	968	1100	7078	121	916
24	12	132	1108	1252	8330	126	1042
25	12	144	1248	1404	9734	144	1186
26	13	156	1413	1582	11316	150	1336
27	13	169	1577	1759	13075	169	1505
28	14	182	1769	1965	15040	175	1680
29	14	196	1960	2170	17210	196	1876
30	15	210	2180	2405	19615	203	2079
31	15	225	2400	2640	22255	225	2304
32	16	240	2651	2907	25162	232	2536
33	16	256	2901	3173	28335	256	2792
34	17	272	3185	3474	31809	264	3056
35	17	289	3468	3774	35583	289	3345
36	18	306	3796	4110	39693	297	3642
37	18	324	4104	4446	44139	324	3966
38	19	342	4459	4820	48959	333	4299
39	19	361	4813	5193	54152	361	4660
40	20	380	5207	5607	59759	370	5030
41	20	400	5600	6020	65779	400	5430

DIRECTIONS:

- Selection process for $\mathcal{N} = 0$ begun by considering 1- and 2- loop $\beta-$functions.
- At 1-loop, all such theories pass because they are coincident (leading N) with $\mathcal{N} = 4$.
- At 2 loops, for the gauge $\beta-$function, a significant number of the models examined survive.
- Checking of Yukawa and ϕ^4 $\beta-$functions at 2 loops is being pursued in a student's PhD dissertation.

Beyond that:

- If all 2 loop requirements are satisfied, at 3 or more loops explicit calculations become impracticable - general arguments based on symmetry may succeed.

- Question of uniqueness of $\mathcal{N} = 0$ theory.

- Use of $\mathcal{N} = 0$ quivers in model building to be discussed later.

Philosophical Question on Renormalization

- Infinite renormalization of QED was greeted with skepticism but soon universally accepted due to exceptionally successful accuracy.

- If the finiteness of $\mathcal{N} = 4$ had been known then would the skepticism have been more persistent?

CONFORMALITY AND

PARTICLE PHENOMENOLOGY

- Hierarchy between GUT scale and weak scale is 14 orders of magnitude. Why do these two very different scales exist?

- How is this hierarchy of scales stabilized under quantum corrections?

- Supersymmetry answers the first question but not the second.

Successes of

Supersymmetry

- Cancellations of UV infinities.
- technical naturalness of hierarchy.
- Unification of Gauge Couplings.
- Natural appearance in string theory.

Puzzles about

Supersymmetry

- The “mu” problem: why is the Higgs at the weak scale not the GUT scale (hierarchy).
- Breaking supersymmetry leads to too large a cosmological constant.
- Is supersymmetry fundamental for string theory?
- There are solutions of string theory without supersymmetry.

Supersymmetry and Grand Unification replaced by Conformality at TeV Scale.

- Will show idea is possible.
- Explicit examples containing standard model states.
- Conformality more rigid constraint than supersymmetry.
- Predicts additional states at TeV scale for conformality.
- Gauge coupling unification.

Conformality as

hierarchy solution

- Quark and lepton masses, QCD and weak scales small compared to TeV scale.

- May be put to zero suggesting:

- Add degrees of freedom to yield GFT with conformal invariance.

- 't Hooft naturalness since zero mass limit increases symmetry to conformal symmetry.

The theory is assumed to be given by the action:

$$S = S_0 + \int d^4x \alpha_i O_i \quad (1)$$

where S_0 is the action for the conformal theory and the O_i are operators with dimension below four which break conformal invariance softly.

The mass parameters α_i have mass dimension $4 - \Delta_i$ where Δ_i is the dimension of O_i at the conformal point.

Let M be the scale set by the parameters α_i and hence the scale at which conformal invariance is broken. Then for $E >> M$ the couplings will not run while they start running for $E < M$. To solve the hierarchy problem we assume M is near the TeV scale.

Large class of d=4 CFTs

- each $SU(4)$ subgroup

- Choice of N.
- 1/N vanishing $\beta-$ functions.
- Finite N?
- Conformal invariance at infra-red fixed point.
- For $\mathcal{N} = 0$ there exists boson-fermion number equality.

Interactions. Gauge fields interact according to gauge coupling which, combined with corresponding theta angle for i th group, is writable as

$$\tau_i = \Theta_i + \frac{i}{4\pi g_i^2} = \frac{\tau d_i}{|\Gamma|}$$

where τ is complex parameter (independent i) and $|\Gamma| =$ order Γ.

Yukawa interactions. Triangles in quiver. Two directed fermion sides and an undirected scalar side.

$$S_{Yukawa} = \frac{1}{4\pi g^2} \Sigma\, d^{abc} Tr \Psi^a_{ij} \Phi^b_{jk} \Psi^c_{ki}$$

in which d^{abc} is ascertainable as Clebsch-Gordan coefficient from product of trivial representaions occurring respectively in $(4 \otimes R_i \otimes R_j^*)$, $(6 \otimes R_j \otimes R_k^*)$ and $(4 \otimes R_k \otimes R_i^*)$.

Quartic scalar interactions. Quadrilaterals in quiver. Four undirected sides. The coupling computable analagously to above.

Large class of d=4 CFTs

- Are they conformal for higher orders in 1/N?

- YES, for $\mathcal{N} = 2$: all such $\mathcal{N} = 2$ theories are obtainable.

- YES, for $\mathcal{N} = 1$: non-renormalization theorems ensure flat directions.

- For the case of $\mathcal{N} = 0$, general answer unknown but is under investigation.

Large class of d=4 CFTs

- Are they conformal for higher orders in 1/N?
- S-duality of underlying type IIB superstring implies $g \rightarrow 1/g$ symmetry.

- Assuming next-leading order in $1/N$ is asymptotically free IR flow at small g *increases* g.

- Consequently IR flow *decreases* g for large g and there must therefore be at least one zero $\beta = 0$ for some finite g. QED.

GENERAL PREDICTIONS.

Consider embedding the standard model gauge group according to:

$$SU(3) \times SU(2) \times U(1) \subset \bigotimes_i SU(Nd_i)$$

Each gauge group of the SM can lie entirely in a $SU(Nd_i)$ or in a diagonal subgroup of a number thereof.

Only bifundamentals (including adjoints) are possible. This implies no $(8, 2)$, etc. A conformality restriction which is new and satisfied in Nature!

No $U(1)$ factor can be conformal and so hypercharge is quantized through its incorporation in a non-abelian gauge group. This is the "conformality" equivalent to the GUT charge quantization condition in *e.g.* $SU(5)$!

Beyond these general consistencies, there are predictions of new particles necessary to render the theory conformal.

