

# Learning non-intrusive data-driven reduced models: application to large-scale systems

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**Workshop on Data-driven & Reduced Order Modeling for Multi-Scale Problems**

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August 31, 2023

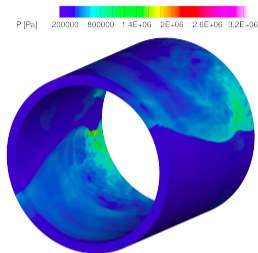
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## Goals

- construct **physics-based data-driven reduced models** of large-scale RDRE simulations with sufficient engineering accuracy
- use these ROMs for downstream tasks such as **design optimization**

## Computational challenges

- the high-fidelity LES simulations are large-scale and computationally very expensive (a single simulation typically requires  $\mathcal{O}(10^6)$  core-hours on supercomputers)
- the resulting training data sets are often sparse
- they comprise down-sampled time instants from the high-fidelity simulation
- only few parametric instances can realistically be simulated to generate training data



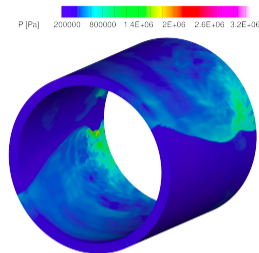
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Part I: constructing ROMs for large-scale simulations for predictions  
beyond the training time horizon

# Discrete operator inference: general idea

- starting point: a **physics-based model**, typically described by PDEs or ODEs
- **variable transformations** that expose polynomial structure in the model
- lens of **projection** to define the form of a structure-preserving low-dimensional model

define the **structure of the reduced model**

## Operator inference learning problem

- non-intrusive learning by inferring reduced model operators from data

$$\operatorname{argmin}_{\hat{\mathbf{O}}} \|\hat{\mathbf{D}}\hat{\mathbf{O}} - \hat{\mathbf{R}}\|_F^2 + \text{regularization}$$

- $\hat{\mathbf{O}}$ : **low-dimensional operators** define the reduced model as a discrete system
- $\hat{\mathbf{D}}, \hat{\mathbf{R}}$ : **data matrix/forcing** from simulation and/or experimental data
- **minimum residual formulation** leads to linear least-squares minimization
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# Steps to perform discrete operator inference

**I. Start:** full-order model. Suppose the process of interest is described through (possibly after applying a lifting transformation) the high-dimensional quadratic discrete model

$$\mathbf{w}[k+1] = \mathbf{A}\mathbf{w}[k] + \mathbf{H}(\mathbf{w}[k] \otimes \mathbf{w}[k])$$

**II. Discrete operator inference:**

1. training data: compute a set of  $n_t$  full-order model solutions (snapshots)
2. training data manipulation (lifting, centering, scaling) to get the snapshot matrix of the transformed variables  $\mathbf{Q} \in \mathbb{R}^{n \times n_t}$
3. subspace identification: determine rank- $r$  reduced basis  $\mathbf{V}_r \in \mathbb{R}^{n \times r}$  via the (thin) singular value decomposition of  $\mathbf{Q}$
4. projection: project each transformed snapshot:  $\hat{\mathbf{Q}}[k] = \mathbf{V}_r^T \mathbf{Q}[k] \in \mathbb{R}^{r \times n_t}$
5. learn reduced operators  $\hat{\mathbf{A}} \in \mathbb{R}^{r \times r}$ ,  $\hat{\mathbf{H}} \in \mathbb{R}^{r \times r^2}$ , and  $\hat{\mathbf{c}} \in \mathbb{R}^r$  via operator inference with regularization

**III. Finish:** reduced-order model. The learned reduced operators define the ROM

$$\hat{\mathbf{q}}[k+1] = \hat{\mathbf{A}}\hat{\mathbf{q}}[k] + \hat{\mathbf{H}}(\hat{\mathbf{q}}[k] \otimes \hat{\mathbf{q}}[k]) + \hat{\mathbf{c}}$$

We then use the reduced model to issue predictions beyond the training time horizon

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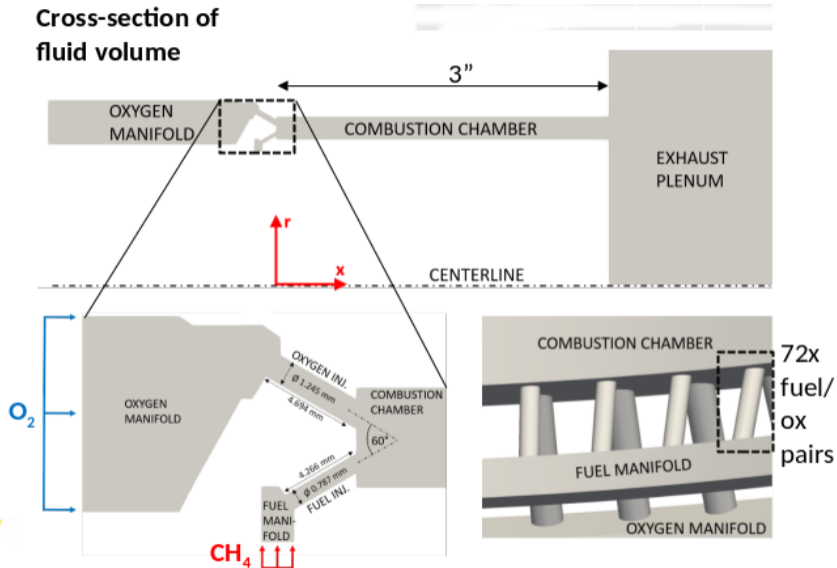
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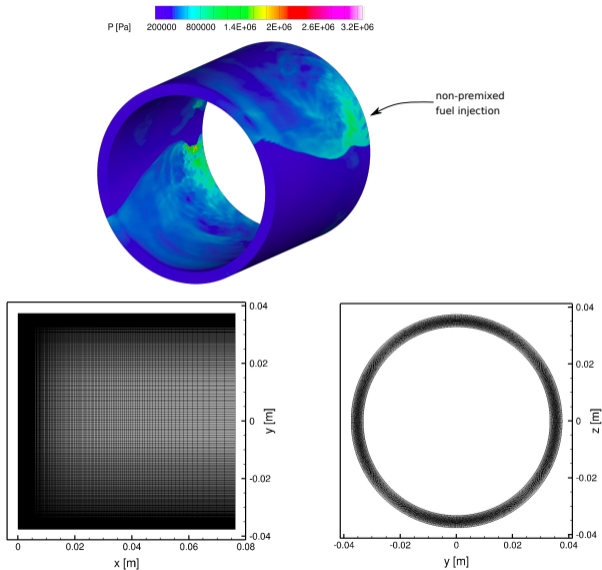
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Numerical demonstration

