Development of a Learning Progression for Students' Conceptions of Size and Scale

Cesar Delgado, Shawn Y. Stevens, and Namsoo Shin

University of Michigan, Ann Arbor

Abstract

The concepts of size and scale are important in the practice and learning of science. Prior research shows many areas of difficulty or lack of knowledge for learners related to size and scale. Thus, there is a need for improved curriculum, instruction, and assessment for size and scale. Learning progressions organized around "big ideas" – such as size and scale - can guide the principled development and alignment of curriculum, instruction, and assessment. This study assesses the accuracy of knowledge about the size of important scientific objects like the atom and the cell through four aspects of size and scale: ordering by size, grouping by size, estimating size relative to a reference object, and estimating absolute size. Participants include 41 students in grades 7-11 in a diverse, low-mid SES public school district, and 6 undergraduates in a research university. The students' consistency across aspects of size and scale was previously assessed, and found to develop in a predictable progression. The accuracy of students' content knowledge of the size of the objects is mapped onto their level of consistency in order to generate a learning progression that describes the growth of conceptual understanding of size and scale.

Development of a Learning Progression for Students' Conceptions of Size and Scale

Size is a characteristic of every object, and is the magnitude or extent of the object. Size is established by comparing the object to another object or a conventionally defined measure, that is, a scale. In turn, scale refers to "the spatial, temporal, quantitative, or analytical dimensions used by scientists to measure and study objects and processes" (Gibson, Ostrom, & Ahn, 2000, p. 219). Size and scale are indissolubly linked; size is defined by comparison to a scale. This study is concerned with what learners know and how they think about the linear measure of length (e.g., the diameter of a red blood cell). The focus is on learners' understanding of the size of "submacroscopic" objects (objects too small to see with the unaided eye).

The concepts of size and scale are important both in science, and in science learning. Scale has been called "the quintessential aspect of every physical theory" (Bazant, 2002), a fundamental conceptual problem in ecology (Levin, 1992) and "one of the major gateways to the modern world of science" (Hawkins, 1978). According to *Science for all Americans*, "Science is a process for producing knowledge. The process depends both on making careful observations of phenomena and on inventing theories for making sense out of those observations." (Rutherford & Ahlgren, 1990, Ch. 1). In both theory and observation, size and scale are important. Concerning theory, size was one of few perceptible characteristics of the atom in classical Greek and seventeenth century European theories of matter (Berryman, 2004; Chalmers, 2005); the atomic nature of matter is arguably the most important scientific hypothesis (Feynman, 1963, I, i 1-2). Concerning observation, every physical object can be characterized partially in terms of its size. Some regular physical objects can be well specified by just three dimensions: size, material, and shape (e.g., a copper sphere with a diameter of 10 cm). In the application of scientific knowledge, scale is a paramount consideration. Objects or organisms of different sizes behave differently, even if scaled up faithfully. As Haldane (1928) pointed out:

> You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, a horse splashes.

US standards documents in science and mathematics identify scale or the related concept of measurement as concepts that pervade science and math, and which can be used to unify student learning across disciplines, topics, and grades; they are tools that help students understand the world (American Association for the Advancement of Science [AAAS], 1993, Ch. 11; National Research Council [NRC], 1996; National Council of Teachers of Mathematics [NCTM], 1989). Thus, size and scale are essential conceptual tools in many traditional content areas in science. As new fields of science and technology emerge, science instruction and curriculum materials need to change accordingly. One such emerging field is nanoscale science and technology (Gilbert, De Jong, Justi, Treagust, & Van Driel, 2002, p. 395). The nanoscale is defined by the size of the objects it studies, between one and 100 billionths of a meter in one or more dimensions. Objects at this scale behave differently than both the bulk (macro-level) materials we are accustomed to, and smaller, atomic-sized objects. The greatly increased surface area-to-volume ratio of nanoscale objects – a size-dependent quantity – is responsible for many of the interesting properties and behaviors of these objects. A “firm grasp on size and scale [is] a prerequisite for any further inquiry into nanoscale science and engineering” (Waldron, Sheppard, Spencer, & Batt, 2005, p. 375).

However, current curriculum and instruction may not be successfully addressing size and scale, for research has identified many areas of difficulty or lack of knowledge for learners that are related to size and scale (e.g., Wilensky & Resnick, 1999; Montello & Golledge, 1998; Tretter, Jones, Andre, Negishi, & Minogue, 2006; Tretter, Jones, & Minogue, 2006; Waldron et al., 2005; Castellini et al., 2007; Waldron, Spencer, & Batt, 2006; Jones, Tretter, Taylor, & Oppewal. 2007). Thus, there is a need for improved curriculum, instruction, and assessment for size and scale.

Recent publications have suggested that "learning progressions" can guide the principled development of effective curriculum, instruction, and assessment for science (Duschl, Schweingruber, & Shouse, 2007; Smith, Wiser, Anderson, & Krajcik, 2006; Wilson & Bertenthal, 2005). Learning progressions are "descriptions of the successively more sophisticated ways of thinking about a topic that can follow one another as children learn about and investigate a topic over a broad span of time (e.g., 6 to 8 years)." (Duschl et al., 2007, p. 214). Learning progressions should be organized around "big ideas" – the central, core concepts in a domain. Size and scale is one of nine big ideas for nanoscale science and engineering (Stevens, Sutherland, Schank, & Krajcik, 2007). While there is some previous research focused on the content knowledge of learners regarding the size of submacroscopic objects, the questions of what characterizes robust conceptual understanding of size, and how this understanding develops, have not been fully addressed.

The goal of our study is to develop an empirically based learning progression that characterizes the development of middle school to undergraduate students' conceptual understanding of important aspects of one-dimensional size and scale. This learning progression for size and scale can guide the design and alignment of curriculum, instruction, and assessment.

Background

Four Important Aspects of Size and Scale

This study focuses on four aspects of size and scale: ordering, grouping, size relative to a reference object, and absolute size. These four aspects can be used to think about size in one, two, or three dimensions, although the focus in this study is on one-dimensional size: length. These four aspects roughly correspond to the words *big* (grouping), *big/bigger/biggest* (ordering), *how big* (absolute size), and *how much bigger* (relative size) (and equivalent words for small).

Ordering. Ordering by size (e.g., atom <cell<ant<human<earth) has been investigated by studies both classic (e.g., Inhelder & Piaget, 1959/1969) and recent (e.g., Tretter, Jones, Andre, et al., 2006; Castellini et al., 2007). Ordering given submacroscopic objects by size is difficult for learners of all age groups (e.g., Waldron et al., 2006). Ordering is a non-quantitative way of expressing the relative size of several objects.

Grouping. Grouping by size involves placing together objects of similar size (e.g. {atom, water molecule}<{bacterium, cell}<{ant, flea}). Expert scientists conceptually group the objects they study into "worlds" that share units and tools used to study them (Tretter, Jones, & Minogue, 2006).

Size relative to another object. Size relative to another object is at the core of measurement (NCTM, 1989; Wiedtke, 1990), and involves a quantitative comparison of the relative sizes of two objects (e.g., an atom is ~50,000 times smaller than a red blood cell in diameter). Unitizing (Lamon, 1994) is a common strategy of experts - they express the size of one object in terms of another, landmark object (Tretter, Jones, Andre, et al., 2006). Vergnaud (1988) reports that not all students at the end of elementary school understand expressions like “three times more”, and that “three times less” was often interpreted as being of a subtractive rather than multiplicative nature (p. 156).

Absolute size. Absolute size involves stating dimensions in terms of a conventionally defined unit (e.g., “an atom is ~0.1 nm in diameter”). Learners of all ages have trouble estimating the size of very small and very large objects (Tretter, Jones, Andre, et al., 2006). Sowder (1992) summarizes various studies showing that students and adults are poor estimators, and students do not understand the nature of estimation.

The four aspects are logically related. Consider the diameters of several sports balls: table tennis (4 cm), tennis (6.5 cm), baseball (7.5 cm), volleyball (21 cm), and soccer (22 cm). Knowing the absolute size of the sports balls informs their ordering by size: table tennis<tennis<baseball<volleyball<soccer; it also allows one to calculate the size of one ball relative to another (e.g., the soccer ball is 22/7.5 = 2.9 times bigger in diameter than the baseball). Conversely, knowing that the diameter of the soccer ball is 5.5 times bigger than the table tennis ball’s, along with the diameter of one ball, allows one to calculate the absolute size of the other. The balls can also be organized into groups, such as {table tennis}<{tennis, baseball}<{volleyball, soccer}, based on their relative or absolute sizes. (Of course, if these sports balls were compared to much smaller and/or larger objects, the balls might all be grouped together instead.)

Relating the four aspects identified above results in greater conceptual understanding. For instance, knowing that a red blood cell’s diameter is around 6 micrometers (absolute size) is of little use in and of itself. This knowledge becomes useful when the learner can, for instance: relate that known diameter to the diameter of atoms, molecules, and viruses (which are smaller), and of dust mites or pollen (which are larger), thus ordering; envision the cell as one of a group of "microscale" objects that are too small to be seen with the unaided eye but can be studied with an optical microscope, and whose size can be conveniently expressed in micrometers (grouping); and to know that the cell is ~50,000 times larger than an atom, ~10 times larger than a typical bacterium, and ~20 times smaller than the thickness of a hair (size relative to other objects).

Our previous work (Delgado, Stevens, Shin, & Krajcik, 2007) has examined whether students understand and use the links among the four aspects of size and scale, by testing whether they provide consistent answers when ordering, grouping, estimating relative size, and estimating absolute size of a set of scientifically relevant objects. For example, a student who – erroneously or correctly - believes that object A is 100 times smaller than object B, is *consistent* if she then estimates the diameter of A as 1/100 mm when informed that object B has a diameter of 1 mm. A student who orders five objects A<B<C<D<E (whether accurately or inaccurately), must include object B in a group that contains A and C, since B is intermediate in size; to not include B in the group is to order and group *inconsistently*.

We found that most middle and high school students provide answers that are *not* entirely consistent, independent of the accuracy of their content knowledge. We also found that not all linkages across aspects are equally common, as Table 1 below shows (see bottom row). Consistency across the two non-quantitative aspects, ordering and grouping, is the most common, present in 40 of 47 students. Consistency between ordering and relative size is the next most common (36/47 students), followed by consistency between ordering and absolute size (30/47). The least common is consistency across the two quantitative aspects, relative and absolute size. Only 15 of 43 students thought the two aspects were strongly related (denoted as absolute-relative conceptual); and just 6 of 47 students were able to produce consistent estimates (absolute-relative procedural).

Table 1.

Consistency among aspects of size and scale.

Order-Group	Order-Relative	Order-Absolute	Absolute-relative (conceptual)	Absolute-relative (procedural)	Level
✔	✔	✔	✔	✔	5 (N=6)
✔	✔	✔	✔		4 (N=8)
✔	✔	✔			3 (N=13)
✔	✔				2 (N=3)
✔					1 (N=6)
					0 (N=4)
N=36	N=30	N=27	N=14	N=6	

Furthermore, we found that 40 of 43 students with complete data could be placed on one of six levels (termed levels 0-5) as shown by the rows in Table 1. Three of the four students with missing data are consistent with these levels, as well. While our cross-sectional design does not follow the learning of individual students over time, these findings strongly suggest and are consistent with a developmental trajectory in which a learner first acquires consistency between ordering and grouping, followed by ordering and relative size, then ordering and absolute size, and finally absolute and relative size.

