The calculation on the previous page clearly shows that particles that were created by $b^\dagger$ contribute oppositely to those created by $a^\dagger$ to the total charge. We concluded in Homework 2 that this charge was electric charge.

2. **a)** We are asked to compute the general, K-type Bessel function solution of the Wightman propagator,

$$D_W(x) \equiv \langle 0|\phi(x)\phi(0)|0\rangle = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\mathbf{p}}}e^{-ipx}.$$

Because x is a space-like vector, there exists a reference frame such that $x^0 = 0$. This implies that $x^2 = -\mathbf{x}^2$. And this implies that $px = -\mathbf{p}\cdot\mathbf{x} = -|p||x|\cos(\theta) = -|p|\sqrt{-x^2}\cos(\theta)$. We can then write $D_W(x)$ in polar coordinates as

$$\begin{aligned}
D_W(x) &= \frac{1}{(2\pi)^3}\int_0^{2\pi} d\phi \int_0^{\pi} e^{i|p|\sqrt{-x^2}\cos(\theta)} \int_0^\infty p^2dp\,\frac{1}{2\sqrt{p^2+m^2}},\\
&= \frac{1}{(2\pi)^2}\int_0^{\pi} d\theta\, e^{i|p|\sqrt{-x^2}\cos(\theta)} \int_0^\infty p^2dp\,\frac{1}{2\sqrt{p^2+m^2}},\\
&= \frac{1}{(2\pi)^2}\int_{-1}^{1} d\xi\, e^{i|p|\sqrt{-x^2}\xi} \int_0^\infty p^2dp\,\frac{1}{2\sqrt{p^2+m^2}},\\
&\quad (\text{where } \xi = \cos(\theta))\\
&= \frac{1}{4\pi^2}\int_0^\infty p^2dp\,\frac{1}{2\sqrt{p^2+m^2}}\frac{1}{i|p|\sqrt{-x^2}}\left(e^{i|p|\sqrt{-x^2}} - e^{-i|p|\sqrt{-x^2}}\right),\\
&= \frac{1}{4\pi^2\sqrt{-x^2}}\int_0^\infty dp\,\frac{p\sin(|p|\sqrt{-x^2})}{\sqrt{p^2+m^2}}.
\end{aligned}$$

Gradsteyn and Ryzhik's equation (3.754.2) states that for a K Bessel function,

$$\int_0^\infty dx\,\frac{\cos(ax)}{\sqrt{\beta^2+x^2}} = K_0(a\beta)).$$

By differentiating both sides with respect to a, it is shown that

$$-\int_0^\infty dx\,\frac{a\sin(ax)}{\sqrt{\beta^2+x^2}} = -\beta K_0'(a\beta) = \beta K_1(a\beta).$$

We can use this identity to write a more concise equation for $D_W(x)$. We may conclude

$$D_W(x) = \frac{m}{4\pi^2\sqrt{-x^2}}K_1(m\sqrt{-x^2}).$$

b) We may compute directly,

$$\begin{aligned}
iD(x) &= \langle 0|[\phi(x),\phi(0)]|0\rangle,\\
&= \langle 0|\phi(x),\phi(0)|0\rangle - \langle 0|\phi(0),\phi(x)|0\rangle,\\
&= D_W(x) - D_W(-x),\\
\implies D(x) &= i(D_W(-x) - D_W(x)).
\end{aligned}$$

Similarly,

$$D_1(x) = \langle 0|\{\phi(x),\phi(0)\}|0\rangle = D_W(x) + D_W(-x).$$

It is clear that both function 'die off' very rapidly at large distances. I was not able to conclude that they were truly vanishing, but they are certainly nearly-so at even moderately small distances.