The minimal extra particle content comes from putting each SM gauge group in one $SU(Nd_i)$. Diagonal subgroup embedding *increases* number of additional states.

Number of fundamentals plus Nd_i times the adjoints is $4Nd_i$. Number N_3 of color triplets and N_8 of color octets satisfies:

$$N_3 + 3N_8 \geq 4 \times 3 = 12$$

Since the SM has $N_3 = 6$ we predict:

$$\Delta N_3 + 3N_8 \geq 6$$

The additional states are at TeV if conformality solves hierarchy. Similarly for color scalars:

$$M_3 + 3M_8 \geq 6 \times 3 = 18$$

The same exercise for $SU(2)$ gives $\Delta N_2 + 4N_3 \geq 4$ and $\Delta M_2 + 2M_3 \geq 11$ respectively.

GAUGE UNIFICATION

- Above TeV scale couplings will not run.

- Couplings of 3-2-1 related, not equal, at conformality scale.

- Embeddings in different numbers of the equal-coupling $SU(N)$ groups lead to the TeV scale unification without logarithmic running over large desert.

Some illustrative examples of model building using conformality.

We need to specify an embedding $\Gamma \subset SU(4)$.

Consider Z_2. It embeds as $(-1,-1,-1,-1)$ which is real and so leads to a non-chiral model.

Z_3. One choice is $\mathbf{4} = (\alpha, \alpha, \alpha, 1)$ which maintains N=1 supersymmetry. Otherwise we may choose $\mathbf{4}$=$(\alpha, \alpha, \alpha^2, \alpha^2)$ but this is real.

Z_4. The only N = 0 complex embedding is $\mathbf{4}$=(i, i, i, i). The quiver is as shown on the next transparency with the $SU(N)^4$ gauge groups at the corners, the fermions on the edges and the scalars on the diagonals. The scalar content is too tight to break to the SM.

To obtain 3 chiral families in $\mathcal{N} = 0$ from abelian orbifolds, consider $\Gamma = Z_p$ with successively increasing $p = 2, 3, 4, 5, 6, 7....$ to access the simplest model.

p = 2 ± 1 real (require ($4 \neq 4^*$ for chirality)

p = 3 No chiral $\mathcal{N} = 0$

p = 4 (i, i, i, i).
Scalars † insufficient for SSB $SU(3)^4 \rightarrow (321)_{SM}$

p = 5 $(\alpha, \alpha, \alpha, \alpha^2)$ $(\alpha, \alpha^2, \alpha^3, \alpha^3)$
In both cases, scalars† insufficient for SSB.

p = 6 $(\alpha, \alpha, \alpha, \alpha^3)$ $(\alpha, \alpha, \alpha^2, \alpha^2)$ $(\alpha, \alpha^3, \alpha^3, \alpha^3)$
Scalars† insufficient for SSB $SU(3)^6 \rightarrow (321)_{SM}$

† scalars, unlike in GUTs, are in prescibed representations.

When we arrive at $p = 7$ there are viable models. Actually three different quiver diagrams can give:

1) 3 chiral families.

2)Adequate scalars to spontaneously break $SU(3)^7 \rightarrow SU(3) \times SU(2) \times U(1)$

and

3) $\sin^2\theta_W = 3/13 = 0.231$

The embeddings of $\Gamma = Z_7$ in SU(4) are:

7A. $(\alpha, \alpha, \alpha, \alpha^4)$

7B. $(\alpha, \alpha, \alpha^2, \alpha^3)*$ C-H-H-H-W-H-W

7C. $(\alpha, \alpha^2, \alpha^2, \alpha^2)$

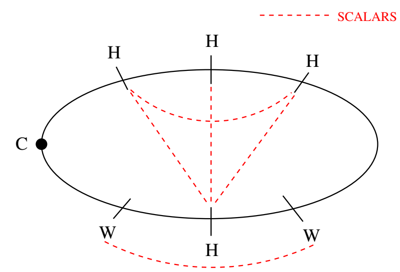
7D. $(\alpha, \alpha^3, \alpha^5, \alpha^6)*$ C-H-W-H-H-H-W

7E. $(\alpha, \alpha^4, \alpha^4, \alpha^4)*$ C-H-W-W-H-H-H

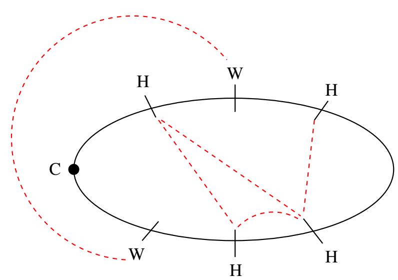
7F. $(\alpha^2, \alpha^4, \alpha^4, \alpha^4)$

* have properties 1), 2) and 3).

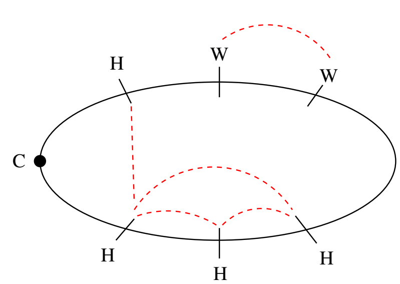
7B $4 = (1, 1, 2, 3)$ $6 = (2, 3, 3, -3, -3, -2)$



7D $4 = (1, 3, 5, 5)$ $6 = (1, 1, 3, -3, -1, -1)$



7E $4 = (1, 4, 4, 5)$ $6 = (1, 2, 2, -2, -2, -1)$



The simplest abelian orbifold conformal extension of the standard model has $SU(3)^7 \rightarrow SU(3)^3$ trinification $\rightarrow (321)_{SM}$.

In this case we have α_2 and α_1 related correctly for low energy. But $\alpha_3(M) \simeq 0.07$ suggesting a conformal scale $M \geq 10$ TeV - too high for the L.H.C.

However, non-abelian orbifolds may yield a simpler model - generalizing a Left-Right symmetric (Pati-Salam) model rather than the S.M.

NON-ABELIAN ORBIFOLDS

We consider all non-abelian discrete groups up to order $g < 32$. There are exactly 45 such groups. Because the gauge group arrived at is $\otimes_i SU(Nd_i)$ we can arrive at $SU(4) \times SU(2) \times SU(2)$ by choosing $N = 2$.

To obtain chiral fermions one must have $\mathbf{4} \neq \mathbf{4}^*$ This is not quite sufficient because for $N = 2$ $\mathbf{4}$ cannot be pseudoreal.

This last requirement eliminates many of the 45 candidate groups. For example $Q_{2n} \subset SU(2)$ has irreps of appropriate dimensions but cannot sustain chiral fermions. because these irreps are , like $SU(2)$, pseudoreal.