This is a clear developmental trajectory, consistent with 43 of 47 participants. However, it is a trajectory that does not incorporate the accuracy of knowledge about the size of scientific objects. Students can be entirely consistent while holding inaccurate ideas about the size of the objects in the tasks. This study continues our investigation by characterizing students' knowledge related to the size of objects (science content knowledge), in terms of the four aspects of interest, and examining whether the level of content knowledge is related to students' level of consistency (which reflects understanding of the logical-mathematical relation among the four aspects of size and scale). In this way, we continue to develop a learning progression for aspects of size and scale that is relevant to science education practitioners, curriculum developers, and researchers.

Research Questions

1) What is students' content knowledge of the size of key scientific objects at different grade levels?
2) How does content knowledge regarding the size of objects map onto the levels of consistency across aspects of size and scale?
3) What does a tentative learning progression for aspects of size and scale look like?

Methods

Instrument

In order to explore how students develop conceptual understanding of size and scale, we created an interview protocol that asks open-ended questions with precise wording, following Patton's (2002) standardized, open-ended format (see Appendix A). This interview is designed to assess each student's ability to order by size, group by size, estimate size relative to a small macroscopic object (the head of a pin), and estimate absolute size, using several key objects. It thus tests specific knowledge of the submacroscopic world via the four aspects of size described earlier. These same tasks also probe whether students are consistent across these aspects. This protocol was assessed for content validity by four science and science education experts with PhDs, and revised after trials with a small number of middle school students. The interview protocol allows for extensive probing and clarifications when needed, and asks the respondent for explanations of their responses, in an effort to ensure that students are responding to the prompts as intended. Interviewer and rater triangulation add to the validity of the instrument.

In the audiotaped interview, respondents are asked for the smallest object they can think of (an aspect of ordering: A < {all other objects being considered}), and the units with which to express its size (related to absolute size). They then order 10 cards (see Appendix B) depicting: an atom, a small molecule, a virus, a mitochondrion, a red blood cell, the head of a pin, an ant, a human, a mountain, and the earth, by the actual size of the objects; after which they group them by size. For four objects (atom, red blood cell, human, earth), students estimate the number of times bigger or smaller the diameter (or height, for human) of each object is than the diameter of the head of a pin. These objects are representative of important size regimes and are also important scientific objects. Their answers are recorded on a sheet (see Appendix C) and available to them for the next task. Finally, students estimate the absolute size for the four objects, given that the diameter of the head of the pin is around 1 mm (an actual pin is shown to them). For the relative and absolute tasks, if the students did not rank atom<cell<pinhead, but did have one of these and another submacroscopic object ranked smaller than pinhead, we substituted objects (e.g., for a student who ordered mitochondrion < virus < cell < atom < molecule < pinhead, we would use atom and molecule to compare to pinhead). However, with students who ranked pinhead as smallest or second smallest, we could not carry out the relative and absolute tasks; they are shown as missing data. The first two questions concerning the smallest object and unit precede the card tasks. The order was chosen so as not to predispose student answers. The use of the same objects across the four tasks allows us to determine whether a student's answers are consistent across aspects.

Participants

We interviewed 41 7-11th grade students at low-mid SES, ethnically/racially diverse small urban public school district; and 6 undergraduates at a research university, all in the Midwestern US. See Table 2 below. We used stratified purposeful sampling (Patton, 2002) in order to obtain results that both generalize to some degree, and shed light on differences by gender, race/ethnicity, and academic ability level (as assessed holistically by their science teachers). We did not interview any 6th, 8th or 12th graders due to logistical constraints.

Table 2

Participants

Grade	Number of Students
7th	8
9th	11
10th	6
11th	16
Undergraduate	6
Total	47

Analysis

Research question 1. In order to assess students' content knowledge of the size of key scientific objects, we created a coding rubric with hierarchical categories based on theory to characterize accuracy of student responses (see Appendix E). Five interview tasks were coded on scales of 0-4 or 0-5: the smallest object respondent knows of; the smallest unit; 10-card ordering; estimates for size relative to a pinhead for atom, cell, human, and earth; and absolute size estimates for the same 4 objects. Additionally, a non-hierarchical coding scheme with 6 levels for grouping was generated. The first author coded all the data, while a second rater coded a part of the data to ensure inter-rater reliability. After one round of coding and editing the rubric for clarity, inter-rater reliability above 80% was obtained on 10% of the data. All differences were resolved by discussion. The interviews were coded from a summary of the recording (see Appendix D for a sample) and the answer-recording sheet (see Appendix C).

We use grade level to organize our analyses because we expect students who are in higher grades to have taken more science courses, and thus to know more factual knowledge about the size of objects like the cell, atom, and earth. Additionally, prior research has described mean student performance by grade or age, so by organizing our analysis by grade we are able to compare our findings to those in the literature.

Research question 2: In order to begin to examine the relationship between the development of consistency across aspects and the growth of content knowledge, we calculate correlation coefficients between these two. We compare the magnitude of the correlation to the correlation between content knowledge and science course or grade. These correlations are calculated for each content task (smallest object and unit, ordering, grouping, etc.), as well as for the overall content knowledge score produced by summing the codes on the content tasks. The grouping task is not included in this analysis, due to its non-hierarchical nature.

Research question 3: The content knowledge of students is organized by level of consistency, resulting in a learning progression that addresses both the increased coherence and consistency of students' ideas about size, and their content knowledge about the size of scientifically important objects.

Results and Discussion

Students' knowledge of the size of key scientific objects at different grade levels.

Next, we present our results and discussion for each task.

Smallest object. Students were asked for the smallest object they could think of. If they responded with a macroscopic object (e.g., an ant or grain of salt), they were prompted for an object too small to see. The results are presented in Figure 1 below.

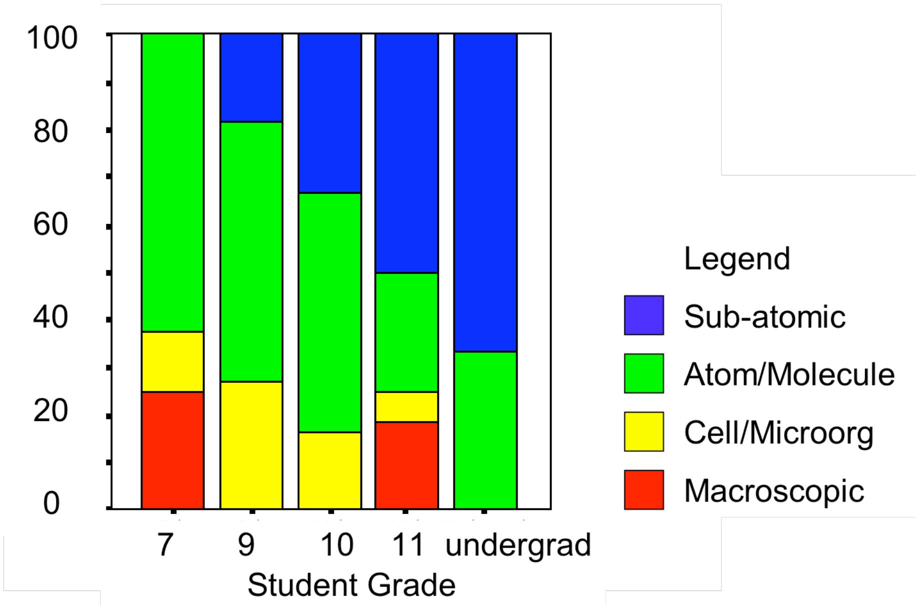


Figure 1. Smallest object respondent knows, by student grade.

Just two 7th grade and three 11th grade students responded with a macroscopic object even after prompting; these are shown in red in Figure 1 above. Three 9th graders and one student in each of 7th, 10th, and 11th grades responded with a cell or microorganism, as shown in yellow above. A large but gradually decreasing percentage of students answered with atom or molecule, while a gradually increasing percentage of students responded with a sub-atomic particle such as electron, proton, or quark. No 7th grade students mentioned sub-atomic particles, which is as expected, since the structure of the atom is usually introduced in high school science classes. All undergraduates responded with atom, electron, or quark. The gradual increase in sophistication of answers is evident.

A recent article presents results from a similar question, presented in a survey format to 495 people of all ages (Castellini et al., 2007). The authors used categories equivalent to ours: small visible object (macroscopic), microscopic object (cell/microorganism), atom, and sub-atomic particle, with an additional category for "answers that were not objects, but measurements, answers that could not be deciphered, and nonsense objects." (p. 184). They reported results by grade bands, including 6-8th grade, 9-10th grade, 11-12th grade, and college-educated. By combining our results for 9th and 10th grade students, we are able to compare our findings with those of Castellini and colleagues, as shown below in Table 3. The findings are broadly similar, with older respondents usually giving better answers (i.e., smaller objects). In both studies, the proportion of sub-atomic responses gradually increases with school level. Castellini and colleagues (2007) found 9-10th graders responding atom at a lower proportion than middle school or 11-12th grade students, but we did not observe this sudden drop. The Castellini survey includes as many as 14% of "nonsense" answers outside our four categories, whereas we did not require such a category; the prompting and clarification available in our interview format precluded such answers.

A survey of 1500 individuals of all ages by Waldron and colleagues (2006) included the question, "What is the smallest thing you can think of?" Around 45% of 11-13 year olds responded with a macroscopic object – a much higher percentage than in the two studies included in Table 3 below, for 7th-8th graders: 16% (Castellini et al., 2007) and 25% (this study). The proportion dropped to around 32% for the 14-17 year old group (roughly corresponding to high school) and to around 29% for 18-22 year olds (corresponding to college ages, though not all respondents in this study were college-educated). Waldron and colleagues (2006) found many more macroscopic answers for these age ranges as well. This survey asked about the smallest

Table 3.

Comparison of findings about smallest object respondent knows, in percentages.

	6-8th *(7th)*		9-10th *(9-10th)*		11-12th *(11th)*		College adult *(undergraduate)*	
	Castellini	*Delgado*	C	*D*	C	*D*	C	*D*
Nonsense	2	*0*	14	*0*	9	*0*	8	*0*
Macroscopic	16	*25*	18	*0*	5	*19*	5	*0*
Microscopic	23	*13*	21	*24*	8	*6*	11	*0*
Atom/molecule	57	*63*	36	*53*	46	*25*	33	*33*
Sub-atomic	2	*0*	12	*24*	32	*50*	45	*67*

Legend: Castellini et al. (2007), unitalicized, header=C; our findings, *italicized*, header=*D*.

Note: Most of the numbers for Castellini were not directly reported in the article and were read from their graph; thus, the numbers reported have a small uncertainty.

object the respondent could *see* immediately before asking about the smallest object the respondent could *think of*, possibly predisposing some respondents to continue thinking of macroscopic objects.

Tretter, Jones, Andre, and colleagues (2006) asked 215 students (from 5th grade to graduate students) to place 26 objects ranging from atomic nucleus to the distance between the earth and the Sun, into size ranges. This task is not directly comparable to responding about the smallest object one can think of; however, middle and high school students aggregately ranked the atom as larger than a grain of rice (on the "absolute" task) and larger than the thickness of a hair (on the "relative" ordering task). This implies that many students must have ranked some macroscopic objects as smaller than some submacroscopic objects.