# Modeling the combustion chamber of a rotating detonation rocket engine



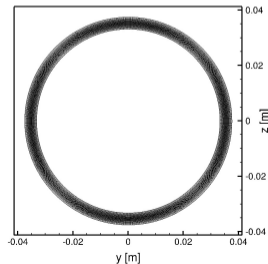
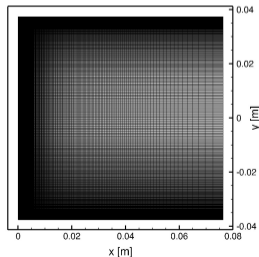
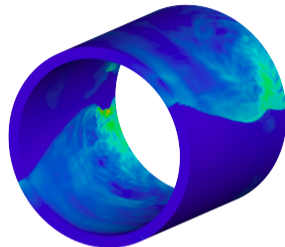
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- LES simulations of the reactive, viscous 3D Navier-Stokes equations
- skeletal chemistry mechanism based on the Foundational Fuel Chemistry Model (FFCM<sub>y</sub>-30)
- non-premixed fuel injection (gaseous methane and oxygen)
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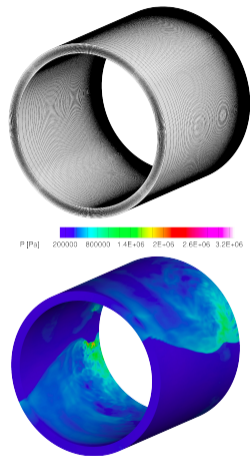
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# Modeling the combustion chamber of a rotating detonation rocket engine

## 1. Generate training data + data manipulation

- $\dot{m} = 0.267 \text{ kg} \cdot \text{s}^{-1}$  and  $\Phi = 1.16$
- 2 ms of full-state solutions generated  $\sim 6\text{M}$  CPU hours on  $> 16\text{K}$  cores
- the original simulation data has been interpolated on a structured mesh comprising  $n_x = 4,204,200$  spatial DoF
- time step size  $\Delta t \sim 10^{-9} \text{ s}$
- available data: 501 down-sampled snapshots over  $[2.50, 3.75]$  ms ( $\sim 4$  periods of two-wave system)
- 18 transformed state variables: specific volume, pressure, 3D velocity, temperature, 12 species mass fractions (full chemistry data)
- training data: snapshot matrix  $\mathbf{Q} \in \mathbb{R}^{76M \times 375}$

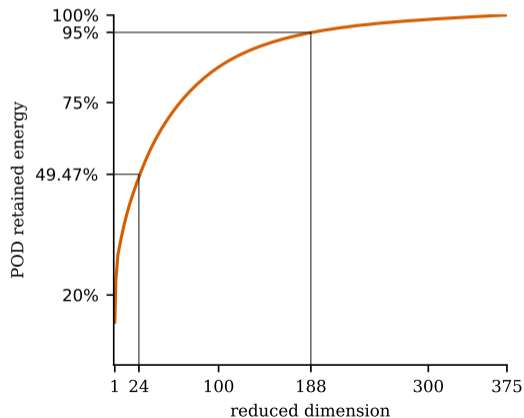
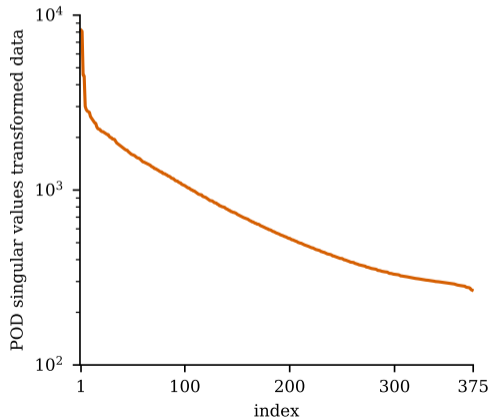


Two dominant co-rotating waves in the quasi-limit-cycle behavior of the flow

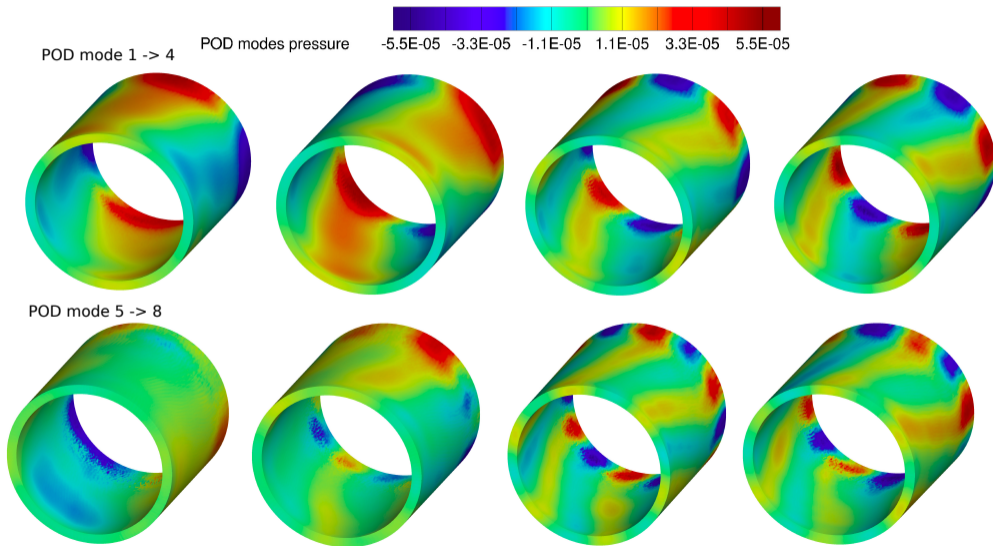
# Modeling the combustion chamber of a rotating detonation rocket engine