This leaves 19 possible non-abelian Γ with $g \leq 31$, the lowest order being $g = 16$. This gives only two families.

The smallest group which allows three chiral familes has order $g = 24$ so we now describe this model.

Using only D_N, Q_{2N}, S_N and T (T = tetrahedral S_4/Z_2) one already finds 32 of the 45 non-abelian discrete groups with $g \leq 31$:

g	
6	$D_3 \equiv S_3$
8	$D_4, \ Q = Q_4$
10	D_5
12	$D_6, \ Q_6, \ \mathrm{T}$
14	D_7
16	$D_8, \ Q_8, \ Z_2 \times D_4, \ Z_2 \times Q$
18	$D_9, \ Z_3 \times D_3$
20	$D_{10}, \ Q_{10}$
22	D_{11}
24	$D_{12}, \ Q_{12}, \ Z_2 \times D_6, \ Z_2 \times Q_6, \ Z_2 \times T$ $Z_3 \times D_4, \ Z_3 \times Q, \ Z_4 \times D_3, \ S_4$
26	D_{13}
28	$D_{14}, \ Q_{14}$
30	$D_{15}, \ D_5 \times Z_3, \ D_3 \times Z_5$

The remaining 13 of the 45 non-abelian discrete groups with $g \leq 31$ are twisted products:

g	
16	$Z_2 \tilde{\times} Z_8$ (two, excluding D_8), $Z_4 \tilde{\times} Z_4$ $Z_2 \tilde{\times} (Z_2 \times Z_4)$ (two)
18	$Z_2 \tilde{\times} (Z_3 \times Z_3)$
20	$Z_4 \tilde{\times} Z_5$
21	$Z_3 \tilde{\times} Z_7$
24	$Z_3 \tilde{\times} Q$, $Z_3 \tilde{\times} Z_8$, $Z_3 \tilde{\times} D_4$
27	$Z_9 \tilde{\times} Z_3$, $Z_3 \tilde{\times} (Z_3 \times Z_3)$

Successful $g = 24$ model is based on the group $\Gamma = Z_3 \times Q$.

The fifteen irreps of Γ are

$1, 1', 1'', 1''', 2,$

$1\alpha, \; 1'\alpha, 1''\alpha, 1'''\alpha, 2\alpha,$

$1\alpha^{-1}, 1'\alpha^{-1}, 1''\alpha^{-1}, 1'''\alpha^{-1}, 2\alpha^{-1}.$

The same model occurs for $\Gamma = Z_3 \times D_4$. The multiplication table (for either case) is shown on the next transparency.

	1	$1'$	$1''$	$1'''$	2
1	1	$1'$	$1''$	$1'''$	2
$1'$	$1'$	1	$1'''$	$1''$	2
$1''$	$1''$	$1'''$	1	$1'$	2
$1'''$	$1'''$	$1''$	$1'$	1	2
2	2	2	2	2	$1+1'$ $1''+1'''$
1α	1α	$1'\alpha$	$1''\alpha$	$1'''\alpha$	2α
$1'\alpha$	$1'\alpha$	1α	$1'''\alpha$	$1''\alpha$	2α
$1''\alpha$	$1''\alpha$	$1'''\alpha$	1α	$1'\alpha$	2α
$1'''\alpha$	$1'''\alpha$	$1''\alpha$	$1'\alpha$	1α	2α
2α	2α	2α	2α	2α	$1\alpha+1'\alpha$ $1''\alpha+1'''\alpha$

etc.

The general embedding of the required type can be written:

$$\mathbf{4} = (1\alpha^{a_1},\ 1'\alpha^{a_2},\ 2\alpha^{a_3})$$

The requirement that the **6** is real dictates that

$$a_1 + a_2 = -2a_3$$

It is therefore sufficient to consider for $\mathcal{N} = 0$ no surviving supersymmetry only the choice:

$$\mathbf{4} = (\ 1\alpha,\ 1',\ 2\alpha)$$

It remains to derive the chiral fermions and the complex scalars using the procedures already discussed (quiver diagrams).

$Q (or D_4) \times Z_3$ Model

VEVs for these scalars allow to break to the following diagonal subgroups as the only surviving gauge symmetries:

$$SU(2)_{1,2,3} \longrightarrow SU(2)$$

$$SU(2)_{5,6,7} \longrightarrow SU(2)$$

$$SU(4)_{1,2} \longrightarrow SU(4)$$

This spontaneous symmetry breaking leaves the Pati-Salam type model:

$$SU(4) \times SU(2) \times SU(2)$$

with three chiral fermion generations

$$3\ [(\ 4,\ 2,\ 1) + (\ \bar{4},\ 1,\ 2)]$$

GAUGE COUPLING UNIFICATION (ABELIAN ORBIFOLD)

With the assumptions of grand unification and low-energy supersymmetry, one achieved a successful gauge unification. The LEP data gives the couplings at the Z-pole as $\alpha_3 = 0.118 \pm 0.003$, $\alpha_2 = 0.0338$ and $\alpha_1 = \frac{5}{3}\alpha_Y = 0.0169$ (where the errors on $\alpha_{1,2}$ are less than 1%.

Using the RG equations

$$\frac{1}{\alpha_i(M_G)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi} \ln \left(\frac{M_G}{M_Z} \right)$$

and, for the MSSM the values $b_i = (6\frac{3}{5}, 1, -3)$, inputting $\alpha_{2,3}(M_Z)$ leads to $M_G = 2.4 \times 10^{16}$ GeV and the *prediction* that

$$\sin^2 \theta = 0.231$$

in excellent agreement with experiment. Indeed this success is the main reason for belief in these two assumptions of low-energy supersymmetry and grand unification,

If we note that, at the Z-pole, the ratio $\alpha_2/\alpha_1 \simeq 2$ as pointed out first in
PHF, Phys. Rev. **D60,** 085004 (1999)
we can reproduce the correct gauge unification. Specifically for the abelian orbifold with $\Gamma = Z_7$ and $N = 3$ it is natural to accommodate one $SU(3)$ factor as $SU(3)_{color}$ and $SU(2)_L$ in a diagonal subgroup of two $SU(3)$ factors. Finally $U(1)$ is in a diagonal subgroup of four $SU(3)$ factors.
This gives the appropriate ratio betwen $\alpha_{1,2}$ and consequently

$$\sin^2\theta = \frac{\alpha_Y}{\alpha_2 + \alpha_Y} = \frac{3/5}{2 + 3/5} = 3/13 = 0.231$$

There is a small correction for the running between M_Z and the TeV scale, but this is compensated by the two-loop correction and the agreement remains as good as for SUSY-GUTs. This is strong encouragenment for the conformality approach.