We also organized our data by the science course students were enrolled in, to look for possible effects of curriculum on students' answers (at the high school level there is some variation in the science course students take in each grade). Figure 2 below shows our findings, with the courses ordered by the sequence in which they are offered at the school district. Fully half of the Biology students responded with a cell or microorganism, substantially higher than for any other class. The proportion of students answering with atom or sub-atomic particle increases from Biology to Chemistry, while cell/microorganisms are no longer represented in Chemistry student responses (although three students in Chemistry answered with a macroscopic object.) Physics students responded with a sub-atomic particle at a higher proportion even than undergraduate students. Thus, it appears that the science course a student is currently taking has a strong impact on the knowledge about objects too small to see that students access when probed. We interpret this finding as suggesting that analyses of student knowledge about the size of objects be organized by their science course in addition to or instead of simply by age or grade level, in order to gain insights about the effects of curriculum and instruction.

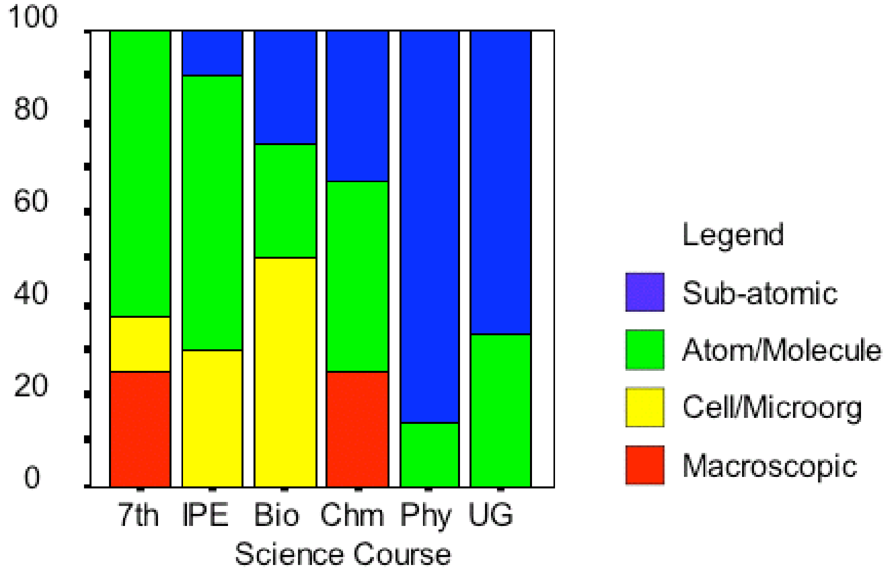


Figure 2. Smallest object respondent knows, by science course.

Smallest Unit. After answering the question about the smallest object they could think of, students were asked about a measurement unit with which to express the size (length or diameter) of that object. Students who replied that they did not know were asked for the smallest unit of measurement they did know of. Figure 3 below shows our findings. The categories for coding include does not know or non-length unit (e.g., nanoliter), "macroscopic" unit (a unit that is of macroscopic size, e.g., a millimeter or inch), a fraction of a macroscopic unit that would be submacroscopic (e.g., a thousandth of an inch), and "submacroscopic" unit (a unit that is of submacroscopic size, e.g., a micrometer or nanometer). Initially, we distinguished between English and metric macroscopic units but these are presented as one in Figure 3 below – only one 7th grader answered with an English unit, the inch. The five students who responded with macroscopic objects are not included in Figure 3, but responded with macroscopic units or did not know (one student was not asked about units.)

Between 40 and 80% of pre-college students at each grade could not provide a unit of length for submacroscopic objects. A few younger students replied using a fraction of a macroscopic unit. While all undergraduates answered with a submacroscopic unit, only two 9th grade and two 11th grade students did so. Surprisingly, all 10th grade students replied with a macroscopic unit or did not know, whereas at least some 7th, 9th, and 11th graders knew of submacroscopic units or thought of using a fraction of a millimeter or inch. Similarly, a graph organized by science course (not shown) revealed that the Chemistry students all replied with a macroscopic unit or did not know. This graph also showed that the pre-college students who knew of submacroscopic units were either in Physics or Integrated Physical and Earth Science courses, again showing the possible influence of current science course upon the knowledge students draw on in addressing the tasks.

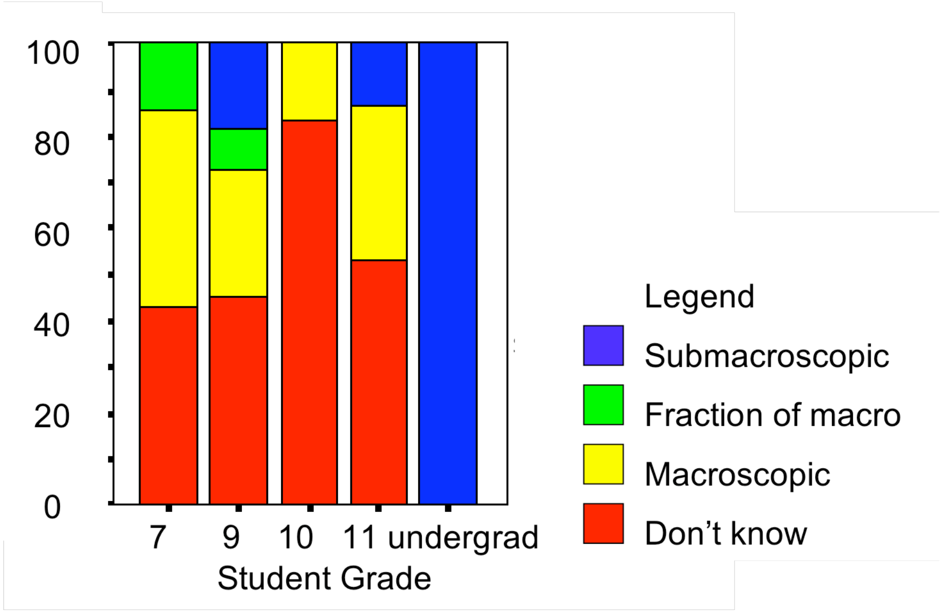


Figure 3. Smallest unit respondent knows, by grade.

This study reveals that while most students in grades 7 through undergraduate may know of objects too small to see, they are likely *not* to know of convenient units with which to express their size. This finding, that students struggle more with units than with objects, appears to contrast with Waldron and colleagues' findings that "Respondents of all ages were more successful ordering units of measure [millimeter, micrometer, and nanometer] than in putting 'germ', 'molecule' and 'atom' in correct size order" (p. 573). This discrepancy may be due to the difference between *producing* and *recognizing* objects or units.

Ordering by size. Students were asked to order by size the objects depicted on 10 cards, ranging from atom to earth. Answers were coded into 6 hierarchical categories. These categories follow the development of scientific knowledge as set out in the *Benchmarks for Science Literacy* (AAAS, 1993). Students first interact with the macroscopic world, then encounter the cell while still in elementary, followed by the atom, molecule, and virus in middle school (mitochondria are not mentioned in the *Benchmarks*). The lowest category (coded 0, and depicted by red in Figure 4 below) corresponds to errors in ordering the macroscopic objects (pinhead, ant, human, mountain, earth), or interspersing macroscopic and submacroscopic objects. The remaining categories require correct ordering of macroscopic objects. Responses that ranked cell smaller than atom were coded 1, shown in yellow in Figure 4 below. Code 2 corresponds to ordering the atom as smaller than the cell, though not as the smallest object. Code 3 was used for responses ranking atom smallest but cell *not* the largest of the submacroscopic objects. Code 4 included responses with atom smallest and cell largest of the submacroscopic objects, but errors among molecule, virus, and mitochondria. Code 5 was used for respondents who had all objects correctly ordered *and* could justify their ordering (students who ordered all correctly but who could not justify their ordering were coded 4). For purposes of clarity of visual presentation, codes 2 and 3 are combined into one category in Figure 4 below; this combined category is shown in green. Similarly, the two top codes are combined into one, shown in blue; only one of these students (an undergraduate) scored the top code by justifying the order.

This figure shows that a gradually increasing proportion of students are able to correctly rank the atom as the smallest and cell as the biggest of the submacroscopic objects, and correctly order the macroscopic objects (shown in blue). Most students at every age group were able to correctly rank the macroscopic objects, but some 7th, 9th, and 11th graders interspersed macroscopic and submacroscopic objects. These students included the two seventh-grade students who had macroscopic responses for smallest object, but also one 9th grade and two 11th grade students who had submacroscopic responses for smallest object but incorrectly ordered pinhead smaller than some submacroscopic objects. Except for undergraduates, every age group has 15% or more students who mistakenly believe that the cell is smaller than the atom. This

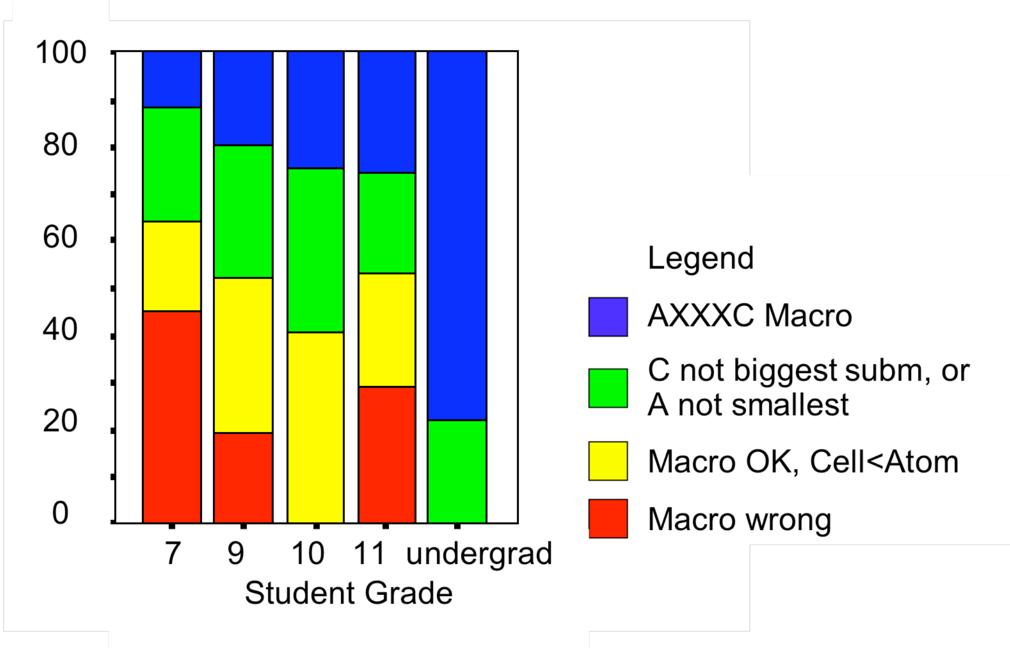


Figure 4. Ten-card ordering task, by grade.

percentage rises to 50% among biology students (graph not shown), possibly showing once more the influence of curriculum and instruction.

The survey by Castellini and colleagues (2006) described earlier asked respondents to rank cell, bacterium, atom, and water molecule. Only 7% of the respondents ordered them correctly, not markedly above chance (1/24 or 4%). This is low compared to our study, in which 4 students (8.5%) correctly ordered atom, molecule, virus, mitochondrion, red blood cell, and the macroscopic objects (albeit, only one justified the ordering). They also report that 45% of respondents correctly identified atom as the smallest object but erred in ordering the remaining objects; this compares to 55% in our study.