## 2. Compute POD basis

- snapshot matrix of transformed variables
- scale and center snapshot data
- compute POD basis  $\mathbf{V}_r \in \mathbb{R}^{76M \times r}$
- low-data regime limits size of the non-intrusive ROM



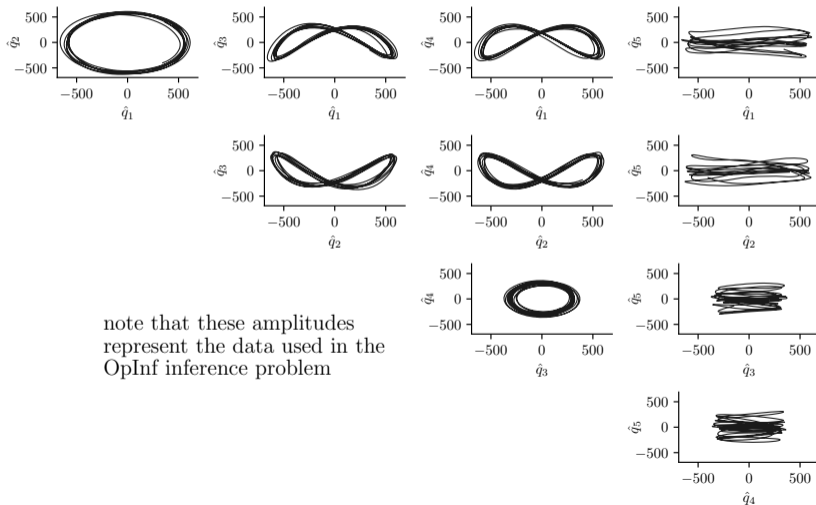
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# Modeling the combustion chamber of a rotating detonation rocket engine

## Two-dimensional phase portraits of POD amplitudes



note that these amplitudes represent the data used in the OpInf inference problem

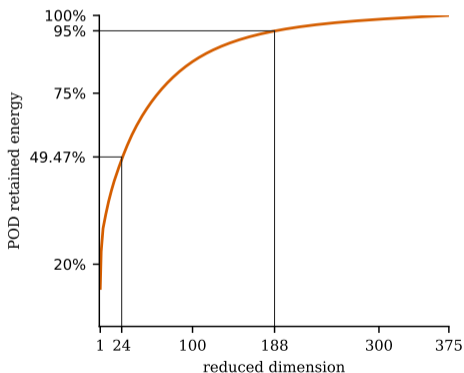
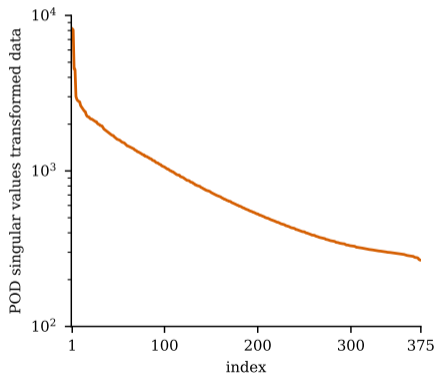
# Modeling the combustion chamber of a rotating detonation rocket engine

## 3. Infer reduced operators

- compute reduced snapshot matrix

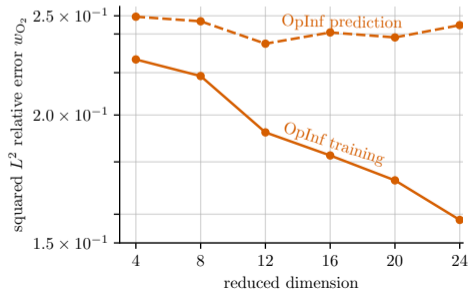
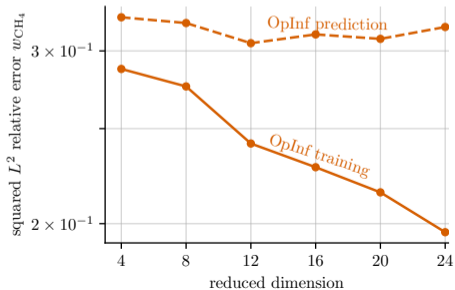
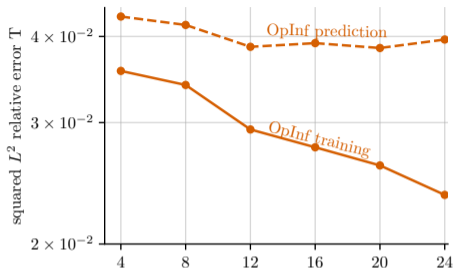
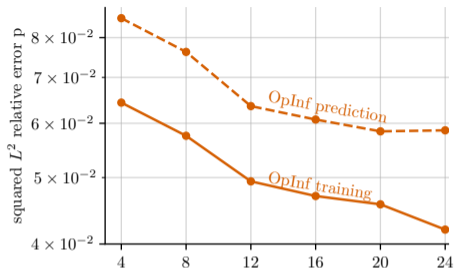
$$\hat{\mathbf{Q}} = \mathbf{V}_r^T \mathbf{Q} \in \mathbb{R}^{r \times 375}$$