GAUGE COUPLING UNIFICATION (NON-ABELIAN ORBIFOLD)

Here we summarize the analysis in:

PHF, RN Mohapatra and S Suh, hep-ph/0104211

This uses a unification at the TeV scale based on $SU(4)_C \times SU(2)_L \times SU(2)_R$ rather than the abelian case which uses trinification $SU(3)^3$.

The orbifold employs $\Gamma = Z_3 \times D_4$. This discrete group has three 2-dimensional irrpes and twelve 1-dimensional ones.

The group D_4 consists of eight rotations which leave a square invariant: two of the rotations are flips about two lines that bisect the square and the other four are rotations through $\pi/2, \pi, 3\pi/2$ and 2π about the perpendicular to the plane of the square.

The low energy group is thus $SU(4)^3 \times SU(2)^{12}$.

We embed the $SU(4)_C$ in r of the $SU(4)$ groups where $R = 1 or 2$ because $r = 3$ leads to loss of chirality.

$SU(2)_{L,R}$ are embedded respectively in p, q $SU(2)$ groups where $p + q = 12$.

Since p, q are integers it is not obvious *a priori* that the value of $\sin^2\theta$ can be acceptable.

The values of the couplings at the conformality scale are:

$$\alpha_{2L}^{-1}(M_U) = p\alpha_U^{-1}$$

$$\alpha_{2R}^{-1}(M_U) = q\alpha_U^{-1}$$

$$\alpha_{4C}^{-1}(M_U) = 2r\alpha_U^{-1}$$

The hypercharge coupling is related by

$$\alpha_1^{-1} = \frac{2}{5}\alpha_{4c}^{-1} + \frac{3}{5}\alpha_{2R}^{-1}$$

Defining $y = \ln(M_U/M_Z)$ we then find

$$\sin^2\theta_W(M_Z) = \frac{p - (19/12\pi)y\alpha_U}{p + q + \frac{4}{3}r + (11/6\pi)y\alpha_U}$$

Here

$$\alpha_S^{-1}(M_Z) = 2r\alpha_U^{-1} - \frac{7}{2\pi}y$$

Using these formulae and $\alpha_S(M_Z) = 0.12$ we find
$sin^2\theta(M_Z) \simeq 0.23$ for p = 4 and r = 2.

The conformality scale is here taken as $M_U \simeq$ 100 TeV, the lower limit necessary to avoid too high a branching ratio for $K_L \to \mu^+ e^-$.

It is highly non-trivial that the gauge coupligs unify correctly.

GAUGE UNIFICATION

The successful derivation of $\sin^2\theta_W \simeq 0.23$ from both the abelian orbifold (based on 333 unification) and the non-abelian manifold (based on 422 unification) is strong support for the conformality approach.

More detailed phenomenological study of the conformality idea is merited.

STRONG-ELECTROWEAK UNIFICATION

was proposed even before the firm establishment of the standard electroweak theory in the early 1970s. Minimal SU(5), both with and without supersymmetry, is ruled out. Such GUTs involve a scale $\sim 10^{16}$ GeV and a GUT hierarchy.

Compactifying IIB on $AdS_5 \times S^5/\Gamma$ leads to candidate semi-simple unification gauge groups.

The following theory has both a bottom-up and a top-down component and leads to several interesting features. Let us begin with bottom-up.

In the SM first consider the electroweak angle $sin^2\theta(\mu)$. At $\mu = M_Z$, its value is measured as 0.231 and with increasing μ it goes through 1/4 at $\mu \simeq 4$ TeV.

We may consider also the ratio $\alpha_3(\mu)/\alpha_2(\mu)$ which is above 3 at $\mu = M_Z$, decreases through 3 at $\mu \simeq 400$ GeV and 2 at $\mu \simeq 140$ TeV. It is 5/2 at exceptionally close to where $\sin^2\theta = 1/4$.

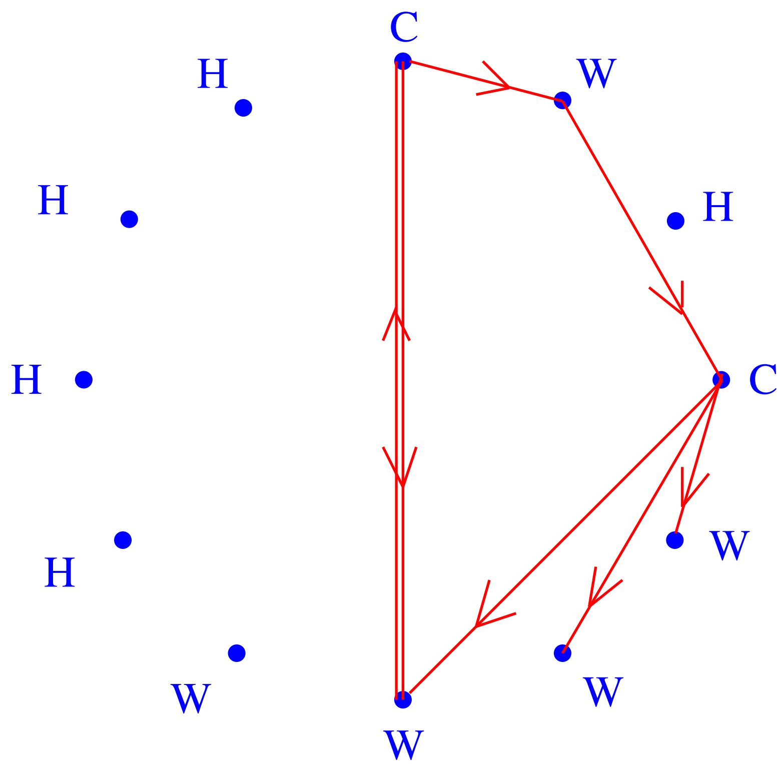
We take this numerology as a hint that in a trinification $SU(3)_C \times SU(3)_W \times SU(3)_H$ the couplings are in the ratio 5 :: 2 :: 2 at $\mu = 4$TeV

This can be achieved by embedding the 333-model in $SU(3)^{12}$ with the C, W, H groups diagonaly embedded in respectively 2, 5, 5 of the $SU(3)$'s. Let us now consider top-down.

Taking as orbifold S^5/Z_{12} with embedding of Z_{12} in the SU(4) R-parity specified by **4** $\equiv \alpha^{A_1}, \alpha^{A_2}, \alpha^{A_3}, \alpha^{A_4})$ and $A_\mu = (1, 2, 3, 6)$.