The study by Waldron and colleagues (2006) mentioned earlier reports that only 15% of 11-13 year olds were able to correctly order germ, molecule, and atom by size. Given that there are only 6 permutations among 3 objects, this proportion is around chance (1/6=17%). In comparison, two out of seven 7th graders (29%) in our study accomplished a similar task - ordering atom, molecule, and red blood cell correctly – within the context of the 10-card sort. It may be that our students are performing at a higher level than Waldron's respondents due to the clarifications and prompts offered by the interview format; however, Waldron and colleagues (2006) report that they found that answers during interviews mirrored their survey responses.

The "relative" ordering task study by Tretter, Jones, Andre, and colleagues (2006) described earlier involved students placing 26 objects into size ranges. From this information, they calculated the mean rank of each object for each grade range. Some of these objects correspond to the ones used in this study: atom, virus, cell, and ant (similar in size to the head of a pin). Reconstructing from their Table 1 (p. 290), the average ordering for middle school students is: cell<atom<virus<ant. This ordering corresponds to a code 1 (yellow in Figure 4

above) in this study. Interestingly, cell was on average ranked as the very smallest object, followed by the thickness of a hair. The average ordering for high school is cell<atom<ant<virus, which corresponds to a code of 0 (red), with bacterium and hair as the smallest and second smallest objects respectively.

Estimating size relative to a pinhead. For the next task, we asked students to estimate how many times bigger or smaller the diameter of an atom, a red blood cell, the earth, and the height of an adult human are compared to the head of a pin (shown to them; diameter 1 mm). Student responses were coded as 1 for accurate if they were within one order of magnitude (ten times) of the accepted value, 0 otherwise, for a total of 4 points possible. Students who did not rank two or more objects smaller than the pinhead were not evaluated on this task; they are excluded from Figure 5 below.

There is a lack of consensus in the literature regarding the range of values that should be acceptable as a function of the magnitude of the value itself. Sowder (1992) provides an argument for allowing a greater range for objects that are a larger number of times bigger or smaller than the reference object:

> … 'reasonableness' can vary with the size of the numbers so that a straight percentage criterion does no seem valid. Clayton's [1988a] example is that estimating a group of 10 to be 5 seems a greater error than estimating a crowd of 100,000 to by 50,000…he proposed a 'Criteria of Reasonableness' (COR) scale…The COR is logarithmic in nature so that the requirements for an acceptable estimate are more exacting for small numbers than for large numbers. (p. 384)

On the other hand, McCrink, Dehaene, and Dehaene-Lambertz (2007) empirically test and confirm Weber's Law, which states that only the ratio of two magnitudes affects their discriminability. We follow this second approach, considering answers with a factor of 10 of the accepted value as correct, independently of the size of the accepted value.

Figure 5 below shows that even undergraduates averaged fewer than three accurate responses, and that there is a trend to improve over successive grades.

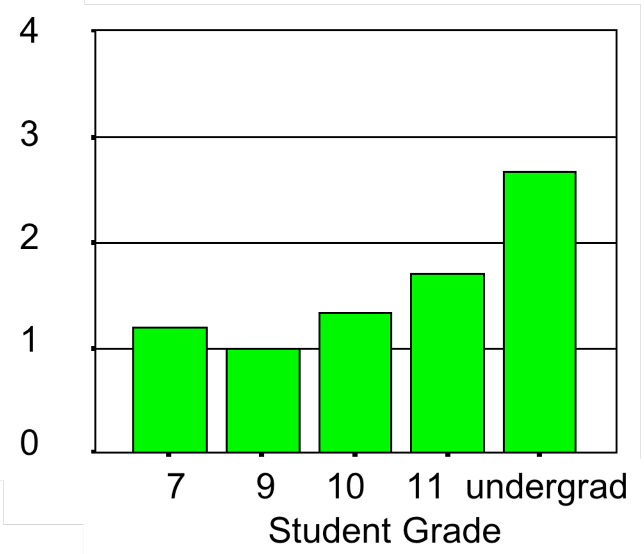


Figure 5. Accuracy of estimates for size relative to a pinhead.

Not all objects were equally easy to estimate for students. Table 4 below shows that the size of red blood cell and human relative to the pinhead were most often correctly estimated, atom and earth the least.

Table 4.

Percentage of students with accurate estimates for each object relative to a pinhead

Object	% of students within 10X
Atom	20
Red Blood Cell	57
Human	50
Earth	29

Estimating absolute size. Students were asked to estimate the diameter of an atom, a red blood cell, the earth, and the height of an adult human. Student responses were coded as 1 for accurate if they were within one order of magnitude (ten times) of the accepted value, 0 otherwise, for a total of 4 points possible. Figure 6 below displays our findings by grade.

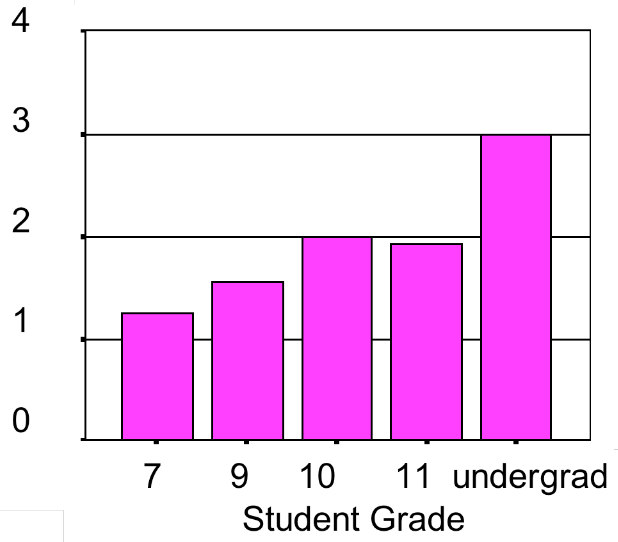


Figure 6. Accuracy of estimates for absolute size.

Students who ranked pinhead as smallest or second smallest are excluded from this graph. All grade levels had equal or better accuracy on the absolute size task than on the relative size task, with a similar trend to improve over time. Undergraduates estimated the absolute size of three out of four objects within an order of magnitude, on average.

Table 5 below shows that accuracy on the absolute task was higher than on the relative task for human and lower for the cell, and similar for earth and atom. Tretter, Jones, and Minogue (2006) likewise found that respondents coming up with objects of a certain size range expressed in metric units (absolute) or body lengths (relative, in the terminology used in this study) did not display a consistent difference in accuracy. In our study, we found that over 90% of the participants were able to estimate a reasonable value for the height of a human, often in terms of feet and inches.

Table 5

Percentage of students with accurate estimates for each object, in absolute size.

Object	% of students within 10X
Atom	24
Red Blood Cell	30
Human	91
Earth	37

Grouping. The analysis of grouping presented a special challenge. Since we asked respondents to group with ten cards, but to estimate relative and absolute size with only four, we were unable to assess consistency between grouping and absolute or relative size tasks. For the accuracy of content analysis, the authors extensively discussed the hierarchical nature of a proposed set of categories, but did not reach an agreement on whether they were truly hierarchical, as we did with the other content tasks. The two lowest categories were less controversial; category 0 consisted of creating groups that were inaccurate, that is, that did not follow the normative size order of the objects. For instance, grouping pinhead with atom and molecule (but excluding virus, mitochondria, and red blood cell) would be coded as grouping incorrectly. The next category included correct grouping that nevertheless mixed macroscopic and submacroscopic objects (e.g., {atom, molecule, vius, mitochondrion, red blood cell, pin}, {rest of cards}). The rationale behind this category is that being able to see and/or touch objects is such a fundamental and phenomenological property that macroscopic and submacroscopic objects should be placed in separate groups. The next proposed categories, 2-5, consisted of the number of correct groups that students formed (that did not mix macroscopic and submacroscopic objects), with category 5 including five-eight correct groups. (Making nine or ten groups means all or all but one "groups" consist of a single object, which is hardly grouping at all). Prior research (Tretter, Jones, Andre, et al., 2006) reports that "In general, the older the participants, the more distinct size categories they conceptualized" (p. 293); however, this interpretation is based not on a grouping task, but on a task in which students assigned objects to predetermined size ranges. Results were aggregated and processed to determine mean difference scores between adjacent objects to generate groups; as the authors note, this approach may obscure individual differences (p. 309). While this study also included a direct grouping task, the number of groups each participant or grade of participants made is not reported. Lack of consensus among the authors of our study regarding whether more groups indeed represented a higher level of knowledge, and a lack of prior research to guide us, led us finally to decide to report on grouping separately. That is, accuracy of grouping is *not* incorporated into the analysis mapping content knowledge accuracy onto consistency level.

The data from our coding scheme for grouping is shown below, in Figure 7. There is a decreasing percentage of students who grouped incorrectly (code 0, in red) or mixing macroscopic and submacroscopic objects (code 1, in yellow). For ease of representation students who made 4 or more groups are shown in blue; their proportion increases over successive grades. The largest percentage of students making just two correct groups was found among undergraduates.

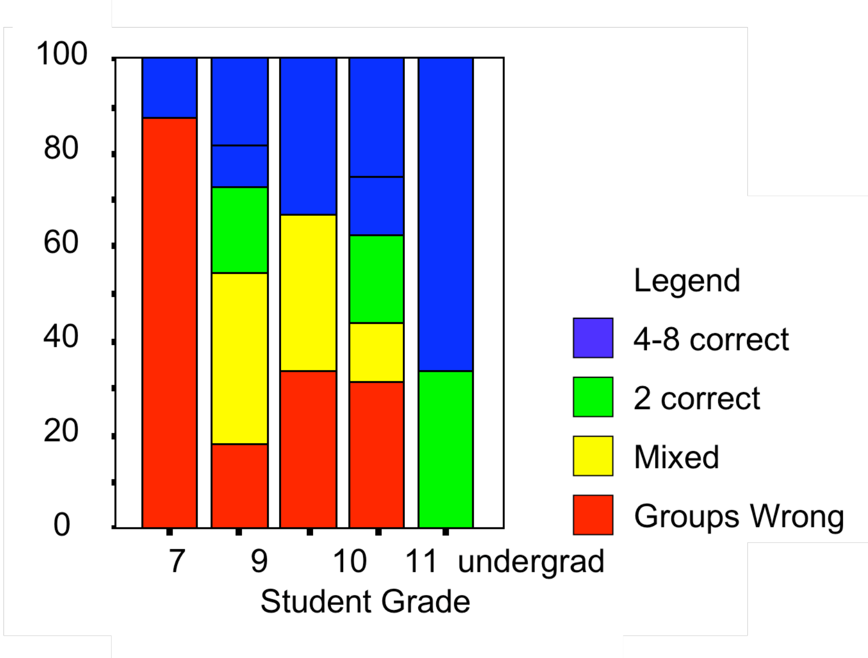


Figure 7. Grouping of ten cards.