- learn a fully discrete quadratic ROM
- solve linear least squares to infer reduced operators

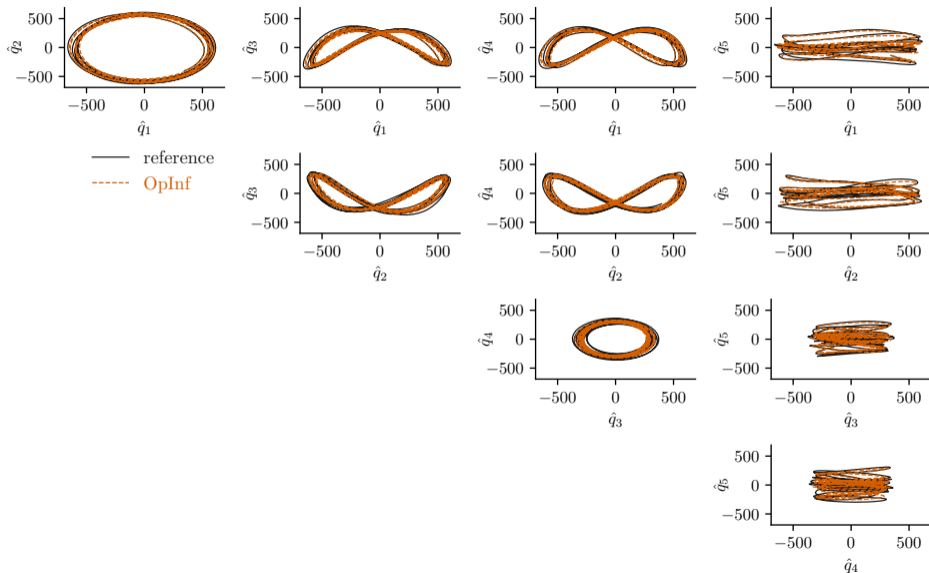


resulting reduced model is completely decoupled from the original CFD code

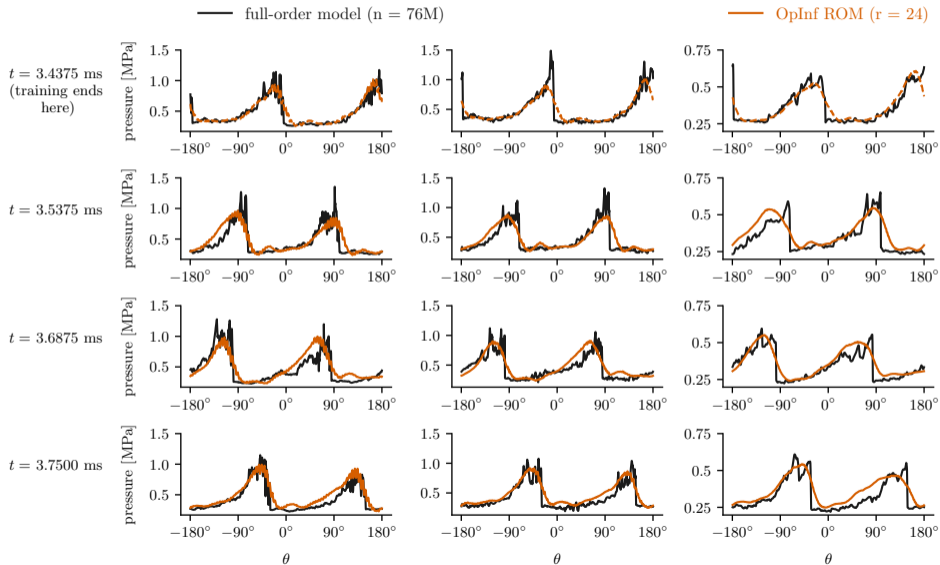
# OpInf reduced model relative error



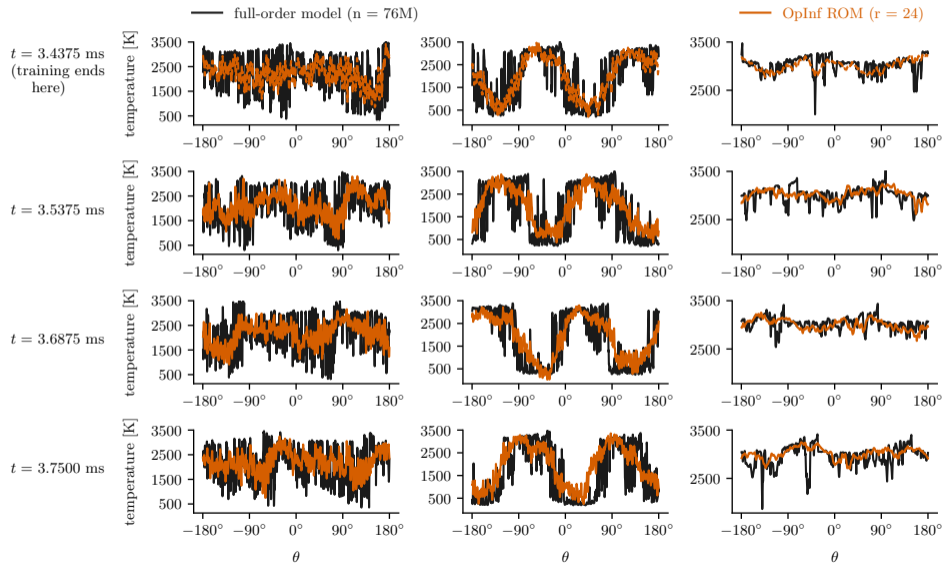
# OpInf predictions for two-dimensional phase portraits