This accommodates the scalars necessary to spontaneously break to the SM. This theory thus predicts TWO numbers: $sin^2\theta(M_Z)$ and $\alpha_C(M_Z)$ whereas usual GUTs predict only ONE of these.

As a bonus, the dodecagonal quiver predicts three chiral families (see next transparency). Also there is no GUT hierarchy.



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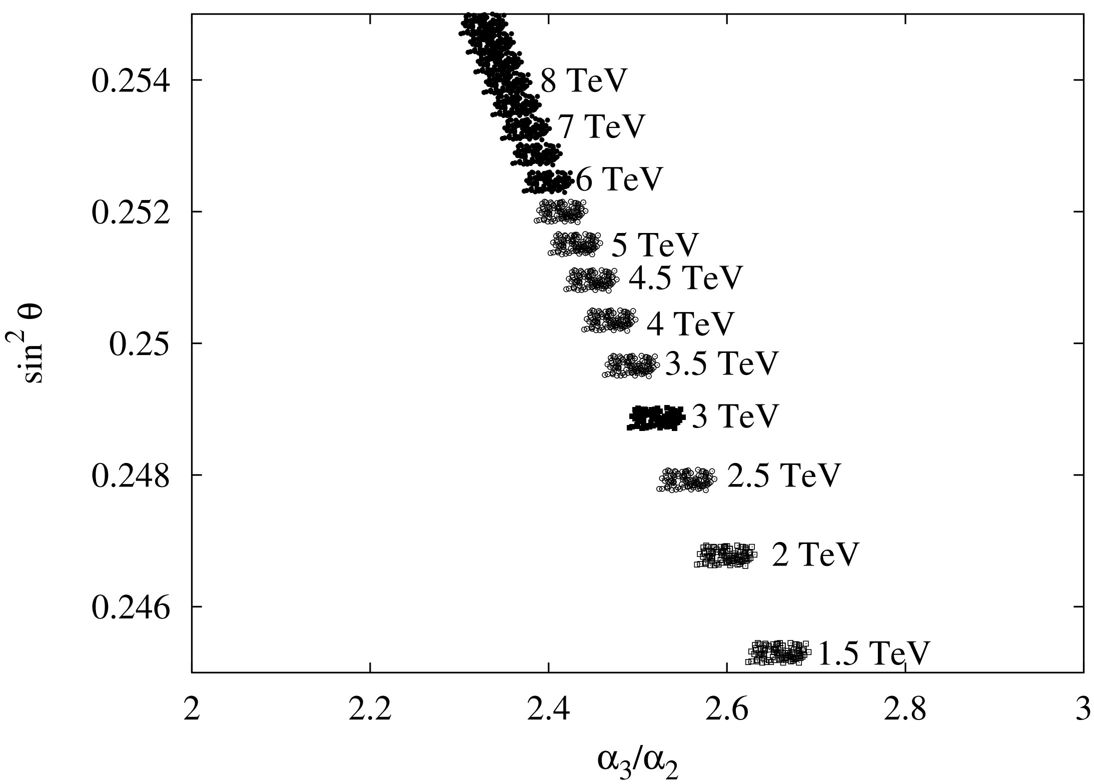
$$A_\mu = (1, 2, 3, 6)$$

$$SU(3)_C \times SU(3)_H \times SU(3)_H$$
$$5(3, \bar{3}, 1) + 2(\bar{3}, 3, 1)$$



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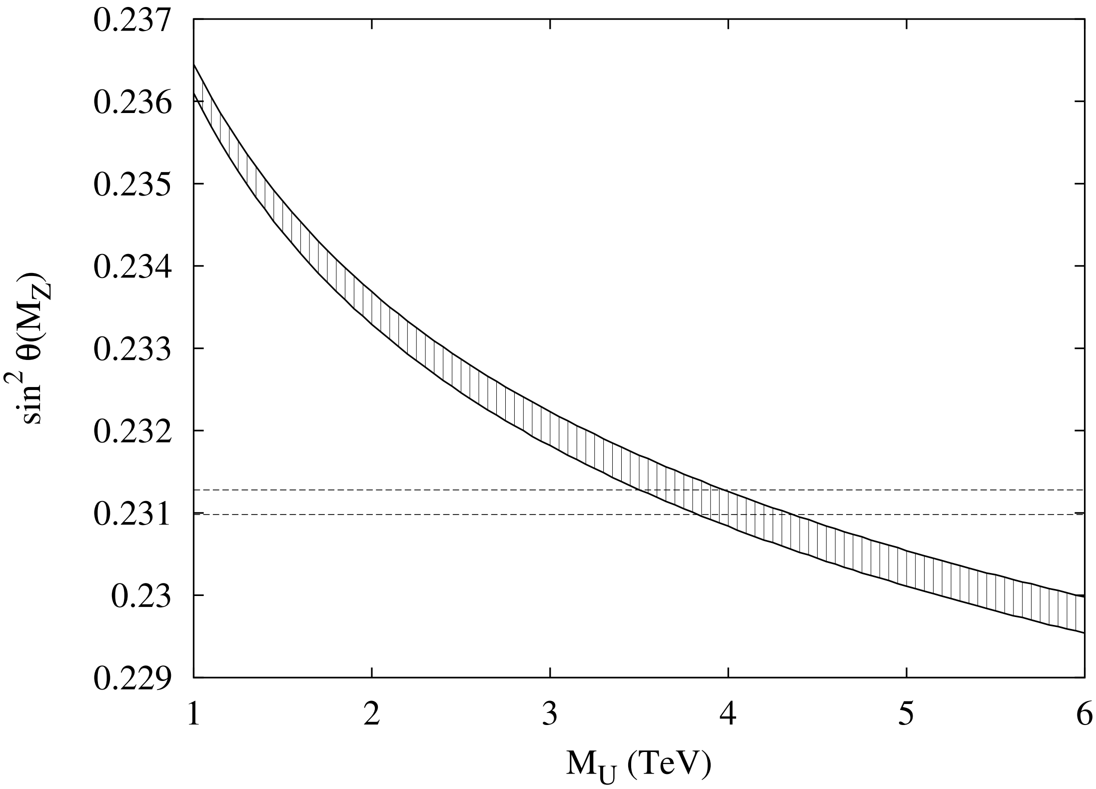
robustness of unification