Mapping content knowledge regarding the size of objects onto the levels of development of consistency across aspects of size and scale

We expected the level of content knowledge examined above to be correlated positively to grade and science course, as science classes are likely to be the main source of information about the size of objects like the atom, the cell, and the earth. The level of consistency, however, is measured independently of the accuracy of content knowledge about the size of the objects. Thus, we did not initially assume that there would be a correlation between accuracy of content and level of consistency. Table 6 below however shows that for each individual content task (smallest object and unit known, ordering, etc.), the correlation with the level on the progression for consistency is *higher* than the correlation with grade or science course (at the 0.01 significance level). Science course is coded by the grade in which it is usually taken, e.g., 7 for 7th grade science, 9th grade for Integrated Physical and Earth Science, etc.; undergraduates were coded as 13. The correlations between grouping and consistency level, grouping and grade, and grouping and course follow the trend shown in Table 6 below.

Table 6.

Correlations between content tasks and consistency level, student grade, and science course.

	Consistency Level	Student Grade	Science Course
Smallest Object	0.473**	0.327*	0.421**
Smallest Unit	0.444**	0.367*	0.316*
Ordering	0.702**	0.287	0.361*
Relative	0.611**	0.433**	0.463**
Absolute	0.602**	0.394**	0.466**

** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).
Students who were not asked to do the relative or absolute task due to their ordering pinhead as smallest or second smallest objects are imputed scores of 0 for relative and absolute tasks.

The correlation between the sum of the content scores and the consistency level is 0.754, higher than the correlation between the sum of the content scores and student grade (0.463) or sum of content scores and science course (0.517; all at the 0.01 significance level). The correlation between content score and consistency level is statistically significantly higher than the correlation between content score and science course ($t=2.56$, $p<0.02$, using Cohen & Cohen's procedure, 1983, p. 57.)

It is important to emphasize that the consistency level is measured independently of accuracy of content knowledge about the size of objects. Thus, the high correlation suggests that there is some mechanism operating between the two measures. While this cross-sectional study cannot characterize this relationship, it might be that understanding the nature of size itself (that is, realizing that the four aspects of size and scale are logically linked) may make it easier to retain information about the size of specific objects by helping students to contextualize it in several ways. For instance, if a student forgets that the diameter of a red blood cell is around 6 micrometers, but realizes the links between aspects of size and scale, not all is lost. She may recall that it is one of a group of objects that can be seen through a microscope though not with the naked eye, and narrow the range of possible values to between around 0.5 and 50 micrometers. This would rely on the grouping aspect of size and scale. If she remembers that a red blood cell is smaller than skin or cheek cells but bigger than a typical bacterium, and remembers the size of one of those, she can narrow the range further. This would employ ordering. If she recalls seeing having observed a blood cell and another object of known size under the same magnification, then she could even estimate the relative size and calculate the absolute size of the blood cell. Thus, she will have various ways to remember the approximate size of the red blood cell, or any other object. On the other hand, a student who does not realize that the aspects are linked will have no recourse but to separately memorize the absolute size, size relative to several other objects, the scale "world" that it is a member of, and position in an ordering of objects. In this hypothesis, the framework of consistency would help students retrieve information through various different pathways. Conversely, the more objects students know the sizes of, the more likely it is that the links will be understood and that they will be available to help estimate the sizes of other objects. This hypothesis will have to be tested using a different experimental design, such as a longitudinal case study of a small number of students over several years or during a focused intervention.

A tentative learning progression for aspects of size and scale.

The findings for Research Question 2 show that students' level of content knowledge about the size of things, and their level of consistency, are significantly and positively correlated. We thus can generate a learning progression based on the level of consistency, augmented with descriptions of the content knowledge students are likely to have at each level. The graphs of content knowledge organized by level of consistency are included as Appendix F. (Note that the large percentages of missing data in the lower levels for relative and absolute size correspond to students who ranked pinhead as smallest or next to smallest; we were unable to carry out the relative size task for cell and atom with these students.) The descriptions of each level below are summarized in Table 7 below.

Level 0. Two 7th graders and two 11th graders fell into this category. Students at this level are not consistent across any pair of the aspects of size and scale. They do not know of subatomic particles, or of units smaller than the millimeter. They are likely to intersperse macroscopic and submacroscopic objects when ordering; in three of four cases, we were unable to evaluate their absolute and relative size accuracy for cell and atom since they ranked pinhead as smallest or second smallest of the objects. However, they can accurately estimate the absolute size of a human, usually in feet and inches.

Level 1. Three students each in 7th, 9th, and 11th grade were in this category. Students at this level are consistent across ordering and grouping only. Like level 0 students, they do not think of subatomic particles when asked about the smallest object they know, and do not know of units smaller than the millimeter. Most can correctly order the macroscopic objects; they may rank atom larger than cell, and if they do rank atom smaller than cell, they do not rank atom as the smallest and red blood cell as the largest of the given submacroscopic objects. Students at this level get zero to two relative estimates right (that is, within an order of magnitude); they may estimate the relative size of cell, human, or earth accurately but not of the atom. They will estimate one or two absolute sizes correctly (usually, the human, and possibly also the earth).

Level 2. Four students in 9th grade fell into this category. They are consistent across ordering and grouping, and ordering and relative size. Students in this level no longer mention macroscopic objects for the smallest think they can think of, and some mention subatomic particles. Most still do not know of units smaller than a millimeter. They order the macroscopic objects correctly, and rank the atom as smaller than the red blood cell, but most still do not rank atom as the smallest and red blood cell as the largest of the given submacroscopic objects. Their estimates for relative size can be accurate for up to two objects. Like level 1 students, they will estimate absolute size accurately for human.

Level 3. This level included two 7th grade, four 9th grade, five 10th grade, and five 11th grade students. Students at this level are consistent across ordering and grouping, ordering and relative size, and ordering and absolute size. They mention atom or sub-atomic particles for the smallest object, but – like students in levels 0-2 – most do not know of submacroscopic units.

They order macroscopic objects correctly and most rank atom as the smallest of the submacroscopic objects; around half also correctly rank the cell as the largest of these. Most can estimate the relative size of one to three objects, most often the human and the cell. All can estimate the absolute size of the human, and most can estimate the absolute size of additional objects with around equal probability for atom, cell, and earth.

Level 4. This level included one 10th grade, five 11th grade, and two undergraduates. These students are consistent across ordering and grouping, ordering and relative size, ordering and absolute size, and understand the link between relative and absolute size – though they are unable to produce consistent numbers for these two aspects of size. Most answer with a subatomic particle in response to the smallest object they know, and some know of submacroscopic units. Most order the atom as smallest and cell as largest of the submacroscopic objects, but either order molecule, virus, and mitochondrion incorrectly or are unable to justify their ordering of these three objects. All can estimate the relative and absolute size of at least one object, and most for two; usually, these are human and cell or earth.

Level 5. This level included one 7th grade and one 11th grade student, and four undergraduates. They are consistent across the four aspects of size and scale. They know of submacroscopic objects and units (except for the 7th grade student), and were as likely to answer with atom as with sub-atomic particle to the prompt for the smallest object they know of. They can order correctly except for molecule, virus, and mitochondrion, or if correct, are unable to justify their ordering of these three objects. They can estimate the relative and absolute size of at least one object correctly, and often as many as all four; atom is the least likely to be accurate.

Table 7.

A tentative learning progression for size and scale.

Level	**Consistency**	**Smallest object**	**Smallest unit**	**Ordering**	**Relative**	**Absolute**
0	None	Atom, cell/ bacterium or macro	Millimeter or don't know	Intersperse macro and submacro	Rank pinhead<atom and/or cell	Rank pinhead<atom and/or cell; human accurate
1	Order-group	Atom, cell/ bacterium or macro	Millimeter or don't know	Macro OK; Atom> cell, or atom or cell out of place	0-2 right (not atom)	Human accurate.
2	Level 1 + order-relative	Submacro	Millimeter or don't know	Macro OK; Atom<cell; atom or cell out of place	0-2 right	Human accurate
3	Level 2 + order-absolute	Atom or subatomic	Millimeter or don't know	Macro OK; atom smallest; errors in other submacro objects	1-3 right (usually human and cell)	Human and one other accurate
4	Level 3 + relative-absolute (conceptual)	Subatomic	Submacro or don't know	Ordering OK except molecule, virus, mitochondrion	1-3 right (human and cell or earth)	1-3 right (human and cell or earth
5	Level 4 + relative-absolute (procedural)	Atom or subatomic	Submacro (μm or nm)	Ordering OK but cannot justify molecule, virus, mitochondrion	1-4 right (human, cell, earth most likely)	1-4 right (human, cell, earth most likely)

Implications and Future Research

This study goes beyond prior research into isolated aspects of size and scale, and builds a theory of how students build conceptual understanding of key aspects of size and scale. As we report in this and a previous paper (Delgado et al., 2007), student participants tend to establish links between aspects of size and scale in a specific order. The level of consistency across aspects that these links enable are correlated significantly to the degree of accuracy of content knowledge about the size of important scientific objects. The cross-sectional design is a first pass at understanding how students' knowledge develops over time, ideally to be supplemented in the future by longitudinal studies that can actually observe students developing consistency and acquiring knowledge of the size of objects. The learning progression that we are developing can guide the principled, iterative development of curriculum and instruction that targets the growth of conceptual understanding of size and scale. Top-down approaches, in which experts design curricular sequences based on the structure of the discipline, do not take into account learners' unexpected and often counter-intuitive difficulties and pathways to understanding (e.g., Hiebert & Carpenter, 1992).

Future work will consist of coding and analyzing another 40 or so interviews with 7th-11th graders from an academic, private, mid-high SES school, and six undergraduates in science or engineering at a research university. We will be testing the robustness of the progression for consistency detected in this study, and seeing if the patterns of content knowledge are similar or different. With the entire corpus of data, we intend to fully characterize the levels of the learning progression, both in terms of consistency across aspects and of content knowledge students are likely to have at each level. Additionally, we will investigate the statistical strength and significance of the relationships explored in qualitative terms in this paper.

Similar efforts to develop learning progressions in other aspects of size and scale, such as area, volume, surface area to volume ratio (SA/V), effects of changes in size or shape on SA/V, and logarithmic and linear representations of scale, can similarly inform science education, particularly if these learning progressions for subsections of the big idea of size and scale can be linked to each other. Finally, this study suggests some possible directions for a greater integration of science and mathematics curricula, which prior research suggests may result in increased learning (e.g., Judson & Sawada, 2000).

References

American Association for the Advancement of Science. (1993). *Benchmarks for science literacy*. New York: Oxford University Press.

Bazant, Z. P. (2002). *Scaling of structural strength*. New York: Taylor & Francis Books.

Berryman, S. (2004). Democritus. *The Stanford Encyclopedia of Philosophy (Fall 2004 Edition)*, E. N. Zalta (Ed.). Retrieved March 15, 2007, from http://plato.stanford.edu/archives/fall2004/entries/democritus

Castellini, O. M., Walejko, G. K., Holladay, C. E., Thiem, T. J., Zenner, G. M., & Crone, W. C. (2007). Nanotechnology and the public: Effectively communicating nanoscale science and engineering concepts. *Journal of Nanoparticle Research, 9*(2), 183-189.

Chalmers, A. (2005). Atomism from the 17th to the 20th century. *The Stanford Encyclopedia of Philosophy (Fall 2005 Edition)*. E. N. Zalta (Ed.). Retrieved March 15, 2007, from http://plato.stanford.edu/archives/fall2005/entries/atomism-modern

Cohen, J. & Cohen, P. (1983). *Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences*. Hillsdale, N.J.: Lawrence Erlbaum Associates. (Second edition)

Delgado, C., Stevens, S., Shin, N., Yunker, M., & Krajcik, J. (2007, April). *The development of students' conceptions of size*. Paper presented at the National Association for Research in Science Teacher Annual Conference, New Orleans, LA.

