3. We are to prove the Gordon identity,

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[\frac{(p'+p)^\mu}{2m} + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(p).$$

Explicitly writing out each term in the brackets and recalling the anticommutation relations of γ^μ, the right hand side becomes,

$$\begin{aligned}\bar{u}(p')\left[\frac{(p'+p)^\mu}{2m} + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(p) &= \bar{u}(p')\left[\frac{1}{2m}\left(p'^\mu + p^\mu - \tfrac{1}{2}\gamma^\mu\gamma^\nu(p-p')_\nu + \tfrac{1}{2}\gamma^\nu\gamma^\mu(p-p')_\nu\right)\right]u(p),\\ &= \bar{u}(p')\left[\frac{1}{2m}\left(p'^\mu + p^\mu - \tfrac{1}{2}\gamma^\mu\gamma^\nu(p-p')_\nu + g^{\nu\mu}(p-p')_\nu - \tfrac{1}{2}\gamma^\mu\gamma^\nu(p-p')_\nu\right)\right]u(p),\\ &= \bar{u}(p')\left[\frac{1}{2m}\left(2p'^\mu - \gamma^\mu\gamma^\nu(p-p')_\nu\right)\right]u(p),\\ &= \bar{u}(p')\left[\frac{1}{2m}\left(2p'^\mu - \gamma^\mu \not{p} - \gamma^\mu \not{p}'\right)\right]u(p).\end{aligned}$$

Now, recall that the Dirac equation for $u(p)$ is

$$\not{p}u(p) = mu(p).$$

Converting this for $\bar{u}(p')\not{p}'$, one obtains

$$\bar{u}(p')\not{p}' = m\bar{u}(p').$$

Applying both of these equations where we left of, we see that

$$\bar{u}(p')\left[\frac{(p'+p)^\mu}{2m} + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(p) = \bar{u}(p')\frac{p'^\mu}{m}u(p).$$

Looking again at the Dirac equation, $m\bar{u}(p') = \bar{u}(p')\not{p}' = \bar{u}(p')\gamma^\mu p'_\mu$, it is clear that

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[\frac{(p'+p)^\mu}{2m} + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(p).$$

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4. **a)** To demonstrate that $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ anticommutes each of the γ^μ, it will be helpful to remember that whenever $\mu \neq \nu$, $\gamma^\mu\gamma^\nu = -\gamma^\nu\gamma^\mu$ by the anticommutation relations. Therefore, any odd permutation in the order of some γ's will change the sign of the expression. It should therefore be quite clear that

$$\begin{aligned}\gamma^5\gamma^0 &= i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^0 = -i\gamma^1\gamma^2\gamma^3 = -i\gamma^0\gamma^0\gamma^1\gamma^2\gamma^3 = -\gamma^0\gamma^5;\\ \gamma^5\gamma^1 &= i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^1 = i\gamma^0\gamma^2\gamma^3 = -i\gamma^1\gamma^0\gamma^1\gamma^2\gamma^3 = -\gamma^1\gamma^5;\\ \gamma^5\gamma^2 &= i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^2 = -i\gamma^0\gamma^1\gamma^3 = -i\gamma^2\gamma^0\gamma^1\gamma^2\gamma^3 = -\gamma^2\gamma^5;\\ \gamma^5\gamma^3 &= i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^3 = i\gamma^0\gamma^1\gamma^2 = -i\gamma^3\gamma^0\gamma^1\gamma^2\gamma^3 = -\gamma^3\gamma^5;\\ &\therefore \{\gamma^5, \gamma^\mu\} = 0.\end{aligned}$$

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b) We will first show that γ^5 is hermitian. Note that the derivation relies on the fact that $(\gamma^0)^\dagger = \gamma^0$ and $(\gamma^i)^\dagger = -\gamma^i$. These facts are inherent in our chosen representation of the γ matrices.

$$\begin{aligned}(\gamma^5)^\dagger &= -i(\gamma^0\gamma^1\gamma^2\gamma^3)^\dagger,\\ &= -i(\gamma^3)^\dagger(\gamma^2)^\dagger(\gamma^1)^\dagger(\gamma^0)^\dagger,\\ &= i\gamma^3\gamma^2\gamma^1\gamma^0,\\ &= -i\gamma^2\gamma^1\gamma^0\gamma^3,\\ &= -i\gamma^1\gamma^0\gamma^2\gamma^3,\\ &= i\gamma^0\gamma^1\gamma^2\gamma^3,\\ &= \gamma^5.\end{aligned}$$