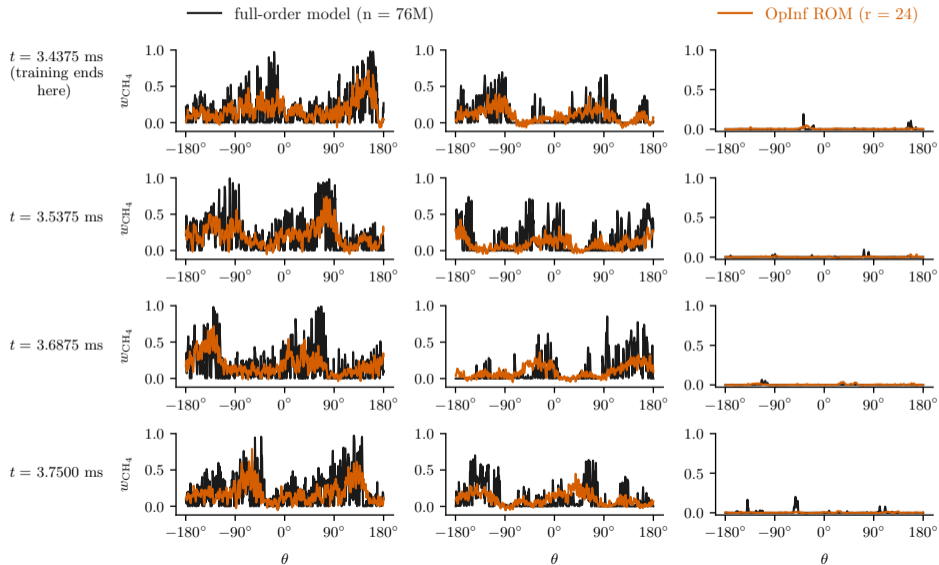
# One-dimensional circumferential pressure profiles



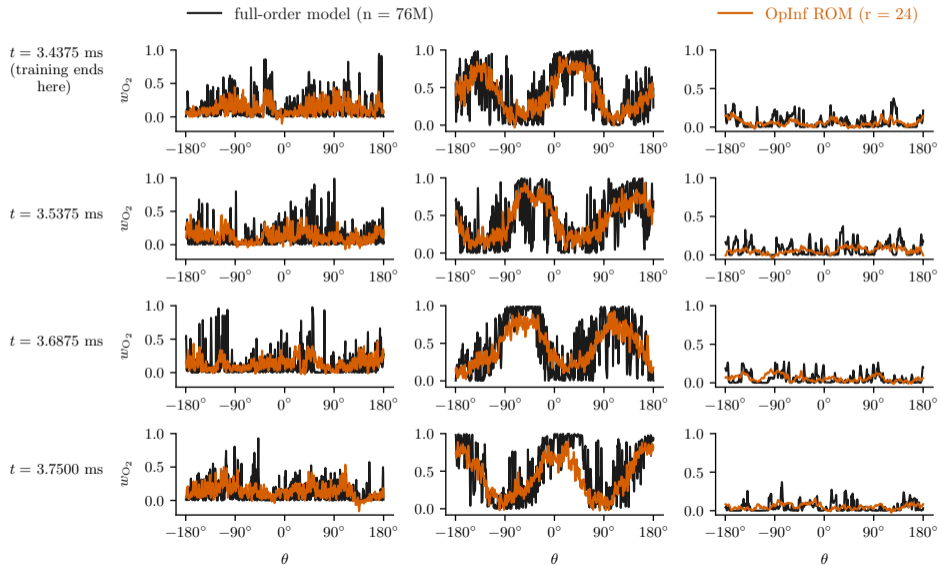
# One-dimensional circumferential temperature profiles



# One-dimensional circumferential fuel mass fraction profiles



# One-dimensional circumferential oxidizer mass fraction profiles





## Part II: towards predictive parametric ROMs of large-scale RDRE combustion chamber simulations

# Parametric discrete operator inference: general idea

- a parametric physics-based model, typically described by PDEs or ODEs
- variable transformations that expose polynomial structure in the model
- lens of projection to define the form of a structure-preserving low-dimensional parametric model

define the structure of the parametric reduced model

## Discrete parametric operator inference learning problem

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# Steps to perform parametric discrete operator inference

**I. Start:** full-order model that depends on a  $d$ -dimensional parameter  $\boldsymbol{\mu} \in \mathcal{D} \subset \mathbb{R}^d$ . Suppose that the process of interest can be described (possibly after a lifting transformation) through the high-dimensional parametric quadratic discrete model

$$\mathbf{w}[k+1] = \mathbf{A}(\boldsymbol{\mu})\mathbf{w}[k] + \mathbf{H}(\boldsymbol{\mu})(\mathbf{w}[k] \otimes \mathbf{w}[k])$$

We seek a to construct a parametric reduced model via OpInf with reduced dimension  $r \ll n$ .

To this end, we consider a quadratic-bilinear model form for the parametric model:

$$\mathbf{w}[k+1] = \underbrace{\mathbf{A}_0 \mathbf{w}[k]}_{\text{linear}} + \underbrace{\sum_{i=1}^d \mu_i \mathbf{A}_i \mathbf{w}[k]}_{\text{bilinear}} + \underbrace{\mathbf{H}_0 (\mathbf{w}[k] \otimes \mathbf{w}[k])}_{\text{quadratic}} + \underbrace{\sum_{i=1}^d \mu_i \mathbf{H}_i (\mathbf{w}[k] \otimes \mathbf{w}[k])}_{\text{quadratic-linear}},$$

which is obtained by assuming an affine dependency on the parameters, Taylor series approximation of the parametric operators etc.

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which is obtained by assuming an **affine** dependency on the parameters, **Taylor series approximation** of the parametric operators etc.