Duschl, A., Schweingruber, H., & Shouse A. (2007). *Taking science to school: Learning and teaching science in grades K-8*. Washington, DC: National Academies Press. [Viewed electronically]

Feynman, R. P. (1963). *The Feynman lecture on Physics*. Reading, MA: Addison-Wesley.

Gibson, C., Ostrom, E., & Ahn, T.-K. (2000). The concept of scale and the human dimensions of global change: a survey. *Ecological Economics (32)* 2, 217-239.

Gilbert, J. K., De Jong, O., Justi, R., Treagust, D. F., & Van Driel, J. H. (2002). Research and development for the future of chemical education. In J. K. Gilbert, O. de Jong, R. Justi, D. F. Treagust, & J. H. Van Driel (Eds.), *Chemical education: Toward research-based practice* (pp. 391-408). Dordrecht, The Netherlands: Kluwer Academic Press.

Haldane, J. B. S. (1928). *On being the right size*. Retrieved April 19, 2006, from http://irl.cs.ucla.edu/papers/right-size.html

Hawkins, D. (1978). Critical Barriers to Science Learning. *Outlook 29*, 3-23.

Hiebert, J., & Carpenter, T. (1992). Learning and teaching with understanding. In Grouws, D. (Ed.), *Handbook of research on mathematics teaching and learning*. New York: Macmillan.

Inhelder, B., & Piaget, J. (1969). *The early growth of logic in the child: Classification and seriation* (E. A. Lunzer and D. Papert, Trans.) New York: W. W. Norton & Co. (Original work published 1959)

Jones, G., Tretter, T., Taylor, A., & Oppewal, T. (2007). Experienced and novice teachers' concepts of spatial scale. *International Journal of Science Education, 30* (3), 2007, pp. 409 – 429

Judson, E., & Sawada, D. (2000). Examining the effects of a reformed junior high school science class on students' math achievement. *School Science and Mathematics*, *100*(8), 19-25.

Lamon, S. (1994). Ratio and proportion: Cognitive foundations in unitizing and norming. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 89-122). Albany, NY: State University of New York Press.

Levin, S. A. (1992). The problem of pattern and scale in ecology. *Ecology 73*(6):1943-67.

McCrink, K., Dehaene, S., & Dehaene-Lambertz, G. (2007). Moving along the number line: Operational momentum in nonsymbolic arithmetic. *Perception & Psychophysics 69* (8), 1324-1333.

Montello, D. R., & Golledge, R. G. (1998). *Summary report from "Scale and Detail in the Cognition of Geographic Information"*. Santa Barbara, CA, May 14-16, 1998. Retrieved April 10, 2007, from http://www.ncgia.ucsb.edu/Publications/Varenius_Reports/Scale_and_Detail_in_Cognition.pdf.

National Council of Teachers of Mathematics. (1989). *Principles and standards for school mathematics*. Reston, VA: NCTM. [Electronic resource]

National Research Council (1996). *National science education standards*. Washington, D.C.: National Academy Press. [Viewed electronically]

Patton, M. Q. (2002). Qualitative research and evaluation methods. Thousand Oaks: Sage.

Rutherford, F. J. & Ahlgren, A. (1990). *Science for all Americans*. New York: Oxford University Press. [Viewed electronically]

Smith, C. L., Wiser, M., Anderson, C. W., & Krajcik, J. (2006). Implications of research on children's learning for standards and assessment: A proposed learning progression for matter and the atomic molecular theory. Focus Article. *Measurement: Interdisciplinary Research and Perspectives, 14*, 1-98.

Sowder, J. (1992). Estimation and number sense. In Grouws, D. (Ed.), *Handbook of research on mathematics teaching and learning*. New York: Macmillan.

Stevens, S. Y., Sutherland, L., Schank, P., & Krajcik, J. (draft, 2007). The big ideas of nanoscience. Retrieved June 2, 2007 from http://hi-ce.org/PDFs/Big_Ideas_of_Nanoscience-20feb07.pdf

Tretter, T. R., Jones, M. G., Andre, T., Negishi, A., & Minogue, J. (2006). Conceptual boundaries and distances: Students' and experts' concepts of the scale of scientific phenomena. *Journal of Research in Science Teaching, 43*(3), 282-319.

Tretter, T.R., Jones, M.G., & Minogue, J. (2006). Accuracy of scale conceptions in science: Mental maneuverings across many orders of spatial magnitude. *Journal of Research in Science Teaching 43*(10), 1061-1085.

Vergnaud, G. (1988). Multiplicative structures. In J. Hiebert and M. Behr (Eds.) *Number concepts and operation in the middle grades* (pp. 141-161)

Waldron, A. M., Sheppard, K., Spencer, D., & Batt, C. A. (2005). Too small to see: Educating the next generation in nanoscale science and engineering. In C. S. S. R. Kumar, J. Hormes, & C. Leushner (Eds.), *Nanofabrication towards biomedical applications* (pp. 375-389). New York: Wiley.

Waldron, A. M., Spencer, D., & Batt., C. A. (2006). The current state of public understanding of nanotechnology. *Journal of Nanoparticle Research 8*, 569-575.

Wiedtke, W. (1990). Measurement. In J. N. Payne (Ed.), *Mathematics for the young child* (pp. 229-249). Reston, VA: NCTM.

Wilensky, U., & Resnick, M. (1999). Thinking in levels: A dynamic systems perspective to making sense of the world. *Journal of Science Education and Technology, 8*(1), 3-18.

Wilson, M., & Bertenthal, M., eds. (2005). *Systems for state science assessment*. Washington, DC: National Academies Press. [Viewed electronically]

Acknowledgments

This research is funded by the National Center for Learning and Teaching in Nanoscale Science and Engineering, grant number 0426328, from the National Science Foundation. Any opinions expressed in this work are those of the authors and do not necessarily represent those of the funding agencies.

Appendix A: Interview Protocol

SIZE AND SCALE INTERVIEW

Bold: Directions or Introduction. Normal: Questions Prompts

Hello. My name is ____ and I'm here from the University of Michigan's School of Education. I want to ask you some questions about the size of things. This interview will be completely confidential, and will not affect your class grade in any way. Your teacher will not hear what you say. Your answers will help us design better science education materials for high school. '**I want to ask you some questions about the size of things. Think of some very small things you know of.** ***(Pause a few seconds.)***

What is the very smallest thing you can think of?

IF RESPONSE IS MACRO	IF AMBIGUOUS ("nucleus/particle")
Can you think of something too small to see with the naked eye?	What do you mean by that? Could you be clearer? What else do you know of that is too small to see with naked eye?

What type of measurement units would you use to express the size of that object? (If necessary, prompt by saying that the width of the table could be expressed in centimeters)

Which is bigger, a bacteria or a water molecule?

Why do you think that?

OK. Take a look at these two sets of cards. (*Lay out cards in two separate sets).* **I'd like you**

to put them in order by the size of the objects, from largest to smallest. You can pick either one of the sets of cards – they both have the same objects. (Demonstrate the size of the head of a pin at this point).

(Record order in which the cards were placed. Code: Earth=E, mountain=MTN, human=H, ant=ANT, pinhead=P, red blood cell=C, mitochondria=O for organelle, virus=V, molecule=MOL, atom=AT. Abbreviations: OK= all correct. MACRO=pin to Earth correct.)

Could you please tell me why you ordered these cards (*the micro and nano cards*) the way you did? *(Select the micro- and nano- cards in pairs.)*

Why did you choose this set of cards?

Could you please place the cards into groups of objects of similar size? Make as many groups as you think makes sense. (Wait for task). Can you tell me how you decided to group these cards together? (Repeat for tape recorder how many groups and what cards in each). What do they have in common? What would you call this group? (Repeat for each group)

Interviewer selects five cards from task 3. These will be atom, cell, pinhead, human, and Earth if they are ordered correctly. If atom and cell are out of order, select one of those, and choose another card in the correct order.

OK. Here are five of the cards you ordered. I want you to think about the length of these objects. For the pinhead, think how wide it is *(Trace width with finger)*. For the person, the height. (*Trace*). For the Earth, atom, and cell, the diameter. (*Trace*). (*Record all answers on worksheet)*

How many times larger is the human than the pinhead*?*

How many times larger is Earth than the pinhead?

How many times smaller is the (red blood cell) than the pinhead?

How many times smaller is the (atom) than the pinhead?

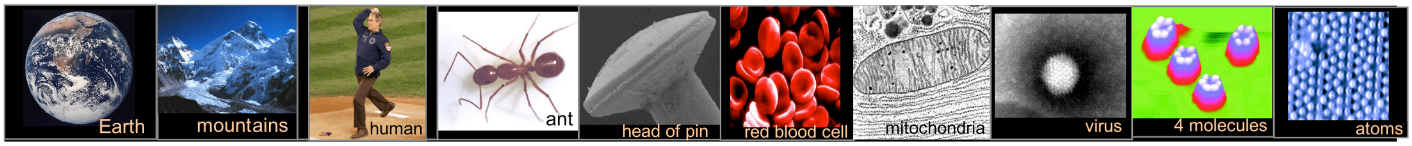
OK, a pinhead is about 1 mm wide. That's a little less than 1/16th of an inch

Would you write down the size of the other objects? *(Pass the student pen and worksheet, and offer scratch paper. Remind student to specify units, if necessary. Ask them to use metric system.)*

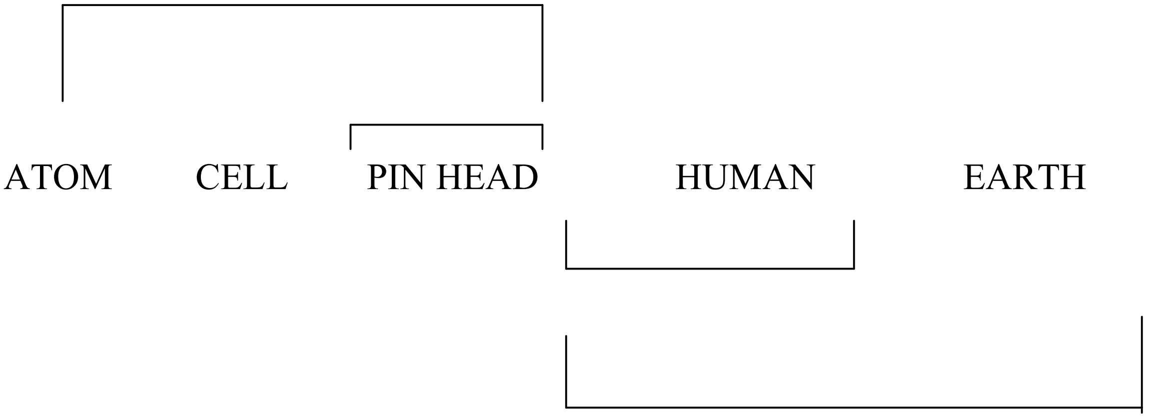
Did you use these numbers here (indicate the relative sizes recorded on sheet) to think about the sizes of the objects? (If yes, ask how; if no, ask if student if s/he thinks the two sets of numbers are related)

If you can think of other ways to express the sizes using different units or different ways of writing the numbers would you please write them below too?