hep-ph/0302074. PHF+Ryan Rohm + Tomo Takahashi



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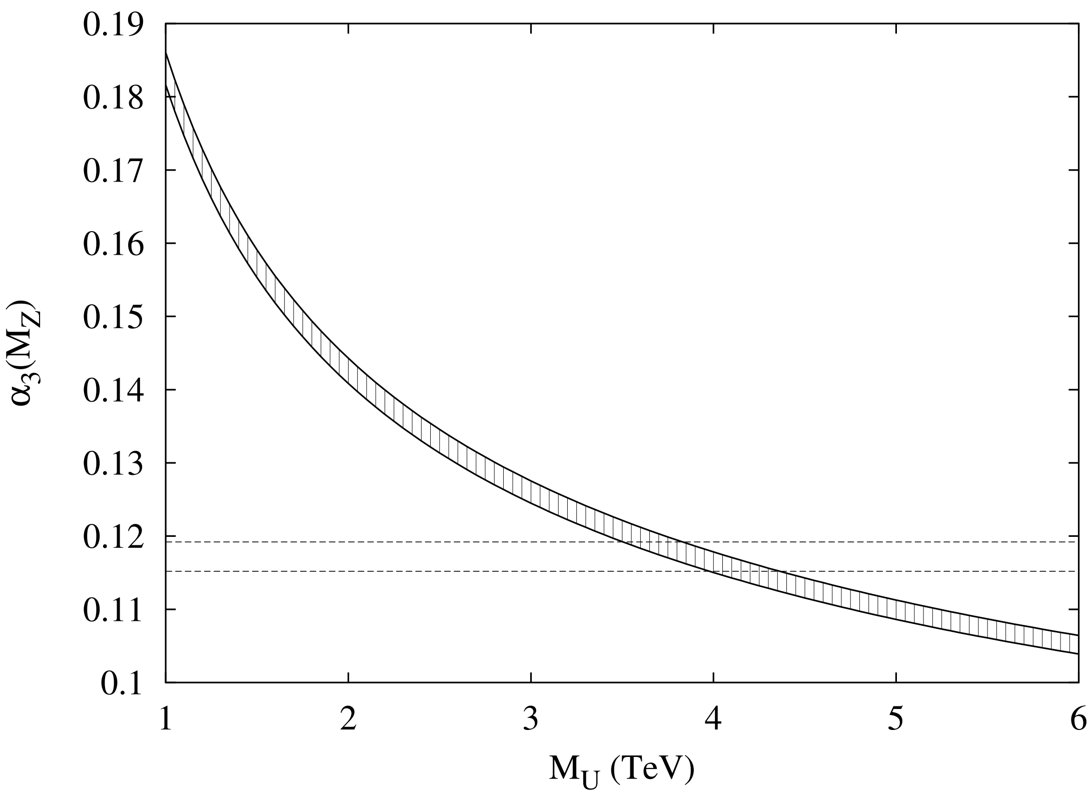
predictivity
hep-ph/0302074
PHF+Ryan Rohm + Tomo Takahashi



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predictivity
hep-ph/0302074
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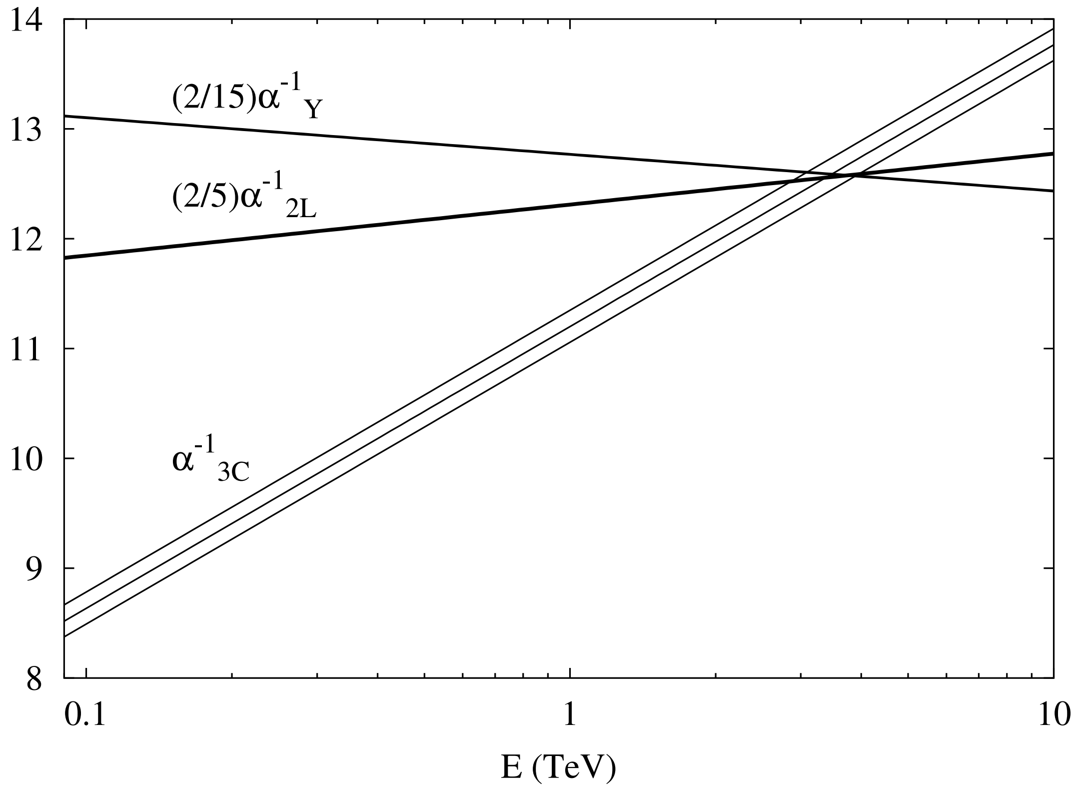