# Steps to perform parametric discrete operator inference

## II. Parametric Oplnf for the quadratic-bilinear ROM:

1. **training data**: compute  $n_t$  full-order model **snapshots** for each training parameter instance  $\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_m \in \mathbb{R}^d$
2. **training data manipulation** (lifting, centering, scaling) to obtain the snapshot matrix of transformed data  $\mathbf{Q} \in \mathbb{R}^{n \times mn_t}$
3. **subspace identification**: rank- $r$  **global** POD basis  $\mathbf{V}_r \in \mathbb{R}^{n \times r}$  using the thin SVD of  $\mathbf{Q}$
4. **projection**: project each snapshot to obtain  $\hat{\mathbf{Q}} = \mathbf{V}_r^\top \mathbf{Q} \in \mathbb{R}^{r \times mn_t}$
5. **learn reduced operators**  $\hat{\mathbf{A}}_i, \hat{\mathbf{H}}_i$  and  $\hat{\mathbf{c}}_i$  for  $i = 0, 1, \dots, d$  via **operator inference with regularization**

# Steps to perform parametric discrete operator inference

**III. Finish:** parametric reduced-order model.

The learned reduced operators define the **parametric quadratic-bilinear reduced model**

$$\hat{\mathbf{q}}[k+1] = \hat{\mathbf{A}}_0 \hat{\mathbf{q}}[k] + \sum_{i=1}^d \mu_i \hat{\mathbf{A}}_i \hat{\mathbf{s}}[k] + \hat{\mathbf{H}}_0 (\hat{\mathbf{q}}[k] \otimes \hat{\mathbf{q}}[k]) + \sum_{i=1}^d \mu_i \hat{\mathbf{H}}_i (\hat{\mathbf{q}}[k] \otimes \hat{\mathbf{q}}[k]) + \hat{\mathbf{c}}_0 + \sum_{i=1}^d \mu_i \hat{\mathbf{c}}_i$$

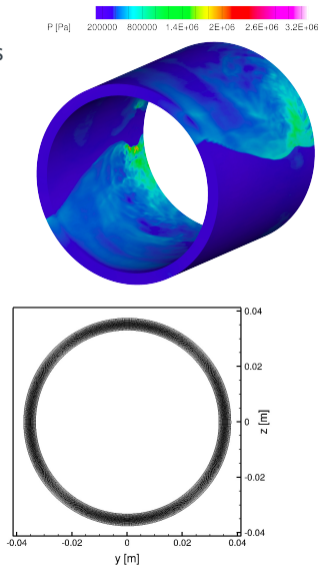
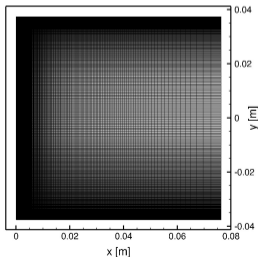
Given  $\boldsymbol{\mu} \in \mathbb{R}^d$ , we use the parametric ROM to issue **parametric predictions** beyond the training set



Numerical demonstration

# Parametric modeling of an RDRE combustion chamber

- LES simulations of the reactive, viscous 3D Navier-Stokes equations
- skeletal chemistry mechanism based on the Foundational Fuel Chemistry Model (FFCM<sub>y</sub>-30)
- non-premixed fuel injection (gaseous methane and oxygen)
- injector design with 72 discrete injector pairs
- parametric variations in the flow conditions



# Parametric modeling of an RDRE combustion chamber

## Parametric description

- parametric variations in flow conditions (mass flow-rate,  $\dot{m}$  [kg · s<sup>-1</sup>], and equivalence ratio,  $\Phi$ )
- these variations are characterized in our model using a scalar parameter  $\mu \in \mathbb{R}$

$$\mu = \frac{\dot{m}}{\dot{m}_0} + \frac{\Phi}{\Phi_0},$$

where  $\dot{m}_0$  and  $\Phi_0$  are the respective maximum mass flow-rate and equivalence ratio

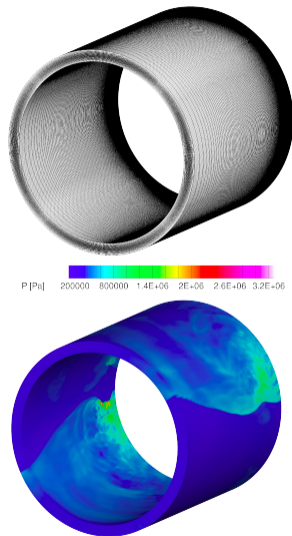
- we have simulation data for three parameter instances

$\mu$	Flow Condition	$\dot{m}$ [kg · s <sup>-1</sup> ]	$\Phi$	Time Interval	Periods	Dominant Waves	Secondary Waves
$\mu_1 = 1.41$	nominal	0.267	1.16	[3.7525, 4.0000] ms	1.97	5	0
$\mu_2 = 1.73$	high $\Phi$	0.266	1.71	[1.8425, 2.0900] ms	1.64	6	8
$\mu_3 = 1.55$	high $\dot{m}$	0.333	1.09	[3.7525, 4.0000] ms	1.97	5	0