Appendix B: Size And Scale Cards



Appendix C: Answer Recording Sheet



ATOM	CELL	PIN HEAD	HUMAN	EARTH
_______	_____	___1 mm__	________	_______

Appendix D: Sample Interview Summary

0099 S&S Summary

6/2/06 – CD

S&S starts at 18:00

Smallest: electron.

Unit: nm. "Nanomicrometer"

Bacteria are bigger because you can see bacteria in water. But there might be bacteria smaller than a molecule of water. Molecule of water is made of two atoms of hydrogen and one of oxygen. There might be bacteria smaller than that.

Photos because recognized Bush.

Atom/mitochondria/virus/molecule/cell/pinhead/ant/human/mountain/earth

(24:00) Blue, green groups: visible to naked eye. Thought about placing them all together, but pin, ant, small so in their own group. Red group: building blocks. Yellow group: made up of red group, but not visible to naked eye.

Red blood cells are made of molecules, viruses are made of atoms, mitochondria are in the nucleus of a cell (or atom?). Viruses have mitochondria. Cells are made up of atoms.

(25:00) I: How many times bigger is human than head of a pin?

R: Oh, seeing as that's like a millimeter, and so, like 6 feet tall, is how many meters? is two meters, so a millimeter is a thousandth – is it OK if I write something?

I: Sure – could you do it on the, here [back of the paper].

R: Hopefully a good estimate. So, 6 feet equals two meters, so if that's one millimeter (asked to speak into recorder). (26:00) So I'm saying 6 feet equals about two meters, and so then, how many millimeters are in a meter. There's a hundred centimeters, er, thirty centimeters in a foot, so thirty times ten, so there's 300 mm in a foot, I think. So then 300 times 6. So then 1800 millimeters in six feet…OK, so assuming all this math is right at [early] in the morning, I say it's like 1800 times. [Asked about Earth to pin] Oh my God (laughs) How many miles is the diameter of the Earth? [NOTE: see how immediately goes to absolute size to calculate ratio, here and above] Are you allowed to tell me how many miles the diameter of the Earth is? I: No. R: OK. Well, what I would do was, I'd think of how many mm in a foot, then I'd just find out how many miles in the diameter, find it out. I'm a math person. You just want an estimate…

I: Do you have any idea what the diameter of the earth might be?

R: We always have the conversion factors in our textbook and we just open it to do all our math problems, so I kind of mindless calculating. (28:00) So let's see, from here to Boston is around 3000 miles [NOTE: actual distance is about ¼ that]. Let's say it's like 10,000 miles (NOTE: 7926 mi is actual value), but I have no idea…so it's got to be like millions of times…

I: If you had a calculator, could you do it?

R: The diameter of the earth? Oh yeah, like the diameter of the Earth? Yeah.

I: Well, I can be your calculator if you need a little bit of help! You've got 10,000 miles. What do you know about miles?

R: There's 5280, or 60? 5260, 80…(NOTE: correct number is 5280)

I: So how many feet is 10,000 miles then? What operation would you do?

R: OK, so this is how many feet per mile, so then times 10,000.

I: OK, and then how would you get to, you've got feet now.

R: OK, feet, and you want it in mm…you multiply by 300. Assuming that's right, but I don't know about that.

I: (mumbles numbers) About 15 billion then.

(Meanwhile, R is calculating with pencil and paper. She uses units at several steps, including mm/ft in a conversion factor)

I helps her notice that there is one zero missing.(31:10)

I: Use numbers here to think of numbers here?

R: Yes. Because it's the conversion factor. I mean, it's the same thing…

I: So this number is dictated by the number here?

R: Yes. Correspondingly. Like compared to this, it's basically writing the same thing.

I: If a person (who did it wrong said they're different?)

R: Once you give it a number, it corresponds, it comes out to be the same thing…

I: Do you think there was a time, when you were smaller, in middle school or elementary, when you didn't know numbers like this had to be connected to numbers like that, or do you think you ALWAYS knew how to do this?

R: Well, probably, every skill is leaned, but I can't remember a specific time like learning it.

I: It seems obvious now.

R: Yeah.

Appendix E: Content Coding Rubric

I. Smallest object respondent knows of

Steps of 100 X starting from the smallest macroscopic object (thickness of a hair; 0.1 mm) are used here.

Codes:

0: non-matter, nonsense, don’t know (e.g., a computer virus)

1: >100 μm (e.g., grain of salt, thickness of hair)

2: 1-100 μm (e.g., cell, microorganism)

3: 10-1000 nm (e.g., part of cell, large protein)

4: 0.1-10 nm (e.g., atom, small-midsized molecule)

5: <0.1 nm (e.g., electron, quark, proton)

Examples:

1) “Mitochondrion”. This is part of a cell, so code as 3.

2) “E. coli”. This is a microorganism, so code as 2.

II. Unit to express the size of that object

Note: the very few (~5%) of students who answer I above with a macroscopic object are reported separately from this coding scheme. Students who answered with a sub-macroscopic object in I above were in most cases probed for the smallest unit they know, which is what this coding scheme analyzes.

Codes:

0: units don’t exist/don’t know/non-length answers (e.g., Newton-meter)

1: Macroscopic-sized unit, English (e.g., inch)

2:Macroscopic-sized unit, metric (e.g., mm, cm)

3: Fraction of a macroscopic unit that would be submacroscopic (e.g., 1/1000 mm)

4: Submacroscopic unit (e.g., Angstrom, nanometer, µm). Accept answer that describe the symbol for micrometer but cannot reproduce it or name the unit). Do NOT accept answers that do not describe a unit of length (e.g., a "nano-")

Examples:

1) "There's a unit that is like a funny u followed by an m. It's smaller than a mm." Code: 4

2) "I don't remember the names of the units, but there are some that are like one-millionth of a mm." Code: 3.

III. Ordering 10 cards by size of the object depicted

Students will first be familiar with macroscopic objects, and thus should be able to order these first. According to standards documents, students should encounter the cell first, from among the submacroscopic objects in this task. Of the remaining submacroscopic objects, the atom is the most central, and also frequently related to size ("the smallest piece of a substance that retains its properties", etc.). Thus, after ordering the macroscopic objects, the second distinction will see if students know that atom<cell in size. Next, comparing the cell to the other submacroscopic objects requires additional content knowledge. Thus, this coding scheme:

Codes:

0: macroscopic objects ordered incorrectly (or macro and submacro objects interspersed)

Remaining codes assume macro objects correctly ordered.

1: atom ordered as larger than cell

2: atom<cell, but atom not smallest of all objects

3: atom smallest, but cell not ranked larger than molecule, virus, mitochondrion

4: atom smallest and cell largest of submacro objects; but molecule, virus, mitochondrion not in the correct order. OR, all objects are in the correct order BUT student admits having guessed, or gives incorrect rationales for placement)

5: all objects in the correct order AND student gives rationale for ranking. (Note: if student admits *guessing*, do NOT code as a 5)

Examples:

1) (1004) cell/atom/molecule/virus/mitochondria/pin/ant/human/mountain/earth

Macro objects are ok (so not coded a 0), but atom ranked larger than a cell. Code: 1.

2) (0091-paraphrased. Order was corrrect) Mitochondria are inside cells, so they are smaller. Viruses can get inside cells. Atoms make up molecules, which make up the virus. I wasn't sure about virus and molecule. Code: 4.

3) (0046) Atom/cell/molecule/mitochondria/virus/pin head/ant/human/mountains/earth

Macro OK (so not 0). Atom ranked smaller than cell, so not a 1. Atom ranked smallest, so not a 2. Cell ranked smaller than molecule, mitochondrion, and virus, so code 3.

IV. Size relative to pinhead for atom, cell, human, earth

There is a little bit of leeway for answers where there is a range of sizes. See below.

Codes:

Atom: The “diameter” of an atom is around 1 Å or 0.1 nm, so the range of acceptable answers is from 0.01 nm (0.1 Å; 10^{-11} m) to 1 nm (10 Å; 10^{-9} m). When compared to a 1 mm diameter pinhead, the acceptable number of times smaller ranges from 100 million to 1 million.

Red blood cell: The range of diameters for blood cells is around 5-10 µm. Thus, the acceptable range of sizes is from 0.5 µm to 100 µm. The acceptable number of times smaller than pinhead ranges from 2000 to 10.

Human: The height of an adult human is usually between 1.5 and 2 m, or 1,500-2,000 times bigger than a pinhead. This makes the range of acceptable answers 150-20,000.

Earth: the diameter of the earth is around 12,700 km, or 12.7 billion times larger than the pinhead. The range of acceptable answers is 1 billion-150 billion.

V. Absolute size for atom, cell, human, earth

This coding scheme will characterize a range of sizes as an acceptable estimate for the size of the four objects: 1 for acceptable, 0 otherwise. These will be added for an overall score from 0-4.

Codes (any units are acceptable. Convert from student’s units, compare to sizes listed below):

Atom: Acceptable answers range from 0.01 nm (0.1 Å; 10^{-11} m) to 1 nm (10 Å; 10^{-9} m).

Red blood cell: The acceptable range of sizes is from 0.5 µm to 100 µm.

Human: The acceptable range of sizes is from 0.15 m to 20 m.

Earth: The acceptable range of sizes is from 1,000 km to 150,000 km.