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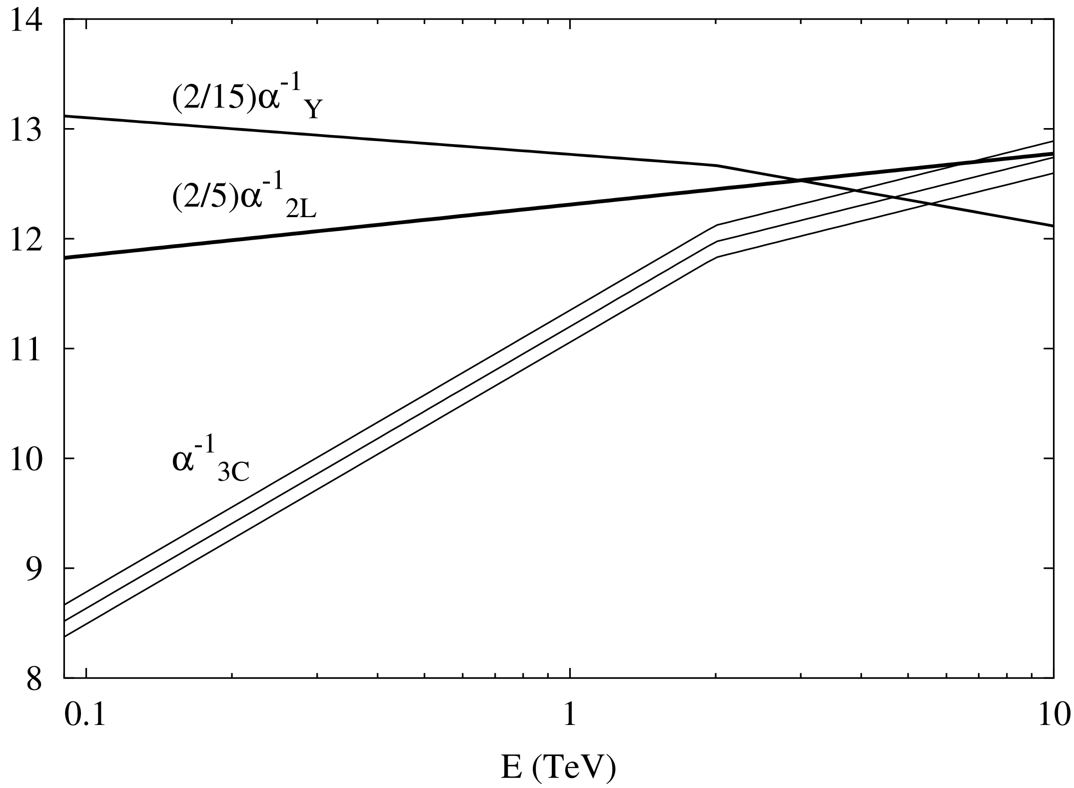
without thresholds: all states at M_U
hep-ph/0302074
PHF+Ryan Rohm + Tomo Takahashi



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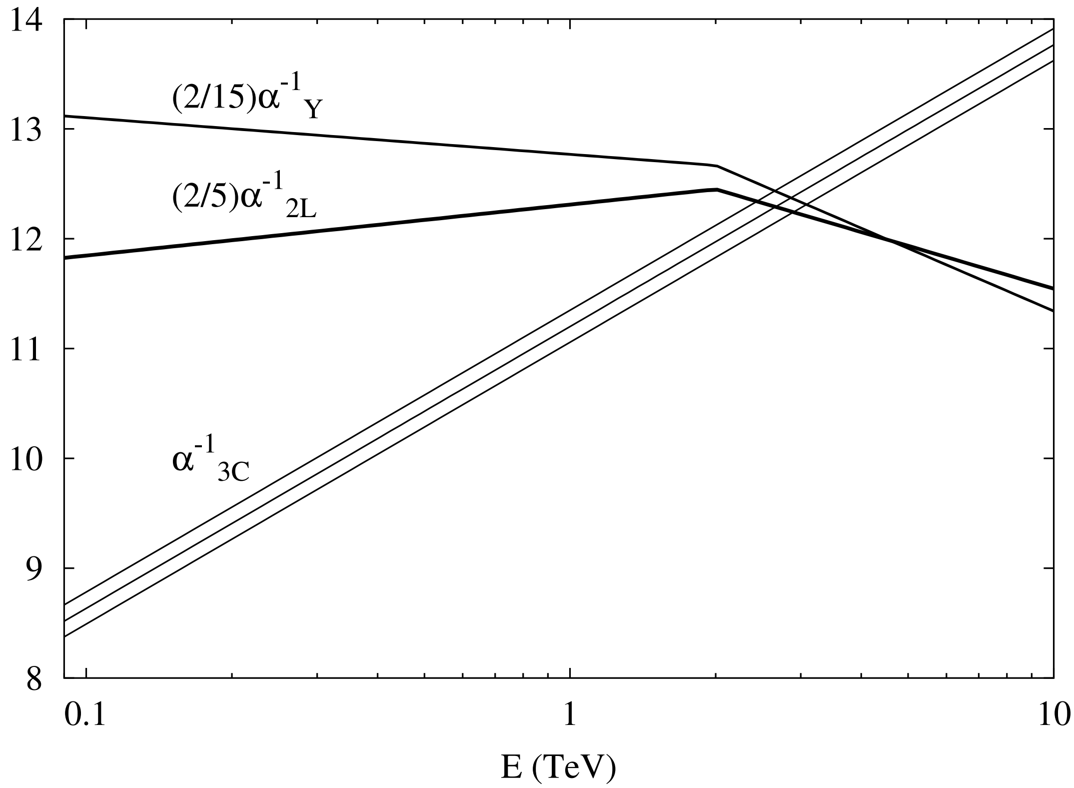
thresholds: CH fermions at 2 TeV
hep-ph/0302074
PHF+Ryan Rohm + Tomo Takahashi



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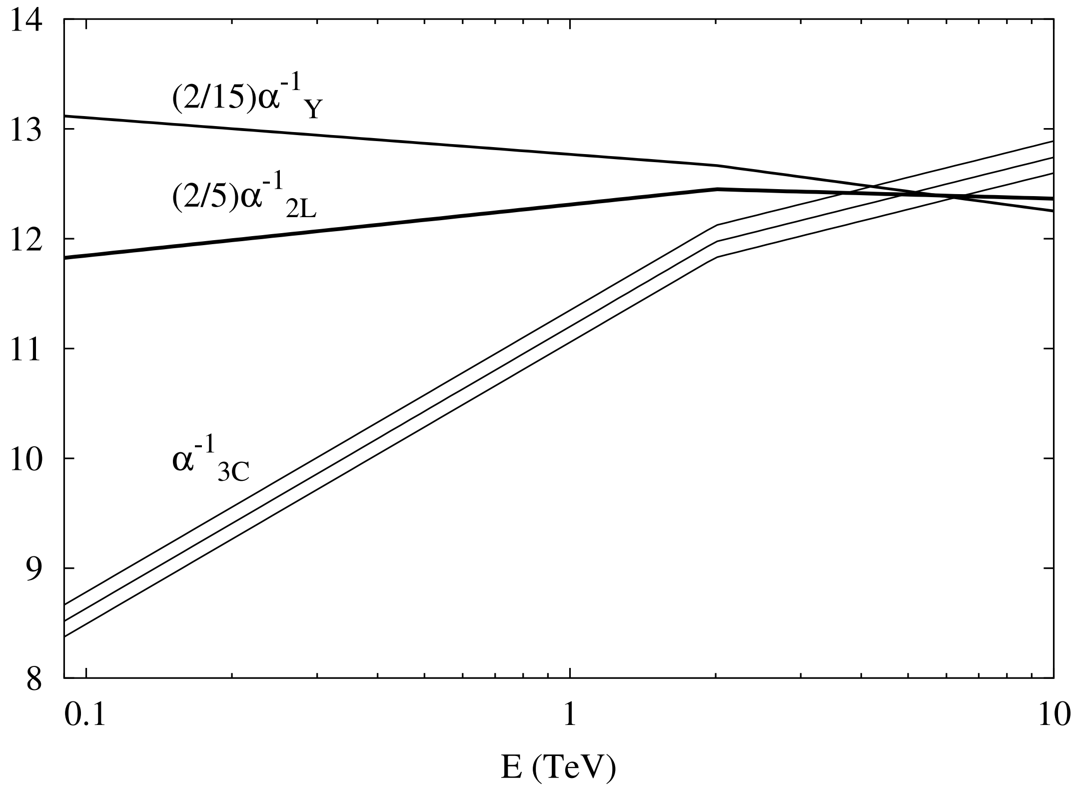
thresholds: WH fermions at 2 TeV
hep-ph/0302074
PHF+Ryan Rohm + Tomo Takahashi