# Parametric modeling of an RDRE combustion chamber

## 1. Generate training data + data manipulation

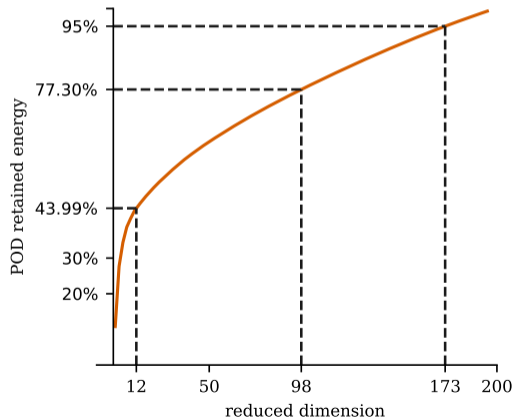
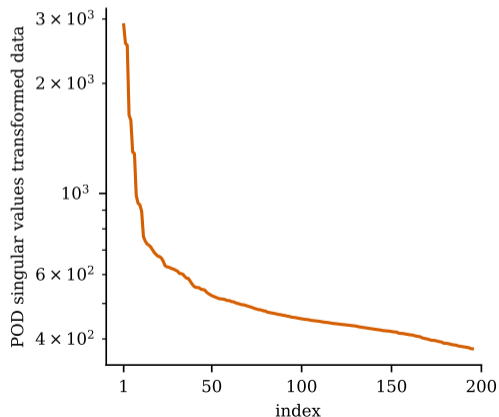
- 2 ms of full-state solutions for all three parameter instances generated  $\sim 6M$  CPU hours on  $> 16K$  cores
- the original simulation data has been interpolated on a structured mesh comprising  $n_x = 4,204,200$  spatial DoF
- time step size  $\Delta t \sim 10^{-9}$  s
- available data: 100 down-sampled snapshots for each parameter instance (about 0.25 ms of physical time)
- 8 transformed state variables: specific volume, pressure, 3D velocity, temperature, 2 species mass fractions (fuel and oxidizer)
- train ROM using data corresponding to  $\mu_1$  and  $\mu_2$ , make ROM predictions for  $\mu_3$
- training data: snapshot matrix  $\mathbf{Q} \in \mathbb{R}^{34M \times 200}$



# Parametric modeling of an RDRE combustion chamber

## 2. Compute (global) POD basis

- snapshot matrix of transformed variables
- scale and center snapshot data
- compute POD basis  $\mathbf{V}_T \in \mathbb{R}^{34M \times r}$
- low-data regime limits size of the non-intrusive ROM



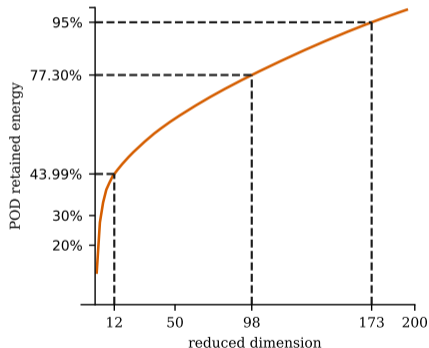
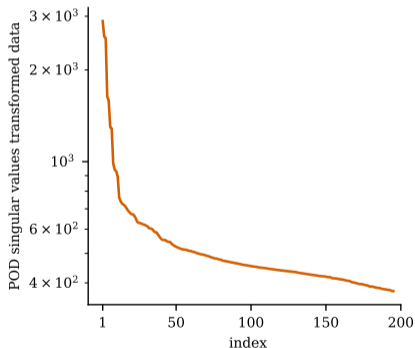
# Parametric modeling of an RDRE combustion chamber

## 3. Inference reduced operators

- compute **reduced** snapshot matrix

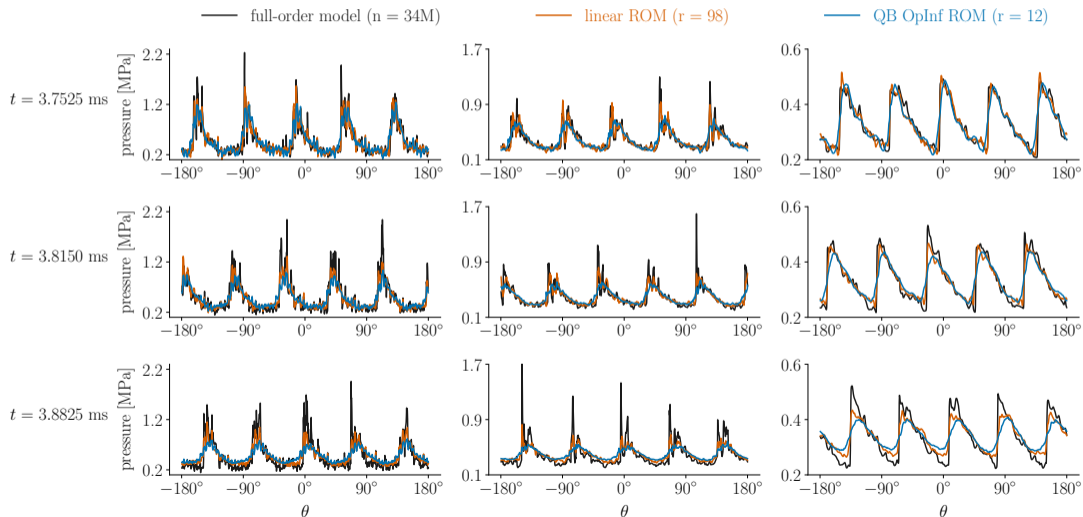
$$\hat{\mathbf{Q}} = \mathbf{V}_r^T \mathbf{Q} \in \mathbb{R}^{r \times 200}$$

- learn a **fully discrete parametric ROM**
- solve linear least squares to infer reduced operators