Appendix F: Graphs of Consistency Level vs. Content Knowledge Accuracy

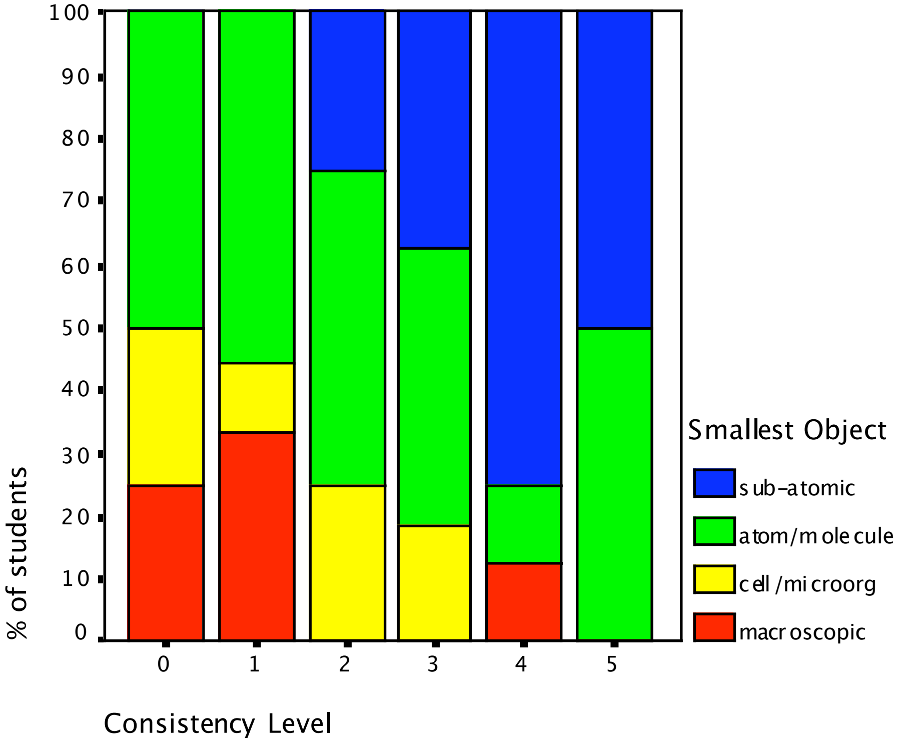


Figure F.1. Level of consistency vs. smallest object

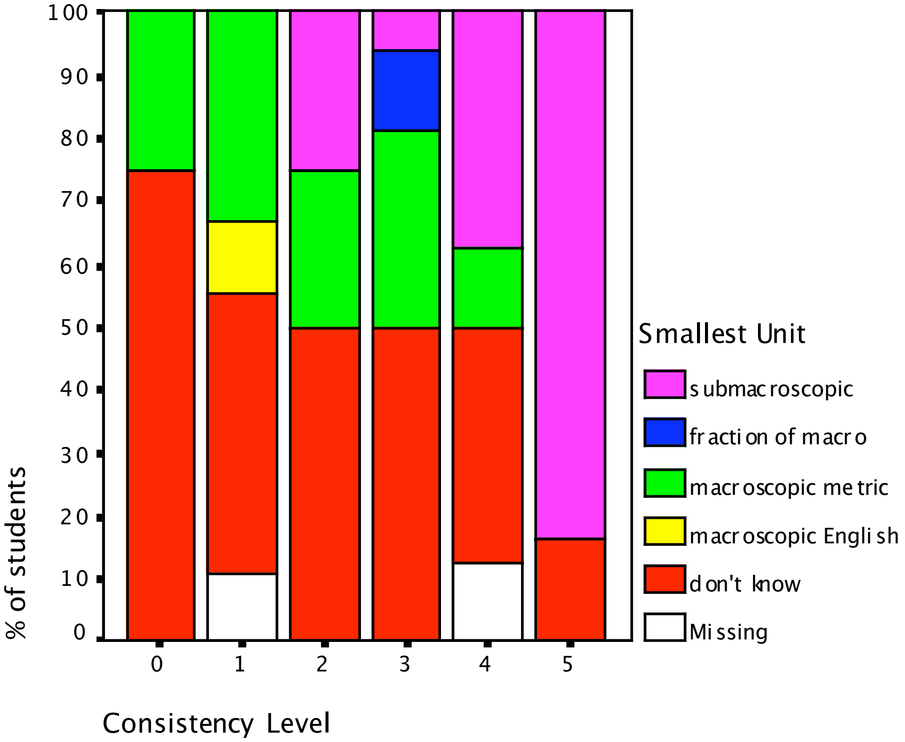


Figure F.2. Level of consistency vs. smallest unit

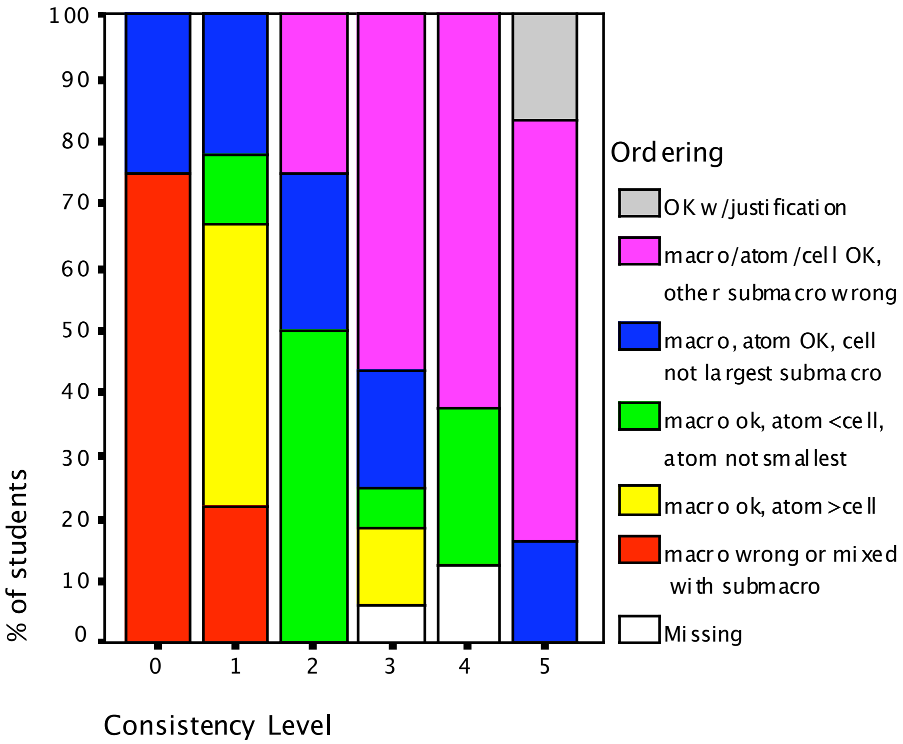


Figure F.3. Level of consistency vs. ordering

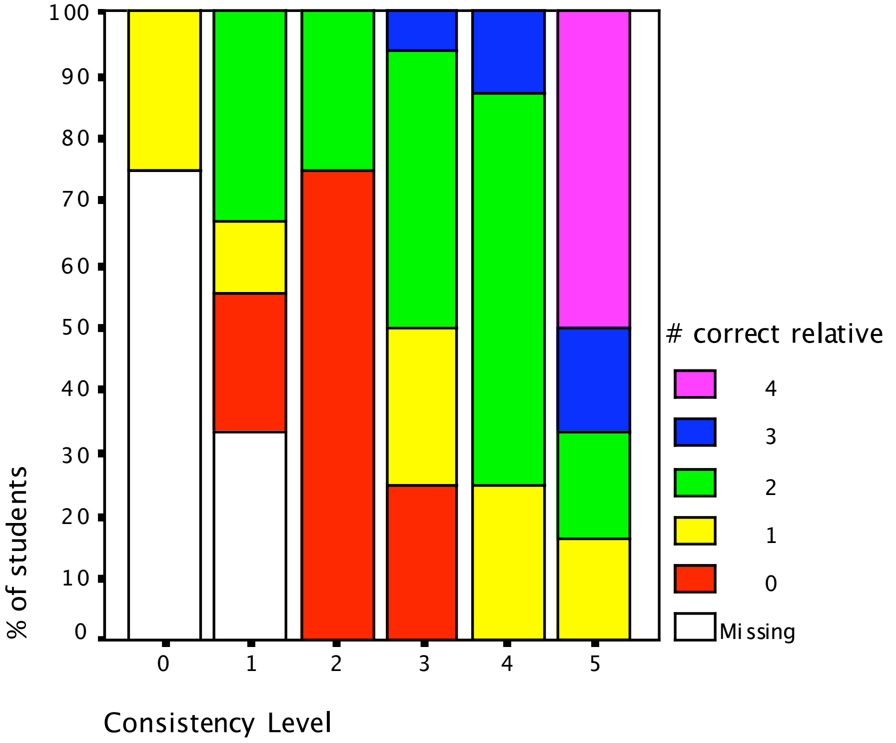


Figure F.4. Level of consistency vs. relative size estimates

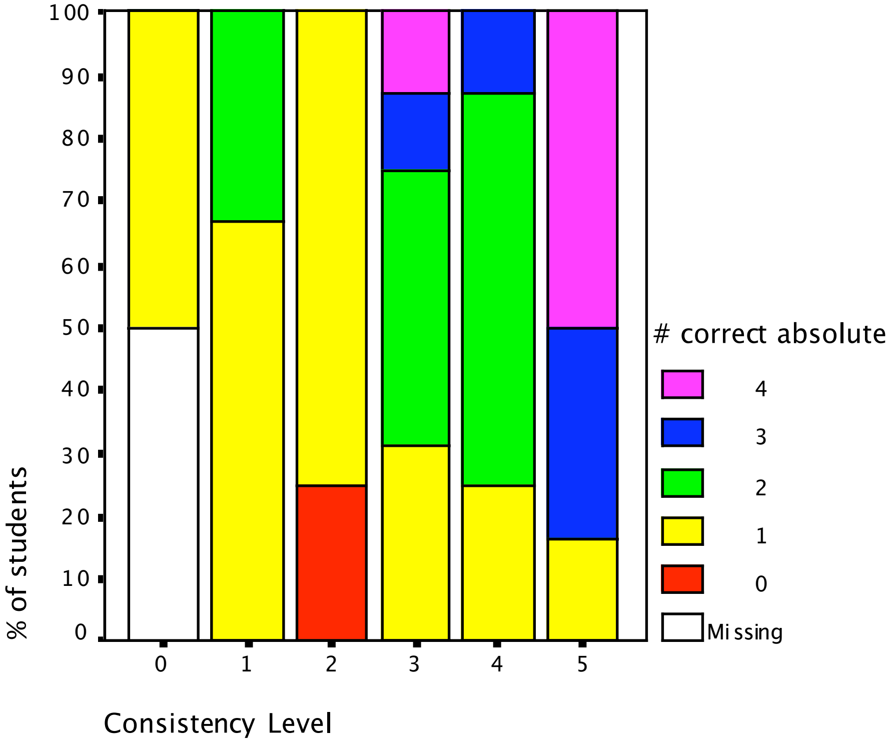


Figure F.5. Level of consistency vs. absolute size estimates.

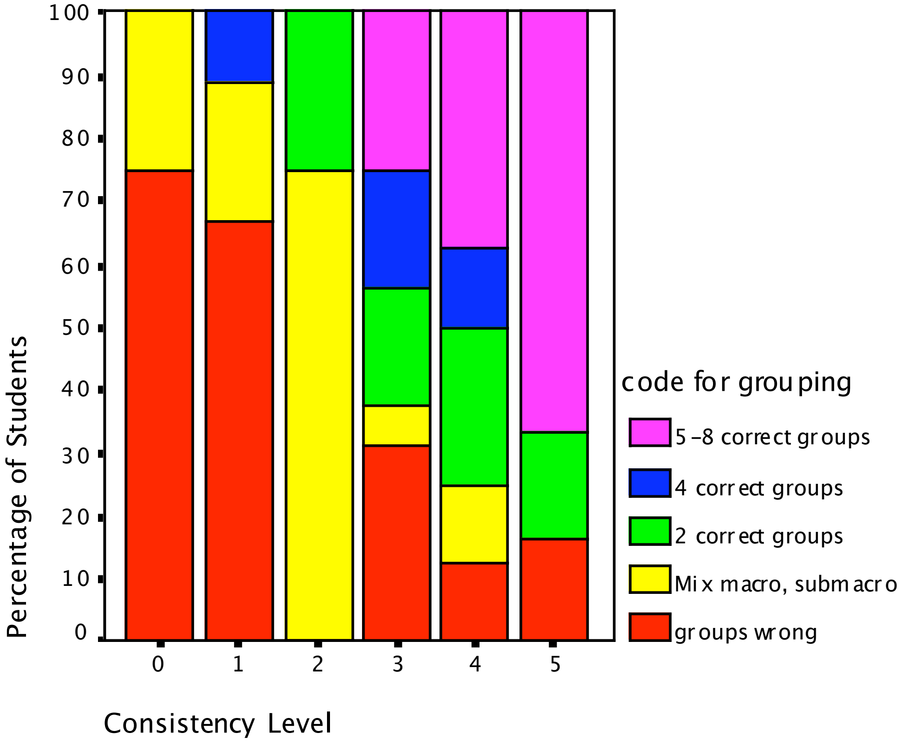


Figure F.6. Level of consistency vs. grouping (non-hierarchical).