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thresholds: CW fermions at 2 TeV
hep-ph/0302074
PHF+Ryan Rohm + Tomo Takahashi

SUMMARY

- Strong-Electroweak Unification at About 4 TeV:
- Predicts **two** numbers ($sin^2\theta$, α_S) whereas SusyGUTs predict only **one**.
- Predicts three families.
- Ameliorates GUT/weak hierarchy.

Classification of abelian quiver gauge theories

We consider the compactification of the type-IIB superstring on the orbifold $AdS_5 \times S^5/\Gamma$ where Γ is an abelian group $\Gamma = Z_p$ of order p with elements $\exp(2\pi i A/p)$, $0 \leq A \leq (p-1)$.

The resultant quiver gauge theory has $\mathcal{N}$ residual supersymmetries with $\mathcal{N} = 2, 1, 0$ depending on the details of the embedding of Γ in the $SU(4)$ group which is the isotropy of the S^5. This embedding is specified by the four integers $A_m, 1 \leq m \leq 4$ with

$$\Sigma_m A_m = 0(\mathrm{modp}) \qquad (2)$$

which characterize the transformation of the components of the defining representation of $SU(4)$.

We

are here interested in the non-supersymmetric case $\mathcal{N} = 0$ which occurs if and only if all four A_m are non-vanishing.

The gauge group is $U(N)^p$. The fermions are all in the bifundamental representations

$$\Sigma_{m=1}^{m=4}\Sigma_{j=1}^{j=p}(N_j, \bar{N}_{j+A_m}) \tag{3}$$

which are manifestly non-supersymmetric because no fermions are in adjoint representations of the gauge group. Scalars appear in representations

$$\Sigma_{i=1}^{i=3}\Sigma_{j=1}^{i=p}(N_j, \bar{N}_{j\pm a_i}) \tag{4}$$

in which the six integers $(a_i, -a_i)$ characterize the transformation of the antisymmetric second-rank tensor representation of $SU(4)$. The a_i are given by $a_1 = (A_2 + A_3), a_2 = (A_3 + A_1), a_3 = (A_1 + A_2)$

It is possible for one or more of the a_i to vanish in which case the corresponding scalar representation in the summation in Eq.(4) is to be interpreted as an adjoint representation of one particular $U(N)_j$. One may therefore have zero, two, four or all six of the scalar representations, in Eq.(4), in such adjoints.

For the lowest few orders of the group Γ, the members of the infinite class of $\mathcal{N} = 0$ abelian quiver gauge theories are tabulated below:

Model No.	p	A_m	a_i	scalar bifunds.	scalar adjoints	chi fermi
1	2	(1111)	(000)	0	6	N
2	3	(1122)	(001)	2	4	N
3	4	(2222)	(000)	0	6	N
4	4	(1133)	(002)	2	4	N
5	4	(1223)	(011)	4	2	N
6	4	(1111)	(222)	6	0	Ye
7	5	(1144)	(002)	2	4	N
8	5	(2233)	(001)	2	4	N
9	5	(1234)	(012)	4	2	N
10	5	(1112)	(222)	6	0	Ye
11	5	(2224)	(111)	6	0	Ye
12	6	(3333)	(000)	0	6	N
13	6	(2244)	(002)	2	4	N
14	6	(1155)	(002)	2	4	N
15	6	(1245)	(013)	4	2	N
16	6	(2334)	(011)	4	2	N
17	6	(1113)	(222)	6	0	Ye
18	6	(2235)	(112)	6	0	Ye
19	6	(1122)	(233)	6	0	Ye

The Table continues to infinity but we stop at $p = 7$:

Model No.	p	A_m	a_i	scalar bifunds.	scalar adjoints	chi fermi
20	7	(1166)	(002)	2	4	N
21	7	(3344)	(001)	2	4	N
22	7	(1256)	(013)	4	2	N
23	7	(1346)	(023)	4	2	N
24	7	(1355)	(113)	6	0	N
25	7	(1114)	(222)	6	0	Ye
26	7	(1222)	(333)	6	0	Ye
27	7	(2444)	(111)	6	0	Ye
28	7	(1123)	(223)	6	0	Ye
29	7	(1355)	(113)	6	0	Ye
30	7	(1445)	(113)	6	0	Ye

Note that there is one model with all scalars in adjoints for each even value of p (see Model Nos 1,3,12). For general even p the embedding is $A_m = (\frac{p}{2}, \frac{p}{2}, \frac{p}{2}, \frac{p}{2})$. This series is the complete list of $\mathcal{N} = 0$ abelian quivers with all scalars in adjoints.

To be of more phenomenolgical interest the model should contain chiral fermions. This requires that the embedding be complex: $A_m \not\equiv -A_m$ (mod p). It will now be shown that for the presence of chiral fermions all scalars must be in bifundamentals.

The proof of this assertion follows by assuming the contrary, that there is at least one adjoint arising from, say, $a_1 = 0$. Therefore $A_3 = -A_2$ (mod p). But then it follows from Eq.(2) that $A_1 = -A_4$ (mod p). The fundamental representation of $SU(4)$ is thus real and fermions are non-chiral[1].

[1] This is almost obvious but for a complete justification, see Frampton & Kephart (2004)

It follows that:

In an $\mathcal{N} = 0$ quiver gauge theory,
chiral fermions are present
if and only if all scalars
are in bifundamental representaions.