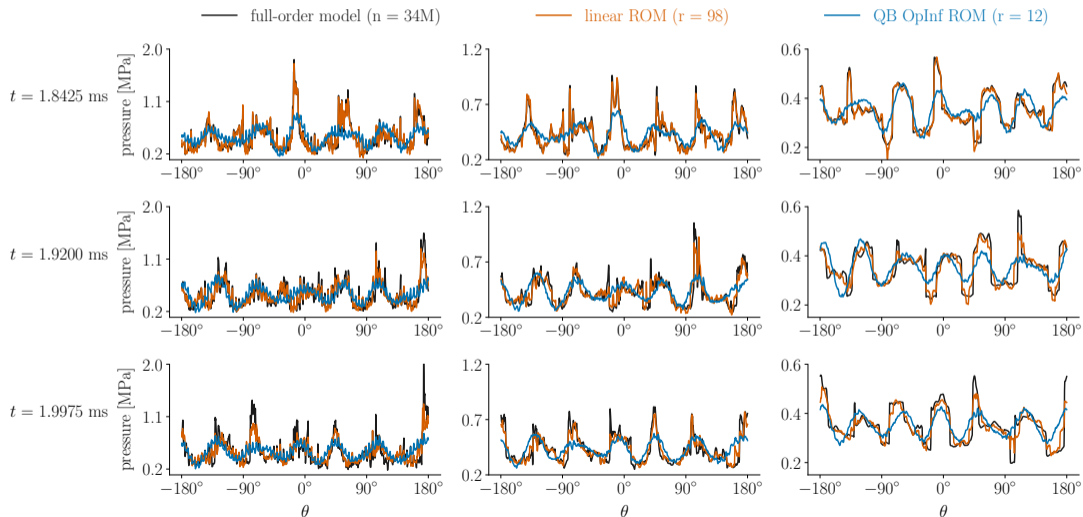


**Due to the scarcity of the available data, we also consider linear parametric reduced models**

# One-dimensional circumferential pressure profiles: 1st training parameter (nominal flow condition)

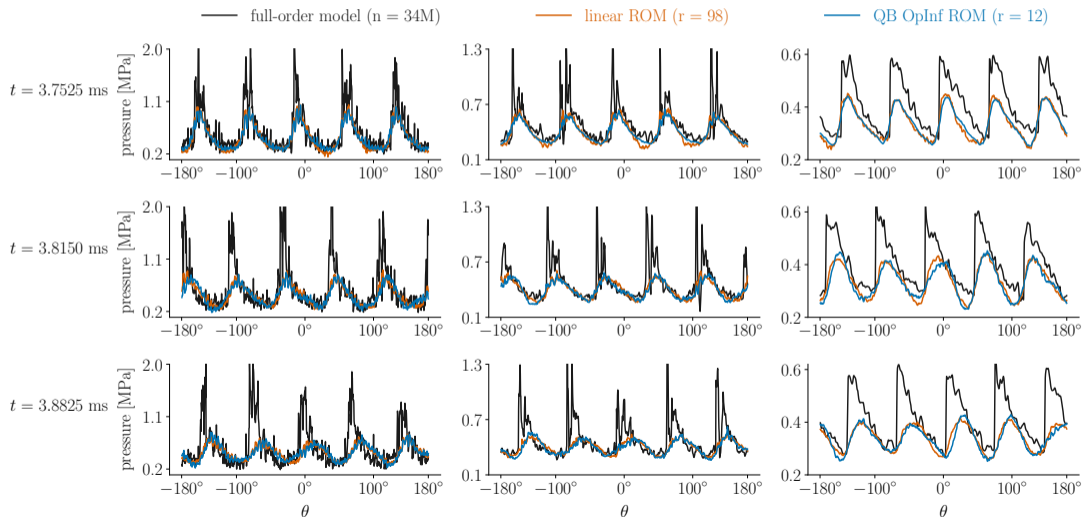


# One-dimensional circumferential pressure profiles: 2nd training parameter (high equivalence ratio)





# One-dimensional circumferential pressure profile prediction outside of the training set (3rd parameter, high mass flow-rate case)



# Outlook

## Domain-decomposed data-driven reduced models

- basis localization can lead to faster singular value decay  $\rightarrow$  smaller  $r_i$ , although now have ROM of dimension  $r_i$  for each subdomain  $i = 1, 2, \dots, k$
- the decomposition also mitigates computational complexity of offline snapshot processing

## Preprocess the training data set via filtering

- filter the snapshots in the training data set – variable by variable – prior to training the ROM
- filtering can reduce overfitting, improve the prediction accuracy of the ROM, and improve the condition of the Operator Inference learning problem

## Summary - Non-intrusive ROMs

- non-intrusive formulation enables rapid deployment, ROM derivation without access to source code, and variable transformations to promote structure
- it moreover enables ROM development across multiple sites without the need to transfer large-scale data sets (e.g., training data generation, data manipulation, and POD basis computation at AFRL, and OpInf ROM construction and postprocessing at UT Austin)
- Operator Inference ROMs are completely decoupled from CFD code and very fast to simulate
- our ROMs evaluate within milliseconds on a laptop computer
- Operator Inference can be used to construct both ROMs for predictions beyond the training time horizon and parametric ROMs
- slow singular value decay is a challenge for linear-subspace approximations
- generation of training data is a major expense; for realistic combustion problems we generally lack sufficient training data to fully resolve the dynamics
- domain decomposition and filtering are promising approaches to address these challenges

## References

- Farcas, I.G., Gundevia, R., Munipalli, R., Willcox, K.E., 2023. *Parametric non-intrusive reduced-order models via operator inference for large-scale rotating detonation engine simulations*. AIAA SciTech Forum 2023, National Harbor MD
- Farcas, I.G., Munipalli, R., Willcox, K.E., 2022. *On filtering in non-intrusive data-driven reduced-order modeling*. AIAA Aviation Forum 2022, Chicago IL
- Qian, E., Farcas, I.G., Willcox, K., 2022. *Reduced Operator Inference for Nonlinear Partial Differential Equations*. SIAM Journal on Scientific Computing 44, A1934–A1959.
- Peherstorfer, B., Willcox, K., 2016. *Data-driven operator inference for nonintrusive projection-based model reduction*. Computer Methods in Applied Mechanics and Engineering 306, 196–215.
- Qian, E., Kramer, B., Peherstorfer, B., Willcox, K., 2020. *Lift & Learn: Physics-informed machine learning for large-scale nonlinear dynamical systems*. Physica D: Nonlinear Phenomena 406, 132401.
- Swischuk, R., Kramer, B., Huang, C., Willcox, K., 2020. *Learning Physics-Based Reduced-Order Models for a Single-Injector Combustion Process*. AIAA Journal 58, 2658–2672.
- McQuarrie, S.A., Huang, C., Willcox, K.E., 2021. *Data-driven reduced-order models via regularised Operator Inference for a single-injector combustion process*. Journal of the Royal Society of New Zealand 51, 194–211.